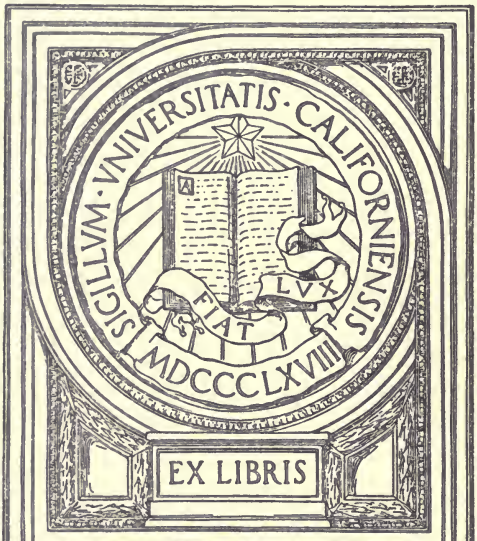
GRAMMAR SCHOOL

ARITHMETIC

CALIFORNIA STATE SERIES

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CALIFORNIA STATE SERIES

Hornbrook Grades R

GRAMMAR SCHOOL

ARITHMETIC

UNIV. OF CALIFORNIA

COMPILED BY
THE STATE TEXT-BOOK COMMITTEE

AND APPROVED BY
THE STATE BOARD OF EDUCATION

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DISTINCTIVE FEATURES

This Arithmetic is designed for use in the last four years of the grammar schools. The method of presentation is the result of long and close observation in the schoolroom, and conforms to the order and manner in which mathematical concepts are most naturally developed in children.

Practical work has been so combined with work of a purely disciplinary character that each reënforces and enhances the value of the other. In business arithmetic, where the practical demands the greater emphasis, the most simple and direct methods of computation are presented. Applications of percentage which are little used in business, but which have a value as a stimulus to thought, are introduced at the point where they will afford the best discipline. Some subjects that have neither a practical nor a high disciplinary value, though found in many text-books, are designedly omitted from this book. The time saved by the omission of such matter is devoted to more fruitful drill on practical exercises.

A carefully planned and continuous system of reviews runs through the book. These reviews take the form, first, of an excursion at the end of each chapter over all the ground thus far traversed, and, second, of a constant correlation of acquired knowledge with concepts about to be developed.

No hard and fast line is drawn between mental and written work. Economy of time and effort is the sole basis of distinction, and this is a self-regulating principle.

Rules and definitions are given as guides in the preliminary stages of acquirement. They are not to be formally memo-

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rized; and, when clear ideas of their contents have been gained, they are to be superseded by rules and definitions of the pupil's own framing.

Constructive work with simple geometrical forms is introduced at intervals whenever the numerical relations of those forms offer valuable material illustrative of arithmetical principles. Exercises to test and develop the pupil's power of visualizing are inserted in every chapter. The pupil's activity is further brought into play by a series of exercises in which he is called upon to supply the conditions for the problems as well as their solution.

Problems involving unknown quantities, which are solved arithmetically only by most complicated processes, are deferred until familiarity with some of the principles governing the use of literal quantities may suggest simpler methods of procedure.

The aim throughout has been to secure a ready skill in dealing with numbers and to develop thought power adequate to the attack of any arithmetical problem that may arise in practical life.

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GRAMMAR SCHOOL ARITHMETIC

CHAPTER I

INTEGERS AND DECIMALS

1. Write an integer of three places.

2. Read: 235
4,235
24,235
124,235

Read "two hundred, thirty five." Do not use "and" in reading an integer.

3. How many figures are used to express the last number in Ex. 2?

4. For what are figures used? Explain.

5. Express a number of two places by the figures 5 and 3. Express another number by the same figures. Which is the greater, and how much?

6. What is the largest integer that can be expressed by using once all the figures 3, 7, and 5? The smallest integer? Find their difference.

7. Find the difference between the largest integer and the smallest integer that can be expressed by using once all the figures 5, 1, and 8.

8. There are six different integers that can be expressed by using once all the figures 1, 2, and 3. Write these numbers in the order of their size and find their sum.

9. Can numbers be expressed without figures?

10. Write in words the number represented by 105.

11. Express in good English the number represented by 228,427. By 699,108.

12. Name all the figures that are used to express number.

13. When the figure 0 stands alone, does it express number? In the sentence "John has 0 marbles," what does 0 express?

14. 0 is called *naught, zero,* or *cipher.* The other nine figures used to express numbers in Arabic notation are called digits. What is the tens' digit of the number 75? Of 235? What is the thousands' digit of the number 8421? Of 29834? Of 127446?

15. In the number 815, which is greater, the hundreds' digit or the tens' digit? How much? What is the sum of all the digits of that number?

16. Write a number the sum of whose digits is 10.

17. Write a number of four places the sum of whose digits is 12.

18. Read:

3
30
300
3,000
30,000
300,000

What is the ratio of 30 to 3? Of 300 to 30? Of each number in the list to the one just before it?

Suggestion to Teacher. If pupils are not familiar with the term "ratio," substitute the question, "30 is how many times 3?"

19. Write a digit and place 0 at the right of it. The result equals how many times the original digit?

20. Placing ciphers at the right of a digit is called annexing ciphers to the digit. Annex two ciphers to 5 and state how many times 5 the result equals.

21. The easiest way to multiply an integer by 10 is to annex one cipher to it. What is the easiest way to multiply an integer by 100? By 1000?

22. Give at sight the following values:

a 35 multiplied by 10 *c* 1000 times 16 *e* 10 times 30000

b 100 times 71 *d* 10 times 3000 *f* 1000 times 50

23. Give at sight the quotient of:

a $40 \div 10$ *c* $4370 \div 10$ *e* $15000 \div 1000$

b $420 \div 10$ *d* $2500 \div 100$ *f* $28000 \div 1000$

24. Give at sight the following values:

a $\frac{1}{10}$ of 520 *c* $\frac{1}{100}$ of 2300 *e* $\frac{1}{1000}$ of 4000

b $\frac{1}{100}$ of 600 *d* $\frac{1}{100}$ of 3100 *f* $\frac{1}{1000}$ of 18000

25. Class Exercise. —— may name a number ending in three ciphers, and the class may give $\frac{1}{10}$ of it. $\frac{1}{100}$ of it. $\frac{1}{1000}$ of it.

26. Multiply 1000 by 1000. A thousand thousands equal a **Million.** How many figures are required to express a million?

27. Read 8,636,448.

Read, 8 million (not millions), 636 thousand (not thousands), 448.

28. Read: 9,240,827. 31,676,201. 125,475,042.

29. Why is it useful to separate a number into periods of three figures each before reading it?

30. Separate into periods and read: 8347621. 98470245. 616823146. 47825001.

31. Class Exercise. —— may write 9 figures on the board in a horizontal line, and another pupil may tell what number they represent.

32. Write in figures, placing a comma after millions and also after thousands: 5 million, 323 thousand, 471. 81 million, 175 thousand, 241. 815 million, 278 thousand, 924.

Millions			Thousands			Units		
Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
4	7	6	8	2	1	0	2	3

33. Class Exercise. Copy on the board the above diagram, placing different figures in the spaces and reading the numbers thus expressed.

34. Write and read a number of seven places, having 3 in the millions' place, 8 in the thousands' place, 4 in the tens' place, and 0 in all the other places.

35. Write and read a number of 8 places, having 2 in the ten-millions' place, 7 in the millions' place, 4 in the units' place, and 0 in the other places.

36. When numbers are expressed in figures they are said to be written in **Arabic Notation.** Write in Arabic notation:

a 323 million, 224 thousand, 24
b 27 million, 960 thousand, 7
c 169 million, 201 thousand, 25
d 41 million, 41 thousand, 41
e 75 million, 75 thousand, 76
f 121 million, 3 thousand, 3

37. Write a number of 7 places whose units' figure is 5. Find $\frac{1}{5}$ of it. $\frac{2}{5}$ of it. $\frac{4}{5}$ of it.

38. Write the largest number that can be written with 4 places. With 6 places. With 9 places. Give the sum of the digits of each of them.

39. How many can you count in a minute?

SUGGESTION TO TEACHER. Find by trial the rate of speed at which different pupils count, timing them by the watch.

40. At your rate of counting, how many could you count in an hour? In a day of 10 hours?

41. Mary Wallace, a little girl living in Philadelphia, counted 75 in a minute. At that rate, how many whole minutes would it take her to count a million? How many whole hours? How many days if she counted 10 hours a day?

42. CLASS EXERCISE. —— may report the number which he can count in a minute. The class may find how many minutes would be required for him to count a million at that rate. How many whole hours. How many days of 10 hours each.

43. Write in Arabic notation:

1st. 435 million, 347 thousand, 526.

2d. The number that is 2 million greater than the 1st.

3d. The number that is 3 thousand less than the 2d.

4th. The number that is 300 thousand more than the 3d.

5th. The number that is 3 more than the 4th.

6th. The number that is 20 thousand less than the 5th.

7th. The number that is 30 million more than the 6th.

8th. The number that is 200 million more than the 7th.

9th. The number that is 40 more than the 8th.

44. Find difference between 1st and 9th number in Ex. 43.

SUGGESTION FOR CLASS EXERCISE. A pupil may write on the board a number containing millions, and the other members of the class may direct modifications as in the previous examples. When the pupil at the board blunders, another pupil may take up his work.

45. Beginning at 2, count by twos to 10. How many numbers did you name?

46. What is the sum of 4 twos? 6 twos? Numbers which are exactly divisible by two are called **Even Numbers.**

47. What is the first even number after 20? How many twos does it equal?

48. Write all the even numbers that can be expressed by one digit.

49. What is the 8th even number? The 12th even number?

50. Divide 1,735,328 by the 7th even number.

51. Can you write an even number which does not end with 0, or 2, or 4, or 6, or 8?

52. Write an even number the sum of whose digits is 9. Find $\frac{1}{2}$ of it. Find $\frac{1}{9}$ of it. Find $\frac{1}{18}$ of it.

53. Write an even number consisting of millions, thousands, and units. Divide that number by 32. By 102. By 104.

54. In 1895 the expenses of the United States government were $356,195,298. The revenues of the government for that year were $313,390,075. How much did the amount spent exceed the amount received?

55. Mention some of the things for which the United States government spends money, and make an example in addition.

56. The cost of the United States army in the year 1895 was $51,804,759. The cost of the navy was $28,797,796. How much did they both cost?

57. Africa contains 11,514,000 square miles, North America 6,446,000 square miles, South America 6,837,000 square miles, Asia 14,710,000 square miles, Australasia 3,228,000 square miles, Europe 3,555,000 square miles, the Polar Regions 4,888,800 square miles. How many square miles of land does the whole world contain?

58. The total exports of the United States in 1895 amounted to $807,538,165; the imports amounted to $731,969,965. How many more dollars' worth of goods were sold to foreign countries than were bought from them?

59. The earth is about 92,800,000 miles from the sun; the planet Mars about 140,000,000 miles from the sun. How much nearer to the sun is the earth than Mars?

60. Multiply a million by a thousand by annexing ciphers.

61. A thousand millions equal a **Billion**. How many figures are required to express a billion?

62. Point off and read:

a 414141414141	*c* 232648648648	*e* 58914367281
b 673673673673	*d* 827345827345	*f* 42781632512

63. Class Exercise. —— may write twelve figures on the board in a horizontal line, and others may tell what number they represent.

64. Write in Arabic notation:

1st. 427 billion, 338 million, 484 thousand, 521.

2d. The number that is 4 billion less than the 1st.

3d. The number that is 2 billion, 7 million, 20 thousand less than the 2d.

4th. The number that is 1 billion, 1 million, and 1 thousand more than the 3d.

5th. The number that is 13,013,013,013 more than the 4th.

65. Class Exercise. —— may write on the board a number containing billions, and the class may direct changes of it as in Ex. 64.

66. Write and read an even number consisting of billions, millions, thousands, and units.

67. Write:

a 98 billion, 348 million, 693 thousand, 207
b 15 billion, 279 million, 427 thousand, 48
c 216 billion, 849 million, 348 thousand, 7
d 821 billion, 326 million, 475 thousand, 75
e 2 billion, 2 million, 2 thousand, 2
f 21 billion, 21 million, 21 thousand, 21
g 78 billion, 78 million, 78 thousand, 78

68. Class Exercise. —— may write on the board numbers consisting of billions, millions, thousands, and units which are given to him by the class.

69. To count a billion takes how many times as long as to count a million?

70. From the time of the establishment of our government in 1789 till 1896 there had been spent for pensions $1,950,403,063 and for interest on public debts $2,791,537,714. How much more had been spent for interest than for pensions? To whom are pensions given? Why?

71. In 1881, the public debt of the United States was $2,077,389,253 and in 1882 it was $1,926,688,678. How much was the debt decreased during the year?

72. In 1894, Europe produced 897,231,061 lb. of wool, North America 342,210,712 lb., South America 397,970,000 lb., Central America 2,000,000 lb., Australia 663,600,000 lb., Asia 258,000,000 lb., Africa 131,925,000 lb. How many pounds of wool were produced that year?

73. Africa has about 127,000,000 inhabitants, North America 89,250,000, South America 36,420,000, Asia 850,000,000, Australasia 4,730,000, Europe 380,200,000, Polar Regions 300,000. What is the entire population of the world?

74. Write the largest number that can be written with 12 figures.

75. Write a number of 15 places and find from the following note how to read it.

The period of figures next higher than billions is called trillions, the next quadrillions, then come quintillions, sextillions, septillions, octillions, nonillions, decillions.

76. Write a number larger than 999 trillions and read it. Why do we seldom use such large numbers?

77. Write the largest number that can be written with 7 places. Find $\frac{1}{9}$ of it. $\frac{2}{9}$ of it. $\frac{1}{21}$ of it.

78. What people in ancient times used letters to express numbers?

79. Copy the Roman numerals and write under each the corresponding Arabic numeral.

I V X L C D M

80. In Roman notation when a letter is repeated its value is repeated. Read: XX. CCC. MMMM. Write in Roman notation: 3. 30. 300. 3000. 50. 500.

81. V, L, and D are not repeated. Can you see why?

82. When a letter of less value is placed after a letter of greater value the sum of their values is represented. Read VIII. XVI. LXVI. CLV. MDCL. Write in Roman notation: 28. 36. 53. 75. 125. 381. 722. 1605. 1620.

83. When a letter of less value is placed before a letter of greater value, the difference of their values is represented. Read: IV. IX. XL. XC. MXCIX. MCM. Write in Roman notation: 14. 49. 99. 144. 579. 714. 1239. 1569. 1889. 1902. 1905. 1907. 1909.

84. A line over a letter denotes that its value is multiplied by 1000. Read: $\overline{\text{M}}$VI. $\overline{\text{V}}$DC. Write in Roman notation: 10051. 5525. 10630. 4324. 8956. 5427. 6385.

85. Write the following numbers in Arabic notation and find their sum: MDIII. MDCCCIV. MDCXX. MCMI. MMDLXV. MDLXII. MCMIV. MCMXIX.

86. Write the following numbers in Arabic notation and find their difference: MCCXCIX and MDCCCXLV.

87. The poet Longfellow was born in MDCCCVII. How many years old was he at the breaking out of the Civil War in MDCCCLXI?

88. How many years elapsed between July 4, MDCCLXXVI, the date of the Declaration of Independence, and July 4th of the present year?

89. Write in Arabic notation, MDC and find $\frac{1}{100}$ of it.

90. Write in Roman notation the following dates:

a The present year.

b 25 years hence.

c 100 years before you were born.

d The year in which our present president was elected.

e The year in which your state was admitted to the Union.

f The year of Dewey's victory in the Philippine Islands.

SUGGESTION FOR CLASS EXERCISE. Let the pupils suggest important dates to be written in Roman notation by the class.

FUNDAMENTAL OPERATIONS AND PROOFS

91. A statement that two quantities are equal is called an **Equation**, as 60 minutes = 1 hour, 14 days = 2 weeks, $8 \div 4 = 2$. Write an equation, using the numbers, 7, 5, and another number.

92. Numbers that are added are called **Addends**. In the equation $3 + 4 = 7$, which numbers are addends?

93. Fill out the following equations and name the addends. $5 + 7 = ?$ $6 + 2\frac{1}{2} = ?$ $\frac{1}{7} + \frac{3}{7} = ?$

94. Give two addends whose sum is 15. 27. $\frac{8}{11}$.

95. Give three addends whose sum is 14. 20. $\frac{6}{7}$.

96. Give three equal addends whose sum is 27. 30.

97. When John has caught 5 more fish he will have caught 7 fish. How many has he caught?

98. If Mr. Reed had $ 325 more, he could buy a farm costing $ 2168. How much money has he?

99. When the sum of two addends is 29, and one of them is 4, what is the other? Give the missing addend when the sum is 29, and the known addend is 21.

100. Class Exercise. Think of two addends and their sum. Then give the sum and one of the addends to the class. The class may find the other addend.

101. In adding 25 and 18 James carelessly wrote 45 as the answer. If either of his addends were subtracted from the number he wrote, would the other addend be found? Explain.

102. Add 13 and 26. If your work is correct, and if one addend is subtracted from your answer, what will be left?

103. Complete these equations. Illustrate with small numbers.

Addend + Addend = Sum − Addend =

104. Add 124 and 354 and prove your work.

To prove the correctness of the addition of two numbers subtract one addend from their sum. If the work is right, the remaining number will equal the other addend.

105. Find sums and prove:

a	*b*	*c*	*d*	*e*	*f*
628	949	639	457	1639	1854
354	848	728	622	2136	237

106. Write an example in subtraction and show which number is the minuend. The subtrahend. The difference.

107. What is a minuend? A subtrahend? A difference?

108. From 728 take 516 — Add subtrahend and difference. If the work is correct, the result will equal the minuend.

109. From 824 Subtract the difference from the minu-
take 512 end. If the work is correct, the result
will equal the subtrahend.

110. Complete these equations. Illustrate.

Minuend − Subtrahend = Min. − Dif. =

Dif. + Sub. =

111. Find the number for which x stands in the following:

	Minuend	Subtrahend	Difference
a	240	x	160
b	x	16	30
c	40	x	10
d	60	50	x

112. Class Exercise. —— may give a minuend and a difference. The class may find the subtrahend.

Suggestion to Teacher. Every pupil should be required to bring to the class his contribution to the class exercise carefully prepared.

113. Class Exercise. —— may give a subtrahend and a difference, and the class may find the minuend.

114. How can you prove an example in subtraction?

115. Subtract and prove:

a	b	c	d	e	f	g
849	623	814	338	599	451	2148
321	517	276	124	378	239	1939

116. Multiply 123 by 3. By 30. By 300. Find the sum of the products. Compare the sum of the products with the product of 123 multiplied by 333.

117. Multiply 275
by 137

1925
8250
27500

37675

In this example of what two numbers is 1925 the product? 8250? 27500? How is 37675 obtained?

SUGGESTION TO TEACHER. Show that in multiplying by a number of two or more places we are finding the sum of the several products of the multiplicand and the number expressed by each figure of the multiplier in its present position; and that in practice the naughts are omitted for the sake of convenience.

118. Multiply a number of 3 places by a number of 4 places, writing out the full partial products. Why are the full products not usually written out?

119. What is a multiplicand? A multiplier? A product? Illustrate.

120. Use 25 as a multiplicand and 17 as a multiplier.

121. If the product of 25 and 17 is divided by 17, what result will be obtained? If the product of 25 and 17 is divided by 25, what result will be obtained?

122. When the product of two numbers is divided by one of the numbers, what result is obtained? Illustrate.

123. The product of two numbers is 35, the multiplicand is 7. Find the multiplier.

124. Take Ex. 123, substituting in turn for 35 the numbers 84, 42, 77, 91, 112.

125. Find the values of x.

	Product	Multiplicand	Multiplier
a	75	x	15
b	x	14	3
c	70	x	10
d	60	6	x

126. Complete the equations. Illustrate.

Multiplicand $\times$ Multiplier = Product $\div$ Multiplier =

Product $\div$ Multiplicand =

127. How can you prove an example in multiplication?

128. Multiply and prove:

a 18×20 b 13×14 c 15×16 d 14×25

129. Multiply and prove:

a	*b*	*c*	*d*	*e*	*f*
836	457	791	625	927	654
125	243	348	244	238	289

130. What is a dividend? A divisor? A quotient? Illustrate.

131. Find quotients of $800 \div 2$, $60 \div 2$, and $8 \div 2$. Add the quotients and compare the sum with the quotient of $868 \div 2$.

SUGGESTION TO TEACHER. Show that in dividing 868 by 2, we divide 800 by 2, then 60 by 2, and then 8 by 2.

132. Divide by 3 first 900, then 60, and then 6, and find the sum of the quotients. Show a shorter way of dividing the sum of those numbers by 3.

133. Divide 400 and 80 and 4 each by 4. Show the usual way of dividing the sum of these numbers by 4.

134. $764 \div 2 = ?$

In dividing 764 by 2 we divide 600 by 2, then 160 by 2, and then 4 by 2.

135. $976 \div 4 = ?$ $(800 \div 4) + (160 \div 4) + (16 \div 4) = ?$

136. Divide 765 by 5, and show how many hundreds, how many tens, and how many units are used in the separate divisions.

137. Divide 5468 by 4, and show how many thousands, how many hundreds, how many tens, and how many units are used in the divisions.

138. Divide 3765 by 5, and show what parts of the number are used in each division.

139. What number divided by 8 will give 2 for a quotient?

140. The quotient of a certain number divided by 8 is 7. What is the dividend?

141. Complete the following equations. Illustrate.

Dividend ÷ Divisor = Dividend ÷ Quotient =

Divisor × Quotient =

142. Find the values of x.

	Dividend	Divisor	Quotient		Dividend	Divisor	Quotient
a	72	4	x	c	x	21	5
b	99	x	11	d	32	x	4

143. Of what number is 11 both divisor and quotient? What number has for divisor and for quotient 7? 12? 13? 15? 17? 21?

144. Use 21 as a divisor and 378 as a dividend. Multiply divisor by quotient, and compare the result with the dividend.

SUGGESTION TO TEACHER. Using small numbers, show that as multiplication and division are reverse processes, they may be used to prove each other.

145. Make a rule for proving an example in division when there is no remainder.

146. Divide each of the following numbers by 23, and prove your work:

322 391 575 759 874 943 1288

147. Use 41 as a divisor and 618 as a dividend. Multiply divisor by quotient, add the remainder, and compare the result with the dividend.

148. Make a rule for proving an example in division when there is a remainder.

149. Divide each of the following numbers by 25, and prove your work:

625 879 579 824 386 758 1028 981

150. Divide 26103 by 12 as follows:

```
12)26103(2175
   24000
   ----
    2103
    1200
    ----
     903
     840
     ---
      63
      60
      --
       3
```

How many times is 12 contained in 24,000? How much remains to be divided by 12 after 24,000 is subtracted from 26,103?

SUGGESTION TO TEACHER. Show the process of dividing each remainder after the successive subtractions, and call attention to the fact that it is more convenient to omit ciphers and that it gives the same result.

151. Divide some numbers by others by long division, writing out all the work. Why is it better usually to omit some of the work?

152. A schoolboy brought this example to his teacher and told her that he had discovered that if the numbers here printed in heavy type were added in the order in which they stand, the result would equal the dividend. He proved his problems in long division in that way. Take an example in long division and prove it in the same way.

```
121)87493(723
   847
   ---
    279
    242
    ---
     373
     363
     ---
      10
```

153. Can you see why adding all the subtrahends and the remainder in long division will give a result equal to the dividend?

SUGGESTION TO TEACHER. Show that in this example 700 times 121 = 84,700, 20 times 121 = 2420, 3 times 121 = 363, and that the sum of these numbers, plus the remainder 10 must equal the dividend.

154. Divide a number by another number that it contains exactly 8 times. Double your dividend and see how the quotient is changed.

155. Work with small numbers and show the truth of the following principles:

PRIN. 1. *Increasing the dividend increases the quotient.*

PRIN. 2. *Decreasing the dividend decreases the quotient.*

PRIN. 3. *Increasing the divisor decreases the quotient.*

PRIN. 4. *Decreasing the divisor increases the quotient.*

156. Some of a milkman's customers buy a pint of milk at a time, and some buy a quart. How many of the customers who buy a pint will together dispose of a gallon? How many who buy a quart?

157. How many pints of water can be drawn from a 20-gallon tank? How many quarts? How many gallons?

158. A 40-gallon tank contains how many times as many pints as a 20-gallon tank?

159. Illustrate the following principles with small numbers:

PRIN. 5. *Multiplying both dividend and divisor by the same number does not change the quotient.*

PRIN. 6. *Dividing both dividend and divisor by the same number does not change the quotient.*

160. Find the product of 8 and 10 and divide that product by 4.

If the following expression of the problem were used, $\frac{8 \times 10}{4}$, and if before multiplying, both 8 and 4 were divided by 4 as follows, $\frac{\overset{2}{\cancel{8}} \times 10}{\underset{1}{\cancel{4}}} = 20$, would the result be the same?

SUGGESTION TO TEACHER. Let pupils prove by trial with many small numbers that canceling common factors in dividend and divisor does not change the quotient.

161. Find the values of the following, canceling when you can:

a $\frac{4 \times 8 \times 6}{2 \times 4 \times 2} = ?$

b $\frac{6 \times 9 \times 18}{3 \times 3 \times 6} = ?$

c $\frac{21 \times 4 \times 6}{7 \times 2 \times 3} = ?$

d $\frac{40 \times 8 \times 10}{20 \times 4 \times 5} = ?$

e $\frac{3 \times 4 \times 5}{7 \times 8 \times 10} = ?$

f $\frac{16 \times 4}{8 \times 8} = ?$

g $\frac{21 \times 28 \times 4}{7 \times 7 \times 6} = ?$

h $\frac{2 \times 8 \times 12}{5 \times 4 \times 6} = ?$

i $\frac{30 \times 7 \times 5}{60 \times 21 \times 15} = ?$

j $\frac{80 \times 4 \times 25}{16 \times 20 \times 5} = ?$

k $\frac{24 \times 7 \times 10}{12 \times 14 \times 20} = ?$

l $\frac{35 \times 6 \times 9}{18 \times 7 \times 5 \times 3} = ?$

Can you show how the process of cancellation depends upon Prin. 6?

162. Cancel and find values:

a $\frac{5 \times 8 \times 10 \times 27}{54 \times 4 \times 20}$

b $\frac{7 \times 9 \times 56 \times 65}{26 \times 49 \times 32 \times 5}$

c $\frac{21 \times 84 \times 6 \times 8}{12 \times 49 \times 18}$

d $\frac{25 \times 6 \times 7 \times 30}{50 \times 9 \times 5}$

e $\frac{16 \times 9 \times 28}{8 \times 36 \times 2}$

f $\frac{42 \times 15 \times 10 \times 6}{25 \times 14 \times 5}$

163. Class Exercise. With $24 \times 30 \times 35$ as the dividend, —— may make a cancellation exercise, and the class may solve it.

164. Class Exercise. With $28 \times 12 \times 36$ as a divisor, —— may give a cancellation exercise for the class to solve.

165. Copy upon the board and read 1,111,111.

166. Point out the figure that expresses a hundred thousand. How many hundred thousand make a million?

167. What part of a million is a hundred thousand?

168. How much is $\frac{1}{10}$ of one hundred thousand? Show the figure that expresses it.

169. How much is $\frac{1}{10}$ of ten thousand? Show the figure that stands for it.

170. Show the figure that stands for $\frac{1}{10}$ of a thousand. $\frac{1}{10}$ of a hundred. $\frac{1}{10}$ of ten.

171. Copy 111.1. The point after the units is called a **Decimal Point.**

172. The number that is $\frac{1}{10}$ of 1 is written .1. Read 111.1.

This should be read 111 *and* 1 tenth.

173. 1 at the right of the tenths' place means $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$. How many tenths and hundredths in .11?

174. 1 at the right of the hundredths' place means $\frac{1}{10}$ of $\frac{1}{100}$ or $\frac{1}{1000}$. How many tenths, hundredths, and thousandths in .111?

175. What does 1 at the right of the thousandths' place mean? What does 1 in the next place to the right mean? In the next?

176. Numbers written at the right of the decimal point are called **Decimals** or **Decimal Fractions.** They decrease in value at a tenfold rate from left to right just as integers decrease. A decimal of one place expresses tenths. What does a decimal of two places express? Of three places?

177. Read:

.1 .2 .9 .07 .007 .09 .009

178. Read .19.

This is read 19 hundredths ($\frac{19}{100}$). A decimal is read like an integer, and then the name of the last decimal place is added to show what kind of fractional parts it represents.

179. Read:

12.1 4.3 8.01 7.02 9.021 .003 .009 .03 .006

180. Write and read a decimal of two places. Of three places. Of four places.

181. Write in words:

.123	4.5	.4	41.41	.103	19.2
.15	3.75	8.6	41.041	21.109	11.025

182. Read:

576.137	432.25	57.41	32.06	75.37	45.81

183. Write in one number an integer of 4 places and a decimal of 3 places. Read it.

184. Write and read a number consisting of an integer of 6 places and a decimal of 2 places.

185. Write and read a number in which there is an integral part of 3 places and a decimal part of 2 places.

186. Write and read a number in which there is an integral part of 5 places and a decimal part of 3 places.

187. Which is greater and how much, the integer 1 or the fraction $\frac{9}{10}$?

Fractions whose denominators are 10, 100, 1000, or 1 with any number of ciphers annexed, may be written as decimals. This way of expressing such fractions is convenient because when thus expressed they may be added, subtracted, multiplied, and divided in the same way as integers.

188. Write as common fractions:

.12	.029	.125	.17	.27	.013	.049	.019

Notice that the denominator of a decimal is not written, but is indicated by the number of places it occupies. It is always 1 with as many ciphers annexed as there are places in the decimal. Thus in .32 the denominator is 100.

189. Write as decimals:

$\frac{13}{100}$ $\frac{45}{100}$ $\frac{4}{10}$ $\frac{9}{10}$ $\frac{17}{100}$ $\frac{127}{1000}$ $\frac{156}{1000}$ $\frac{23}{100}$

190. Write as decimals:

$\frac{3}{10}$ $\frac{19}{100}$ $\frac{7}{10}$ $\frac{279}{1000}$ $\frac{7}{100}$ $\frac{9}{100}$ $\frac{29}{100}$ $\frac{3}{100}$

191. Express $\frac{9}{10}$, using only one figure. Express $\frac{9}{100}$ by two figures. $\frac{9}{1000}$ by three figures.

192. Write as common fractions:

.6 .06 .027 .125 .0004 .00025 .000005

193. If you cut a string into ten equal parts, in how many places must you cut it? What is each part called? Express it as a decimal.

194. How many times must a string be cut to divide it into 100 equal parts? Into 1000 equal parts?

195. Show on a ruler .1 of 10 in. .1 of 5 in.

196. If 20 pupils were in a class and .1 of them were dismissed, how many would remain?

197. How much is .1 of 100? .01 of 100? Why can you not easily show on a ruler .001 of an inch?

ADDITION OF DECIMALS

198. Read and add:

a	*b*	
462.001	321.12	725.375
25.01	56.	409.003
63.475	81.07	361.1
181.312	73.22	448.0035
692.436	195.87	772.6

199. In adding decimals why is it best to arrange them so that the decimal points are in a vertical line?

200. Write and add:

a 57 and 123 thousandths, 181 and 28 hundredths, 49 and 3 tenths.

b 167 and 4 tenths, 2128 and 4 hundredths, 396 and 4 thousandths.

c 821 and 47 hundredths, 526 and 47 thousandths, 2936 and 1 tenth.

d 674 and 37 hundredths, 25824 and 128 thousandths, 1948 and 4 tenths.

e 666,666 and 6 tenths, 7,777,777 and 77 hundredths, 88,888,888 and 888 thousandths.

201. Add:

a 7.3 in. and 4.7 in.

b 8.4 sq. yd. and 5.6 sq. yd.

c 3.35 sq. ft. and 4.95 sq. ft.

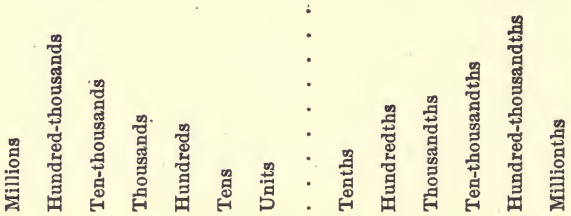
202. .1 is how many times as great as .01?

203. In the expression 333.333, which 3 expresses the greatest value? The least value?

204. 3 in the first integral place expresses how many times as much as 3 in the first decimal place?

205. 5 in the first decimal place expresses how many times as much as 5 in the third decimal place?

206. Which figures in a decimal stand for the greater value, those near the decimal point or those far away from it?



207. Class Exercise. Copy the diagram, placing a figure in each decimal place, and read the numbers thus expressed.

208. How much is $\frac{1}{10}$ of .01? $\frac{1}{10}$ of .001? $\frac{1}{10}$ of .0001? $\frac{1}{10}$ of .00001?

209. Give values of x:

a $x = 1$ in the 4th decimal place.

b $x = 1$ in the 6th decimal place.

c $x = 4$ in the 5th decimal place.

d $x = 7$ in the 4th decimal place.

e $x = 6$ in the 6th integral place.

f $x = 2$ in the 6th decimal place.

210. Add, and write the sum in words:

a	b	c
1.235	24375	325685
123.5	2.4375	.325685
.1235	.24375	3.25685

211. Write and read a decimal of 4 places. Of 5 places. Of 6 places.

212. Which integral place is occupied by millions? Which decimal place by millionths?

213. Give the places of the following: Thousands, thousandths, ten-thousands, ten-thousandths, hundreds, hundredths, hundred-thousands, hundred-thousandths.

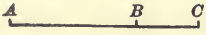
214. The expression .3 shows that some unit is divided into 10 equal parts, and that 3 of those parts are taken. What does .003 show? .0003? .00003? .000003?

215. When we have .3 of an inch, what unit has been divided into 10 equal parts? Explain the expression ".7 of a foot." ".17 of a dollar."



216. If A and C were each 3 in. from B, how far apart would they be?

217. If A and C were each 122.57 mi. from B, how far apart would they be?



218. If A is 7.7 in. from B, and C is 3.8 in. from B, how far apart are A and C?

219. Class Exercise. Let pupils give different lengths (with decimals) to AB and BC, and find distance from A to C.

220. By rail the distance from Nashville, Tenn., to Evansville, Ind., is 155.07 mi., and from Evansville to Chicago, Ill., 287.15 mi. Mary Allen lives in Nashville. How far will she travel in going from her home through Evansville to Chicago and returning by the same route?

221. How many cents equal 1 hundredth of a dollar? .17?

222. Add:

a	b	c	d
\$ 48.33	\$ 75.25	\$ 81.39	\$ 813.45
76.48	38.60	47.50	425.15
13.15	49.76	86.72	327.40

Suggestion to Teacher. Call the attention of pupils to the fact that they have been using decimals of a dollar in their work with dollars, dimes, and cents.

SUBTRACTION OF DECIMALS

223. Subtract:

a	b	c	d	e
446.35	674.37	821.42	123,478.008	964,821.88
29.78	338.49	365.17	1,939.981	283,464.79

224. Which is greater and how much, .6 of a dollar or 60¢? \$.4 or 45¢? \$.5 or 48¢? \$.7 or 70¢?

225. Which is greater, .1 or .10? .4 or .40? .50 or .5? .7 or .700?

226. Write a decimal. Annex a cipher to it, and tell how the value of the decimal is affected.

Without changing values,

227. Change to hundredths: .7. 2.1. 45.3.

228. Change to thousandths: .25. .4. 8.1. 2.56.

229. Change to ten-thousandths: .125. 2.4. .17.

230. Change to hundred-thousandths: .4758. 3.56. .9.

231. Change to millionths: .85674. 18.35. 42.7. .489. . 49.

232. Write the expression .3. Place a cipher between the decimal point and the figure 3. How is the value of the expression .3 changed by placing the cipher?

233. .5 of a dollar equals how many cents? .05 of a dollar equals how many cents? Find their difference.

		a	*b*	*c*	*d*
234.	From	175.5	691.15	436.4	827.3
	take	20.35	420.615	125.25	121.125

235. Mr. Adams had 75.1 acres of land and sold 13.4 acres. How many acres had he left?

236. From 195.35 sq. yd. take 37.15 sq. yd.

237. Find the values of:

a	1 − .04	*c*	2 − 1.8	*e*	800 − .390
b	300 − .08	*d*	6001 − 40.683	*f*	8602 − 304.407

238. Write decimally 11 tenths. 4125 thousandths.

239. 11 − 11 tenths = ? 113 − 113 tenths = ?

240. 117 − 134 thousandths = ? 8 tenths − 436 thousandths = ?

241. 297 − 4138 thousandths = ? 480 thousand − 483 thousandths = ?

242. From one take three hundred seventy-one thousandths.

243. From two and three hundred forty-seven thousandths take eight hundredths.

244. From ten thousand take ten thousandths.

245. From ten millions take ten hundredths.

246. From five hundred take five hundredths.

247. From eight hundred thousand take eight thousandths.

248. From five tenths take five hundredths.

249. Use 12.75 as a minuend with 3.50 as a subtrahend and read the difference.

250. A merchant bought goods for $ 89.35 and sold them for $ 125.75. How much did he gain?

How much is gained on goods:

251. Bought for $ 129.37, sold for $ 178.12?

252. Bought for $ 363.48, sold for $ 429.95?

253. Bought for $ 428.35, sold for $ 516.81?

254. Bought for $ 596.47, sold for $ 731.97?

255. Bought for $ 1028.50, sold for $ 1296.75?

256. Bought for $ 1534.81, sold for $ 2346.55?

257. Bought for $.17, sold for $.23?

How much is lost on goods:

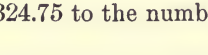
258. Bought for $ 275.37, sold for $ 179.33?

259. Bought for $ 186.38, sold for $ 175.47?

260. Make problems about buying and selling.

261. Add 875.15 to itself.

262. Add 324.75 to the number that is 4 more than 324.75. Add 324.75 to the number that is .2 more than 324.75. Add 324.75 to the number that is .25 more than 324.75.



263. Draw the line XZ 4 in. long. Mark the point Y, 1 in. from Z. How long is the line XY?

264. If XZ were 8.8 inches and YZ were 2.2 inches, how long would the line XY be?

265. If XZ were 10 in., and XY were 7 in., how long would YZ be?

266. If XY were 7.8 in., and XZ were 10.9 in., how long would YZ be?

267. Harold stands 7.8 rd. directly east of a certain point, and his brother Stanley stands 15.6 rd. directly west of it. How far apart are the boys? Represent by lines.

268. Stanley and Harold measured lines on the floor. Harold started in a corner and measured 3 ft. along by the side wall. Stanley measured 5 ft. from the same corner in the same direction. How far apart were the ends of their lines?

SUGGESTION TO TEACHER. Let two boys take the parts of Harold and Stanley for the benefit of those who cannot imagine the conditions.

269. In problem 268 if Harold's line were 7.6 ft. long and Stanley's 9.8 ft. long, what would be the distance between the ends of their lines?

270. Stanley was at one end of a side wall of a room 21 ft. long, and Harold was at the other end. Stanley walked 7 ft. in the direction of Harold, and Harold walked 2 ft. toward Stanley. How far apart were they then?

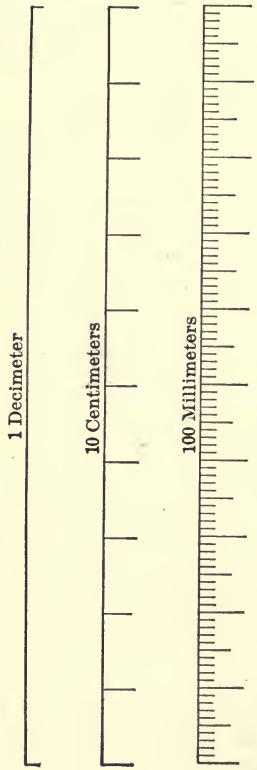
SUGGESTION TO TEACHER. For a class exercise let children find length or width of the schoolroom, imagine or enact movements like those in the previous problems, and find distances.

271. A room is 30 ft. long. If Stanley walked from one corner of it 7.1 ft. toward Harold who is at the other end of the same side wall, and Harold walked 7.9 ft. toward him, how far apart would they be?

272. High-water mark at a certain town on the Ohio River was 38.3 ft. one year and 42.1 the next year. How much higher did the river rise the second year than the first?

273. Find the cost of the materials for a Thanksgiving dinner at the house of Mr. Smith. Turkey $1.75; oysters $.55; potatoes $.05; other vegetables $.10; bread $.05; pickles $.10; jelly $.10; plum pudding $.40; mince pies $.20; milk $.10; coffee $.05; salt, pepper, sugar (estimated) $.05; nuts and raisins $.30. The family consisted of Mr. and Mrs. Smith and six children. What was the average cost for each person?

274. Plan an ordinary dinner and its cost.



NOTE TO TEACHER. For the following work pupils must be provided with rulers showing the decimeter, centimeter, and millimeter.

275. How many centimeters in 3 decimeters? In $5\frac{1}{2}$ decimeters?

276. How many millimeters in 5 centimeters? In $7\frac{3}{5}$ centimeters?

277. How many millimeters in a decimeter?

278. Draw on the board a line 10 dm. long. Its length is 1 meter. A decimeter equals what part of a meter? A centimeter equals what part of a meter?

279. The Latin word "centum" means 100. How many cents make a dollar? How many centimeters make a meter?

SUGGESTION TO TEACHER. Let some pupils make meter sticks, marking the subdivisions of 1 dm. and of 1 cm. Let others mark off a meter and its subdivisions on ribbon or tape. Keep the best of these as a part of the school apparatus.



280. What is meant by the perimeter of a figure? Draw a square centimeter. How many centimeters in its perimeter? How many millimeters?

281. Draw a square decimeter. How many decimeters in its perimeter? How many centimeters? How many millimeters?

282. Draw a line on the board 1 decimeter and 6 centimeters long. Lengthen it 4 centimeters. How many decimeters long is it now? How many centimeters?

283. A millimeter is what part of a centimeter? Of a decimeter? Of a meter?

284. M. stands for meter; dm. for decimeter; cm. for centimeter, and mm. for millimeter. Can you see why?

SUGGESTION TO TEACHER. Let pupils find in metric measurements the length of room, book, desk, writing tablet, pencil, penholder, door, blackboard, or any other object.

285. Draw a line 1.3 cm. long. How many millimeters long is it?

286. How many millimeters in 3 cm.? In 7 cm.? 9 cm. and 4 mm.? 2 dm.? 3 dm. and 4 cm.? 5 dm. and 2 cm.?

The metric system is a very convenient way of measuring, because a unit of each denomination is $\frac{1}{10}$ of a unit of the next higher. As it is used in government service, every child should learn it.

287. Compare 1 cm. with $\frac{1}{2}$ in. 1 dm. with 4 in. 1 m. with 1 yd.

288. Draw the rectangle *ABCD*, making the base 8.4 cm. and the altitude 5.6 cm. How long is the perimeter? Describe a rectangle.

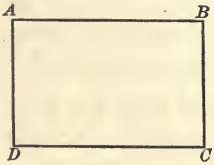


FIG. 1.

289. Draw in your rectangle the line *AC*. A line drawn from one angle of a figure to another angle that is not next to it is called a **Diagonal**. Draw as many diagonals as you can in your rectangle *ABCD*. Which diagonal is the longer?

290. Draw a triangle. Can you draw a diagonal of the triangle? Explain.

291. Two boys were in diagonally opposite corners of a room. The length of the diagonal was 35 ft. If each walked 3.5 ft. toward the other, how far apart would they be?

SUGGESTION TO TEACHER. Let children find the length of the diagonal of the floor. Two pupils may stretch a string from opposite corners at a convenient height parallel to the floor, and hence perpendicular to the intersection of the side walls, and then measure the string. Use the terms "diagonal," "parallel," and "perpendicular," and let children measure and adjust until parallelism is secured. Let pupils give to the class problems similar to Ex. 291.

292. By measuring, find the length of the diagonals of the cover of your arithmetic. Of the top of your desk.

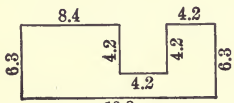


FIG. 2.

293. How long is the perimeter of Fig. 2 if the dimensions are centimeters?

NOTE TO TEACHER. The expression "Fig. 2" is used for the sake of brevity instead of the more exact expression, "The figure represented by Fig. 2." This contraction is used throughout the book.

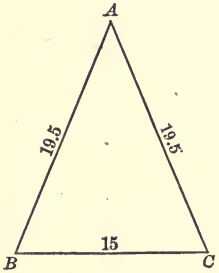


FIG. 3.

294. How long is the perimeter of the triangle ABC? The measurements are given in centimeters.

295. Triangles that have two sides equal are called **Isosceles Triangles.** Draw two lines of the same length meeting at a point. Join the ends of the lines by a straight line. What kind of a triangle have you drawn?

296. Draw an isosceles triangle whose equal sides are each 7 cm. long.

297. The side of a triangle upon which it is supposed to stand is called its **Base.** One of the equal sides of Fig. 3 is how much longer than its base?

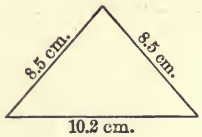


FIG. 4.

298. What is the sum of the equal sides of Fig. 3? The sum of the equal sides is how much more than the base?

299. How long is the perimeter of Fig. 4? The sum of the equal sides is how much more than the base?

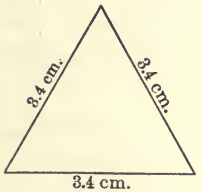


FIG. 5.

300. How long is the perimeter of Fig. 5? What kind of a triangle is it? Why?

301. How long is the perimeter of Fig. 6? Is it an isosceles triangle? Explain. The sum of the two shorter sides is how much more than the longest side?



FIG. 6.

302. The side AB is how much longer than the side AC? Than the side BC? The sum of AB and AC is how much more than BC? The sum of AB and BC is how much more than AC? The sum of AC and BC is how much more than AB?

303. Name the denominations in order from a millimeter to a meter.

304. Express in decimal form the part which one unit of each lower denomination is of one meter.

305. In the perimeter of a square decimeter, how many centimeters? How many millimeters?

306. If there are 5 sq. cm. in a row, how many square centimeters are there in a rectangle composed of 3 rows? 7 rows?

307. If there were 5 sq. cm. in a row, how many rows would it take to make a perfect square?

308. If there were 5 sq. cm. in a row, how many rows would it take to make a rectangle containing 30 sq. cm.? 40 sq. cm.?

309. How many rows would it take to make a perfect square if in each row there were 3 sq. cm.? 4 sq. cm.? 6 sq. cm.?

310. How many square millimeters in a square centimeter?

311. Draw a square decimeter. How many square centimeters in it? How many square millimeters?

312. Draw on the floor a square meter and divide it off into square decimeters. Divide one of the square decimeters into square centimeters. Can you easily divide square centimeters on the floor into square millimeters? Explain.

313. In a square meter, how many square decimeters? How many square centimeters? How many square millimeters?

314. Express in decimal form the part which one unit of each lower denomination is of one square meter.

315. John had 10 cents. He spent 6 cents for a ball and 3 cents for a top. How much had he left? His father had $537.84. He bought a horse and a carriage for $300, and a set of harness for $19.75. How much had he left?

316. One day a bank cashier paid out seven thousand-dollar bills. On the next he paid out seven hundred-dollar bills. How much more did he pay out on the first day than on the second?

317. A coat that cost $9.75 was sold for $12.50. How much was gained?

318. A coat that cost $14.75 was sold for $13.50. How much was lost?

319. A ball that cost 6 cents was sold so as to gain 1 cent. For how much was it sold? A horse that cost $115 was sold so as to gain $17.35. For how much was it sold?

320. By selling a horse for $475.50, Mr. Smith gained $87.75. How much did the horse cost? Make similar problems.

321. Mr. Cox spent $237.38 in May, and $348.31 in June. How much more did he spend in June than in May?

322. Mr. Ward deposited $89.25 in the bank on Monday, and on Tuesday $48.55. On Wednesday he drew out $105.35. How much remained to his credit in the bank?

323. Have you ever seen a bank? If so, describe it.

324. I had two notes due me, one of $420 and another of $266.66. How much was still due me after $389.50 was paid?

SUGGESTION TO TEACHER. Show promissory note and explain its use.

325. Mr. Gage bought a piece of land of Mr. Wood and gave him his note for $700. When the interest on the note amounted to $38.75, Mr. Gage paid $500 on it. How much did he still owe?

326. When $56.25 interest was due on the amount Mr. Gage then owed he paid $175.25. How much did he still owe?

MULTIPLICATION OF DECIMALS

327. Multiply 1.2 by 3.

$$\begin{array}{r} 1.2 \\ 3 \\ \hline 3.6 \end{array}$$

When 2 tenths are multiplied by 3, the result is 6 tenths, just as 2 units multiplied by 3 are 6 units, or as 3 times 2 oranges equal 6 oranges.

328. When a decimal is multiplied by a whole number, there are as many decimal places in the product as there are in the multiplicand. Multiply 4.75 by 5. By 20. By 30.

329. Give rapidly the products obtained by multiplying each of the following numbers in succession by each integer between 1 and 13: .6. .8. .9. 1.2. .12. .012.

		a	*b*	*c*	*d*	*e*
330.	Multiply	478.37	21.175	9.35	2.3	24.7
	by	6	11	36	24	81

331. What is the cost of a dozen hats at $3.75 apiece?

332. Multiply 1.28415 by the third even number.

333. All numbers that are not even are called **Odd Numbers.** Write in order the first eight odd numbers and find their sum.

334. Every odd number ends with one of 5 digits. Name them.

335. How many of the first 19 numbers are odd?

336. Multiply 8.8571 by the fourth odd number.

337. Multiply 16.754 by the seventh odd number.

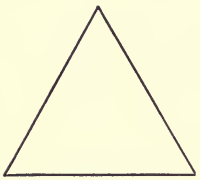


FIG. 7.

338. An **Equilateral Triangle** is a triangle all of whose sides are equal, as Fig. 7. If each side of Fig. 7 were 8.75 in. long, how long would its perimeter be?

SUGGESTION TO TEACHER. Show the following method of constructing an equilateral triangle. Draw a line of convenient length for base BC as in Fig. 8. With B as a center and BC as a radius, draw an arc. With C as a center and CB as a radius, draw a second arc intersecting the first at A. Draw AB and AC. Erase construction lines.

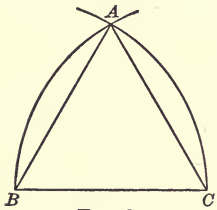


FIG. 8.

339. Construct an equilateral triangle each side of which is 4 in. How long is its perimeter? How long would the perimeter be if each side were 8.25 in.? 4.875 in.?

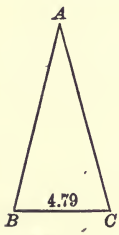


FIG. 9.

340. In the triangle ABC each of the equal sides is twice as long as the base. What kind of a triangle is it? How long is the perimeter?

341. Construct an isosceles triangle each of whose equal sides is twice the base.

SUGGESTION TO TEACHER. Let pupils use the same method of construction as is used for equilateral triangles except that each arc should be drawn with a radius twice as long as the base.



FIG. 10.

342. In the triangle *ABC*, *AB* and *AC* are each 3 times as long as *BC*. How long is the perimeter? The sum of the equal sides is how much more than the base?

343. Construct an isosceles triangle each of whose equal sides is 3 times as long as the base. The perimeter is how many times as long as the base?

344. Construct an isosceles triangle whose base is 4.5 inches and each of whose equal sides is 6.5 in. The perimeter is how much longer than the base?

345. Construct an isosceles triangle whose base is 3 in. and the sum of whose equal sides is 9 in.

346. How much will a dozen knives cost at \$.87 apiece?

347. John had \$.15 which was $\frac{1}{3}$ of what he needed to buy a music book. What was the price of the book?

348. .00256 is $\frac{1}{7}$ of what number? $\frac{1}{11}$ of what?

349. If a dozen knives are bought at \$.67 apiece and sold for \$1.00 apiece, how much is gained?

350. The rent of a house is \$17.50 per month. How much is the rent for a year?

351. A man spends on the average \$.25 a day for cigars. How much does he spend in a leap year?

352. NEW YORK, Sept. 12, 1898.

MR. WM. H. MORSE

Bought of THOMAS D. LONG,

25 lb. Sugar	@	\$.05	\$1.25
17 lb. Coffee	@	.25	4.25
6 lb. Tea	@	.87½	5.25
			\$10.75

Received Payment,

THOMAS D. LONG.

353. Make a bill similar to Ex. 352, in which the price of the sugar is 6¢ per lb., the coffee 30¢, and the tea 95¢.

354. Imagine that you are a clerk in a store where a customer buys the following bill of goods. Make out the bill and receipt it.

8 yd.	Gingham	@	\$.37½ per yd.
9 yd.	Binding	@	.07 per yd.
1 yd.	Percale	@	.11 per yd.
2	Fans	@	.75 each.

SUGGESTION TO TEACHER. Get bill heads from merchants and let them be copied in the following exercises.

355. Imagine yourself to be a clerk in a grocery store. Make out and receipt a bill of goods bought by Mr. James Jones.

356. Make out and receipt a bill of goods bought in (*a*) a dry goods store. (*b*) A shoe store. (*c*) A music store. (*d*) A toy store. (*e*) A clothing store.

357. Make out a meat bill for Mr. Walter Smith for the week ending Saturday, Sept. 8, 1900.

358. When 4.8 is changed to 48, by what is it multiplied? By what must 4.8 be multiplied to make it 480?

SUGGESTION TO TEACHER. Show the method of multiplying by any power of 10 by moving the decimal point to the right, and of dividing by any power of 10 by moving the decimal point to the left.

359. Multiply 1.357 by 10. By 10000. By 100000.

360. Divide 125.7 by 10. By 100. By 1000. By 100000.

361. Add 1.25 to 100 times itself.

362. Add to 3.25 the number that is $\frac{1}{100}$ of it.

363. Subtract from 875 the number that is $\frac{1}{10}$ of it.

364. Subtract .213 from 1000 times itself.

365. 3.78 is how much less than 1000 times itself?

366. How do you multiply an integer or a decimal by 10? By 100? By 1000? By any number expressed by 1 with one or more ciphers annexed?

367. How is an integer or a decimal divided by 10? By 1000? By any number expressed by 1 with one or more ciphers annexed?

368. How much is .1 or $\frac{1}{10}$ of 30? .2 of 30?

369. How much is .01 or $\frac{1}{100}$ of 300? .02 of 300?

370. How much is .01 of 375? .02 of 375?

371. Write an integer of three places and find .03 of it.

372. Write an integer of four places and find .7 of it.

373. When an integer is multiplied by a decimal there are as many decimal places in the product as there are in the multiplier. Multiply 325 by .7. By .13. By .125.

374.

	a	*b*	*c*	*d*	*e*
Multiply	275	283	413	671	1289
	1.1	.17	1.01	.21	.001

375. How much is .25 of a square 8 in. in dimensions?

376. With 375 as a multiplicand and .31 as a multiplier, what is the product?

377. With 145 as a multiplicand and .41 as a multiplier, what is the product?

378. How many places and in which direction must the decimal point be moved in order to divide 125.7 by 100, or to find $\frac{1}{100}$ of it?

379. Find .01 of 217.25. Of 365.7. Of 412.137.

380. How much is .01 of 225.7? .03 of it? .08 of it? Compare the number of decimal places in the products with the number of decimal places in multiplicand and multiplier.

381. When an integer is multiplied by a decimal, how must the product be pointed off?

382. How must the products be pointed off when a decimal is multiplied by an integer?

383. When a decimal is multiplied by a decimal, *the product contains as many decimal places as there are decimal places in both multiplicand and multiplier.* Multiply .05 by .5.

384. Find products:

a 1.57 × .3	*f* 84.2 × .43
b 14.5 × .7	*g* 1.32 × 4.1
c 41.42 × 6	*h* 6.71 × .11
d 2.42 × 1.21	*i* 3.41 × .701
e 3.43 × 6.41	*j* 1.2 × .41

385. If there are not as many figures in the product as there are decimal places in both multiplicand and multiplier, ciphers must be prefixed to the product before pointing it off. Explain.

386. Multiply .15 by .3. .35 × .07. .002 × .7. .021 × .008.

387. To square a number is to multiply it by itself. Square: 1.5. .16. 2.3. .009. .18. 1.9. 3.2. .051. 2.8. 4.08. .025.

388. Draw a square whose dimensions are 1.5 dm. How many square decimeters in it?

389. How many square decimeters in a rectangle 1.3 dm. long and 1.2 dm. wide? Represent.

390. How many square inches in a square whose dimensions are 1.25 in.? Represent.

391. How many square inches in a rectangle 7.5 in. long and 3.5 in. wide?

392. A lot cost $687.50, and the house which stood upon it cost 4.5 times as much. How much did the house cost? How much did both cost?

393. Kate drew a rectangle 4.5 in. long and 2.75 in. wide. Anna drew a rectangle 3.5 times as large as Kate's rectangle. How many square inches in Anna's rectangle?

394. One hundredth of anything is called 1 per cent of it. Per cent is written %, as 4% means .04. Write as per cent: .17. .07. .03½. $\frac{99}{100}$. .50. $\frac{13}{100}$. .61.

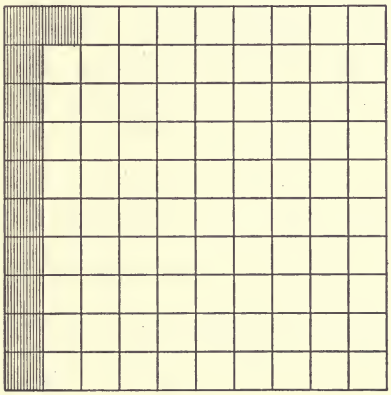


FIG. 11.

395. What per cent of Fig. 11 is shaded? Unshaded?

396. BOARD WORK. Draw a square decimeter and mark it off into square centimeters. Shade 13% of it. What per cent of it is unshaded?

397. Shade $\frac{1}{5}$ of it. What per cent is unshaded?

398. Shade the following parts of the figure and tell in each case what per cent is unshaded:

$\frac{1}{4}$	.3	39%	.4	$\frac{1}{2}$	57%	.6
.7	$\frac{3}{4}$	78%	$\frac{4}{5}$	.9	97%	100%

399. What per cent of a dollar is 1 cent? 3 cents? 21 cents?

400. George had a dollar and lost 5% of it. How many cents had he left? How many cents had he left when he had spent another 5% of the dollar?

401. Of 100 words that John wrote in a spelling test, 13 were wrong. What was his per cent on that test?

402. When your record in an examination is 99%, how many hundredths of your work are correct? How many hundredths are wrong?

403. Class Exercise. —— may name a number less than 100, and the class may tell what per cent it is of 100, and how many per cent of 100 it lacks of being 100.

404. As 6% of anything is .06 of it, we may find 6% of any number by multiplying it by .06. Find 6% of 44.

405. In the same way find 6% of: 28. 39. 63. 144. 135.

406. How would you find 7% or any other per cent of a number? Illustrate.

407. A company of soldiers consisted of 100 men. 7% of them were mustered out. How many soldiers remained?

408. Find 5% of: 14. 24. 75. 1.83. 6.44. 3.72. 8.49.

409. Mr. Miller bought $960 worth of goods, and, in selling them, gained 15%. How much did he gain? How much did he receive for them?

410. Mr. Low bought $3125.50 worth of goods and sold them at a loss of 2%. How much did he lose? How much did he receive for them?

411. Make problems about buying goods and selling them at a certain per cent of gain or of loss.

412. How many per cent of anything is the whole of it? $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{3}{4}$ of it?

413. Fill out the following and learn:

The whole = 100% $\qquad$ $\frac{1}{4}$ = ——%

$\frac{1}{2}$ = ——% $\qquad$ $\frac{3}{4}$ = ——%

414. Find 25% of 64 by multiplying it by .25. Find 25% of 64 by taking $\frac{1}{4}$ of it.

415. Find the values of each of the following in two ways, first by multiplying by the decimal fraction which the per cent equals, and then by a common fraction which it equals. 25% of 48. 50% of 12. 75% of 8. 75% of 24. 75% of 32.

416. Find in the shortest way how many men equal 50% of 14 men. How many bu. in 75% of 12 bu.?

417. How many inches in 50% of a foot? 25%? 75%? How many quarts in 25% of a gallon? How many quarts in 75% of a peck? How many ounces in 25% of a pound? 50%? 75%?

DIVISION OF DECIMALS

418. Divide .75 into 3 equal parts. If 75 cents are divided equally among 3 persons, will the "25" which each receives be 25 cents or 25 other things? If .75 of anything are divided into 3 equal parts, one of these parts will be 25 what?

419. When a decimal is divided by an integer, there are as many decimal places in the quotient as there are in the dividend. Divide 9.24 by 7.

In dividing a decimal by an integer by short division place the decimal point of the quotient directly under that of the dividend as soon as it is reached.

$$7\overline{)9.24}\quad 1.32$$

In this case the quotient of 9 units divided by 7 is 1 unit with a remainder of 2 units. The decimal point should be placed after the 1 unit before the division is continued.

420. By 7 divide: 2.583. 1.0332. 4.1328.

421. Find values of x:

a	b	c	d
$x = \frac{19.64}{4}$	$x = \frac{38.82}{6}$	$x = \frac{5.76}{8}$	$x = \frac{21.60}{6}$
e	f	g	h
$x = \frac{343.7}{7}$	$x = \frac{13.25}{5}$	$x = \frac{848.8}{8}$	$x = \frac{1.989}{9}$

422. Find the length of one side of an equilateral triangle whose perimeter is 7.5 in.

423. By 9 divide: 8.811. 34.569. 672.3. 4712.31

424. $12,384.75 were divided among 5 heirs. How much did each receive?

425. In one week Mr. A. earned $123.66. What were his average earnings for each working day of the week?

426. Find $\frac{1}{4}$ of .0076.

4)$\underline{.0076}$
.0019

Queries. How many tenths in $\frac{1}{4}$ of .0? How many hundredths in $\frac{1}{4}$ of .00? How many thousandths in $\frac{1}{4}$ of .007? How many ten-thousandths in $\frac{1}{4}$ of .0036?

427. Find $\frac{1}{2}$ of .008. Of .016. Of .246.

428. By 8 divide: .01728. .002016. .12102. .025832.

429. Divide .12 by 9, carrying the division to three places of decimals.

9)$\underline{.120}$
$.013\frac{1}{3}$

Annexing a cipher to .12, we have .120, which is equal in value to .12. .120 divided by 9 equals $.013\frac{1}{3}$.

430. How many ten-thousandths in the quotients of the following?

$\frac{1.34}{6}$ $\frac{87.1}{9}$ $\frac{.128}{7}$ $\frac{.542}{8}$ $\frac{76.4}{3}$

431. Divide to three places of decimals: $\frac{7.75}{4}$. $\frac{82.1}{3}$. $\frac{64.8}{5}$.

432. Divide 22.75 by 13.

$$13\overline{)22.75}\ (1.75$$
$$\underline{13}$$
$$97$$
$$\underline{91}$$
$$65$$

In dividing a decimal by an integer by long division, write the quotient above the dividend and place the decimal point of the quotient above the decimal point of the dividend as soon as it is reached.

433. By 21 divide: 8.82. 26.04. 10.353. 4.1349.

434. By 32 divide: 5.44. 1.632. .11424. 20.48.

435. By 24 divide: .3456. .5184. .241584. .5544.

436. Divide 55.44 by 44. By 28. By 77.

437. If $3.15 were divided among 15 boys, how much would each receive?

438. If hats are bought at $8.64 a dozen, how much does one hat cost?

439. If $250 were divided equally among 3 men, how many dollars and cents would each man receive?

440. If the following sums of money were divided equally among five persons, how many dollars and cents would each person receive?

$124 $661 $946 $12823 $67847

441. What is $\frac{2}{3}$ of 18.24? Of 17.52? Of 86.25?

442. If 768.32 acres of land were divided equally among 16 men, how many acres would 3 men receive?

443. If 11 doors cost $19.25, how much would 2 doors cost?

444. Multiply 549.36 by $3\frac{1}{2}$. By $4\frac{1}{3}$. By $8\frac{1}{9}$. By $12\frac{1}{4}$.

445. $46.125 \times 2\frac{1}{5} = ?$ $46.125 \times 3\frac{1}{5} = ?$

446. How much will one knife cost at $9.00 a dozen? At 10.50 per dozen? At $15.00 per dozen?

447. How much is gained on each hat by buying hats at $20 a dozen, and selling them at $2.00 apiece?

448. How much is gained on each quart of milk:

a Bought at $.28 a gallon, sold at $.08 a quart?

b Bought at $.25 a gallon, sold at $.07 a quart?

c Bought at $.30 a gallon, sold at $.09 a quart?

449. In buying milk at $.20 a gallon and selling it at $.06 a quart, how many quarts must a milk dealer sell to gain $1.00?

450. In buying balls at $1.00 a dozen and selling them for $.10 apiece, how much is gained on each ball?

451. When 45 yards of calico are bought for $1.35, and sold at $.05 a yard, how much is gained on each yard?

452. Divide 8.64 by 2. If both dividend and divisor were ten times as large as they are, what would the quotient be?

SUGGESTION TO TEACHER. The principle, "Multiplying both dividend and divisor by the same number does not change the quotient," should be thoroughly reviewed and illustrated before the following work is done.

453. Divide 3.76 by .2.

If both dividend and divisor are multiplied by 10, we have $37.6 \div 2$. This is similar to previous problems.

454. By the following rule perform this example in division of decimals, and give reason for the rule. $1.96 \div .4$.

To divide by a decimal —

Move the decimal point of the divisor to the right until the divisor is an integer. Move the decimal point of the dividend an equal number of places to the right, annexing ciphers if necessary. Divide, and point off as many decimal places in the quotient as there are then in the dividend.

455. Divide each of the following by .09:

1.125 12.33 43.119 62.91 4.815

456. Use .06 as a divisor with the following dividends:

221.4 13.2 54.6 91.2 .636 5940 2100

457. Find values of x:

a b c d e

$$x = \frac{78.3}{.03} \quad x = \frac{96.8}{.008} \quad x = \frac{49.7}{.14} \quad x = \frac{37.5}{.15} \quad x = \frac{88.4}{.08}$$

f g h i j

$$x = \frac{168}{.35} \quad x = \frac{78}{.15} \quad x = \frac{65.4}{.12} \quad x = \frac{165}{.22} \quad x = \frac{89.1}{.006}$$

458. A music teacher earned $100 in a month, giving lessons at $1.25 each. How many lessons did she give?

459. At 75¢ a yard, how many yards of lace can be bought for $12.75? For $23.25?

460. Find quotients:

Dividend	Divisor	Dividend	Divisor
a 2.25	1.5	*f* 2.057	12.1
b 2.75	2.5	*g* 3.144	1.31
c 137.5	1.25	*h* 539.6	14.2
d 396	1.2	*i* 114.92	.221
e 4.84	1.1	*j* 603.2	.232

461. How long is a rectangle which is

a 5 ft. wide, and contains 35 sq. ft.?

b .3 ft. wide, and contains .75 sq. ft.?

c .7 ft. wide, and contains .77 sq. ft.?

d .5 cm. long, and contains .125 sq. cm.?

e .9 in. long, and contains .72 sq. in.?

MISCELLANEOUS EXERCISES

1. Add, 1248.375, 115.67241, 3935.5428, and 138.463249.

From 13 thousand and 21 thousandths take 11 hundred and 4 hundredths.

3. Multiply .246 by .89.

4. Divide 243.26647 by .98.

5. Write in Arabic notation and find the sum: MI, MV, MX, ML, MC, MD.

6. Find the sum of all the numbers less than 100 that are expressed in Roman notation by 2 letters. By 3 letters. By 4 letters. By 5 letters. By 6 letters. By 7 letters.

7. Subtract from 100 the number less than 100 that is expressed in Roman notation by 8 letters.

8. Express decimally and add: 137 and 17 hundredths, 23 thousand 67 and 19 ten-thousandths, 38 thousand 5 and 11 millionths.

9. From 256 thousand 17 and 15 thousandths take 128 and 129 ten-thousandths.

10. How many square feet in a rectangle 1.75 ft. long and 1.25 ft. wide? How long is its perimeter?

11. How wide is a rectangle that contains 1.92 sq. in. and is 1.6 in. long? How long is its perimeter?

12. 1.44 is how many times .0012?

13. A merchant bought $2125.75 worth of goods, and sold them so as to gain 12% of the cost. How much did he gain?

14. Mr. Duncan bought goods that cost him $1226.35, and sold them so as to gain 16%. For how much did he sell them?

15. A man died, leaving $12,000. He willed 50% of it to his wife, 30% to his daughter, and the rest to a library. How much did each receive?

16. Thomas bought a dime's worth of ice cream, which was only 50% of the amount he wanted. How many cents' worth of ice cream did he want?

17. Jennie has 7 cents, which is 25% of her sister's money, and 50% of her brother's money. How many cents has each of them?

18. $45.75 is 25% of how many dollars?

19. How many pounds in 25% of a ton? In 10%? 20%?

20. Mr. Wade invested $870, and gained 10% on it in a year. How much had he at the end of the year?

21. Mr. Brooks invested $9000, gained 10% on it in the first year, and added the gain to his capital. He gained 10%

on that amount in the second year, and added it to his capital. During the third year he increased his capital by 10%. Find how much he had at the end of each year. How much more than his original investment had he at the end of the third year?

22. How much is 100% of 2 watermelons? Of 4 chairs?

23. A chair that cost $3 was sold at a gain of 100%. For how much was it sold?

24. What number is as much greater than 10 as 10 is greater than 8?

25. What number is as much more than 20 as 20 is more than 17? As much less than 20 as 20 is less than 21?

26. What is the average of 10 and 16, or what number is as much greater than 10 as it is less than 16?

To find the average of two numbers, divide their sum by 2. To average three numbers, divide their sum by 3. To average four numbers, divide their sum by 4, etc.

27. What is the average of 18 and 20? Of 4 and 50? Of 9, 21, and 24? Of 8, 12, and 25?

28. If you stand 98% on an arithmetic test, 95% on a spelling test, and 92% on a geography test, what is your average per cent?

29. Joseph worked 9 problems on Monday, 12 on Tuesday, and 12 on Wednesday. How many problems a day did he average?

30. What is the average of 8.48, 10.24, and 4.96?

31. Mr. Harris earned $25.37 in one week, $38.75 the next week, $31.25 the next week, and $40.50 the next week. How much were his average earnings during the four weeks?

32. High-water mark at a certain town on a large river was 48.3 ft. one year, 50.5 ft. the next, and 47.6 ft. the next. What was the average?

33. What was the average height of a river for four successive days, if on the first day it was 33.9 ft. high, on the next 34.3 ft., on the next 34.9 ft., and on the next 35.1 ft.?

34. Mr. Howe invested $ 36,000 in business. At the end of 8 years his capital was $ 64,000. What was his average gain per year?

NOTE TO TEACHER. The following work requires a Fahrenheit thermometer and an explanation of its use.

35. How many degrees are there between the freezing point and the boiling point on the Fahrenheit thermometer?

36. The temperature at noon for four successive days in July was 90°, 91°, 92°, and 84°. Find the average temperature.

37. The temperature at noon for five successive days in January was 21°, 19°, 15°, 18°, and 27°. Find the average temperature.

38. The temperature of our bodies is about 98°. How much above freezing point is that? Below the boiling point?

39. How many degrees does the temperature fall when it changes from 57° to 3° below the freezing point?

40. What is the decrease in temperature from 20° to 1° above 0? From 20° to 1° below 0? From 15° to 5° below 0?

41. The temperature at Minneapolis one winter day was 11°. Before night it fell 20°. What was the temperature then?

42. The next day it rose 15°. What was the temperature?

43. The next morning it was 4° below 0. How much had it fallen?

44. Mrs. A. bought 19¢ worth of groceries and offered $ 1.00 in payment. The clerk gave her 1¢ and said, "Twenty." Then he gave her a nickel and said, "Twenty-five." He then gave her a quarter saying, "Fifty." He ended by giving her a half dollar and saying, "One dollar." In the same way find how that amount of change could be given with different coins.

45. CLASS EXERCISE. —— may tell a story of a purchase made and payment offered. Members of the class may show different ways of making change.

46. Draw two horizontal lines and two vertical lines.

47. Lines which lie in the same direction are called **Parallel Lines.** Find parallel lines in the room. In your book. Name some capital letters that have parallel lines when printed.

48. Think of your own name printed in capitals. Can you see any parallel lines in it?

49. How many pairs of parallel lines has a rectangle? Are there any parallel lines in a triangle?

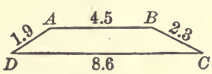


FIG. 12.

50. A four-sided figure that has only 2 parallel sides is called a **Trapezoid.** *AB* and *CD* are parallel. How long is the perimeter of the trapezoid *ABCD*, the measurements representing inches?

51. The sum of the parallel sides of Fig. 12 is how much more than the sum of the non-parallel sides?

52. Draw trapezoids of different shapes.

53. Two lines meeting at a point form an **Angle,** $\angle$. The point where the lines meet is called the **Vertex** of the angle. Draw an angle and mark its vertex *A*.

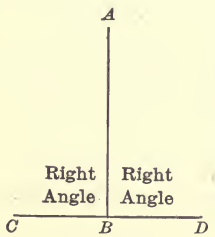


FIG. 13.

54. When one straight line meets another straight line so as to make two equal angles, the angles are called **Right Angles.** What letter is at the vertex of each angle in Fig. 13?

55. Place two pencils so as to show two right angles.

56. Fold a strip of paper so that the line of the fold makes right angles with the edge.

57. Cut out a paper circle and fold it into fourths. What kind of angles are made by the folds?

58. Find right angles made by lines in the surfaces of the room or of objects in it.

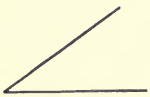


FIG. 14.

59. An angle less than a right angle is called an **Acute Angle.** Draw an acute angle.

SUGGESTION TO TEACHER. As children naturally judge of the size of an angle by the length of the lines that form it, pupils should draw and cut out a right angle, and by applying it to given angles, find out whether they are acute, right, or obtuse.

60. Draw a trapezoid and mark the acute angles.

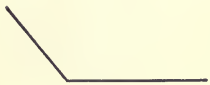


FIG. 15.

61. An angle greater than a right angle is called an **Obtuse Angle.** Draw an obtuse angle.

62. Draw a trapezoid and mark obtuse angles and acute angles.

63. What kind of angles has a rectangle? An equilateral triangle?

64. Draw a trapezoid that has two right angles. Name the other two angles.

65. In the printed words "ADMIRAL DEWEY," how many right angles are there? Acute angles? Obtuse angles?

66. If the name of the county in which you live were printed in Gothic type like the words "ADMIRAL DEWEY," how many right angles would there be in it? How many acute angles? How many obtuse angles?

SUGGESTION TO TEACHER. Splints or toothpicks are useful in the following exercises.

67. With 3 lines make 2 right angles; 2 obtuse angles; 2 acute angles. Show the vertices of the angles.

68. With 2 lines make 4 angles, and tell of what kind they are.

69. With 3 lines make 12 angles, and tell their kinds. Make 10 angles. 9 angles.

70. With 4 lines make 16 angles. 20 angles. 24 angles.

71. With 5 lines make 4 angles. 5 angles. 20 angles.

72. Class Exercise. —— may tell how many angles he can make with a certain number of lines, and the class may make them.

73. A triangle that has a right angle is called a **Right Triangle.** Draw a right triangle.

74. Draw an isosceles triangle. The angles at the base are equal. What kind of angles are they?

75. Draw an isosceles triangle on paper. Cut it out and fold it so that the equal sides coincide. Cut along the line of the fold, and you have two equal triangles. What kind of triangles are they?

76. How long would the perimeter of one of these right triangles be if the base were 20 in., perpendicular 1 in. longer than the base, and hypotenuse 8 in. longer than the perpendicular?

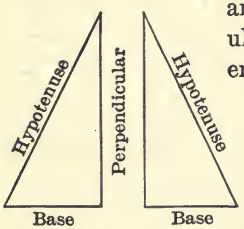


Fig. 16.

77. Place together the two triangles you have made so that they form a rectangle. If the area of that figure were 28 sq. in., what would be the area of each right triangle?

78. A figure drawn upon a flat surface is called a **Plane Figure.** Can you draw a plane figure on the surface of a ball? Of a slate? Of a piece of gas pipe?

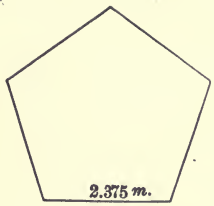


FIG. 17.

79. A plane figure bounded by five straight lines is called a **Pentagon.** When (as in Fig. 17) the lines are all equal and make equal angles, the figure is called a **Regular Pentagon.** What kind of angles has a regular pentagon?

80. Find the length of the perimeter of the pentagon represented by Fig. 17.

81. How long is one side of a regular pentagon whose perimeter is 9.15 in.?

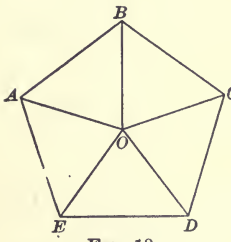


FIG. 18.

82. Draw a pentagon that is not regular.

83. The pentagon in Fig. 18 is divided into triangles by equal lines drawn from its center to the vertices of its angles. What kind of triangles are thus formed? How many of them? Each triangle is what part of the pentagon? What % ?

84. What % of the pentagon is the figure *ABCO*? *AEDO*? *BCDEO*? *CDEABO*?

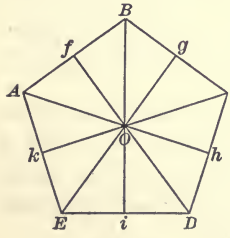


FIG. 19.

85. Figure 19 differs from Fig. 18 in having the lines *Og*, *Oh*, etc., drawn from the center of the pentagon to the middle point of each side. They are perpendicular to the sides. Each right triangle thus formed is what part of the pentagon? What % ?

86. What % of the pentagon is *AOB*? *EOi*? *BOhC*? *ABCh*?

87. Give the outlines of a figure which is 70% of the pentagon. 40%. 90%.

88. A butcher bought a hog weighing 375 lb. at $.03 a lb. How much did it cost?

89. He sold 15 lb. of it at 2¢ per lb., 50 lb. at 5¢ and the rest at 12¢ per lb. How much did he receive for it?

90. A farmer sold 15 doz. eggs at 18¢ a dozen, receiving for them sugar at 6¢ a pound. How many pounds of sugar did he receive?

91. A fruit dealer buys 29 doz. oranges for $ 8.70. How much does he pay for each orange?

92. If he sells the oranges at the rate of 5¢ apiece, how much does he gain on each orange? How much on all the oranges?

93. What number multiplied by 9 will give the same product as 12 multiplied by 6?

94. Mr. Hale had $ 5728 and paid 25% of it for a farm. How much did the farm cost? He sold the farm for $ 1200. How much did he lose?

95. He left the other 75% of his money in the bank until it had gained $ 472 interest. How much money had he then, including the money he received from his farm?

96. A grocer bought 185 barrels of flour at $ 3.75 a barrel, and sold it all for $ 740. How much did he gain?

97. A miller bought 35 bu. of wheat for $ 22.75, and sold it at $.61 a bushel. How much did he lose?

98. A farmer had an orchard of 276 trees. One year they averaged 13 bu. of apples to each tree. What was the value of that season's crop at $.75 a bushel?

99. The next year the trees averaged 9 bu. per tree, and the apples brought $.80 a bushel. What was the value of the crop that year?

100. A merchant's profits in January, 1899, were $1428.75. In January, 1900, his profits were 20% less. What were his profits in January, 1900?

101. Mr. Strong had $975.85 in a bank; he drew a check on the bank for $625.47. How much money had he remaining in the bank?

SUGGESTION TO TEACHER. Show bank checks. Explain their use and let pupils copy and fill them out for use in imagined transactions.

102. If you had $65.87 in a bank, and should draw a check for $38.45, how much of your money would be left in the bank?

103. Mr. Gale had $1225 in a bank. He drew $12.25 every Saturday night for 10 weeks. How much had he left in the bank?

104. Aline deposited $11.75 in a savings bank in February. She drew out $3.25 in March and $2.95 in April. She deposited $14.45 in May. How much had she then in the bank?

105. Mr. Davis bought a stove worth $18.75. The dealer allowed him 2¢ a pound for an old stove, weighing 195 lb. He gave a check on the bank for the balance. What was the amount of the check?

106. There were 276 houses on the street. A postman delivered 3 letters at 28 of the houses, 2 letters at 41 of the houses, and 1 letter at 105 houses. At how many houses were no letters delivered?

107. There were 559 books in a school library, which was an average of 13 to each pupil. How many pupils were there in the school?

108. In February of a common year, Mr. Fisk's family burned a ton of coal in 14 days. At $8.50 per ton, what was the cost of the coal for that month?

109. A lot is in the form of a trapezoid. One of the parallel sides is 16.8 rd. long, and the other is twice as long. Of the

sides that are not parallel, one is 19.7 rd. long, the other is 15.4 rd. long. How long a fence is required for the whole lot?

110. Mr. Lee started to Denver with $ 300. He paid $ 47 for railroad fare, his hotel bill was $ 4 a day for a week, other expenses $ 7.50, and his return ticket was $ 47. How much did he spend? How much had he left?

111. Passengers were first carried on railroads in the United States in the year MDCCCXXVIII. How many years have we had railroads?

112. Square: 13. 1.5. 1.7.

113. Ella had a flower bed a yard square. She divided it into square feet and placed a rosebush in the middle of each square foot. How many rosebushes had she? Represent.

114. Draw a square decimeter and show into how many square centimeters it can be divided. Into how many square inches can a square foot be divided?

115. How many square inches in 5 squares whose sides are each 11 in. long?

116. How long is the perimeter of a square, a side of which is 3.1 in. long? What is its area?

117. Draw a right triangle. If its base were 7.5 in., its perpendicular 10 in., and its whole perimeter 30 in., how long would the hypotenuse be? What would be its area?

118. Find the length of the perpendicular of a right triangle whose perimeter is 90 in., its base 22.5 in., and hypotenuse 37.5 in. Find the area of the triangle.

CHAPTER II

PROPERTIES OF NUMBERS

1. When an integer can be divided by another number without a remainder, it is said to be divisible by that number. Is 9 divisible by 5? Give a reason for your answer.

2. Choose an even number and illustrate this statement: An even number is an integer that is divisible by 2.

3. Choose an odd number and illustrate this statement: An odd number is an integer that is not divisible by 2.

4. Name the first even number after 10. How many twos does it equal?

5. Square the third odd number. Square the sixth even number. Multiply the seventh even number by the fifth odd number. Find the difference between the sixth odd number and the eighth even number.

6. Find $\frac{4}{5}$ of the fifth even number. Find 7% of the fourth odd number.

MULTIPLES AND FACTORS

7. A **Multiple** of a number is the product obtained by multiplying it by an integer. Thus 5 is the first multiple of 5, 10 is the second multiple of 5. Give quickly the first twelve multiples of

3 4 5 6 7 8 9 10 11 12

8. Figure 1 represents two lots of land owned by Mr. Smith and Mr. Brown. The fence between the lots is 150 ft. long

and cost 7¢ a foot. How much of this expense should each man pay? Give reasons for your answer.

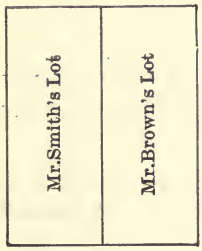


FIG. 1.

9. Mr. Smith and Mr. Brown decided to take away the fence and leave a strip 10 ft. wide on each side of the line where it had stood. This strip is used as a common playground by the children of both families. How many square feet in their common playground?

10. A fine park in Boston is called Boston Common. What does the word "common" mean in this case? "20 is a common multiple of 10 and 5." What does that statement mean?

11. A number which is a multiple of two or more numbers is called their **Common Multiple**. Give several numbers that are common multiples of 2 and 3. Of 3 and 7. Of 4 and 5.

12. Of what two numbers besides itself and 1 is 15 a multiple? 10? 35? 21? 22? 33? .

13. Write all the numbers of which 6 is a common multiple. 8. 14. 16. 12. 24. 40. 36.

14. Give two numbers which multiplied together make the product 18. 20. 27.

15. The numbers that make a product are called the **Factors** or **Divisors** of that product. 12 has 3 pairs of factors, 1×12, 2×6, and 3×4. Give all the pairs of factors of 24, except the pair of which 1 is the least factor.

SUGGESTION TO TEACHER. Let pupils find factors of a number by using as a trial divisor each number in succession, beginning with 2. Lead them to see that as soon as the quotient which they obtain is less than the divisor they use, it is unnecessary to try any more numbers, as they will merely get the same pairs of factors stated in reverse order.

16. Give all the pairs of factors of 45. 28. 36. 60. 72.

17. Give all the numbers of which 30 is a multiple. 66.

18. Class Exercise. —— may give a number which is a multiple of some other numbers, and the class may find all its factors.

COMPOSITE NUMBERS

19. A number that is the product of two or more integers is called a **Composite Number.** Give three composite numbers and their factors.

20. What number is composed of the factors 2 and 11? 3 and 11? 2, 3, and 11? 7 and 7? 2, 3, and 7? 2, 2, and 3?

21. What factors compose 77? 40? 18? 42?

22. Name an even composite number, and give factors that compose it.

23. Name an odd composite number, and give factors that compose it.

24. Name a composite number that is a multiple of 5, and give its other divisors.

25. Make and keep a list of all the composite numbers less than 41.

26. Write a composite number whose tens' digit is 2, and give its factors.

27. Write the following numbers and their factors:

a A composite number whose units' figure is 5.

b The first composite number after 26.

c A composite number between 30 and 40 that is not a multiple of 5.

d A composite number between 30 and 40 that is not a multiple of 2 nor of 5.

e A composite number between 20 and 30 that is not a multiple of 2, 5, nor 7.

28. Divide .00168 by the 3d composite number.

29. Find 5% of the 9th composite number.

30. Multiply the 8th composite number by .009.

PRIME NUMBERS

31. A number that has no integral factors except itself and 1 is called a **Prime Number.** Think of each of the numbers from 2 to 10 and tell which of them are prime.

2 is the first prime number, as 1 is considered neither prime nor composite.

32. No prime number of more than one place ends in 2, 4, 6, 8, 0, or 5. Can you tell why?

33. Make a list of all the prime numbers less than 50 in their order.

SUGGESTIONS TO TEACHER. Show pupils how to find prime numbers less than 50 by examining each number to see if it can be exactly divided by 2, 3, 5, or 7. Develop the fact that there is no need of dividing even numbers because they are all divisible by 2; nor numbers that end in 5, because they are divisible by 5. Lead pupils to see that if a number will not contain 2, it will not contain 4, 6, 8, or any number of twos; that if it will not contain 3, it will not contain 9, 15, 21, or any number of threes, and so on.

34. Group the prime numbers less than 50 as they are found in each ten numbers, as:

1st ten { 2, 3, 5, 7 } 2d ten { 11, 13, 17, 19 } Keep the list.

35. Find the sum of all the prime numbers that are expressed by one digit.

36. Divide 13.5 by the 2d prime number.

37. Divide the 3d prime number by .8.

38. Find the difference between 26.4 and the 11th prime number.

39. Multiply the 8th prime number by .0004.

40. Find 6% of the 9th prime number.

41. Write the first 30 numbers in two lists, one of prime numbers, the other of composite numbers. Leave out the number 1.

42. Find the sum of all the composite numbers less than 15.

43. Find the sum of all the primes less than 20.

44. What prime number is nearest to 20?

45. What two prime numbers are near to 12?

46. 15 is half way between two prime numbers. What are they?

There are three pairs of these numbers.

47. What prime number is nearest to the 2d multiple of 5? To the 8th multiple of 5?

Suggestion for Class Exercise. Let children select prime numbers and give a clew to them, and let the class identify them.

48. Find the difference between the prime number nearest to 20 and the prime number nearest to 8.

49. Class Exercise. —— may name a number larger than 50 which he thinks is prime, and the class may see if he is right.

Note to Teacher. The following method of finding prime numbers less than 100 is very useful:

Write the first hundred numbers as on p. 67, omitting 1 because it is considered neither prime nor composite. Counting from 2, the first prime number, strike out as composite every second number because it is a multiple of 2; counting from 3, strike out as composite every third number. Lead the pupils to discover why it is unnecessary to strike out the multiples of 11 or of any larger primes in finding the prime numbers less than 100.

This device, which is an adaptation of the well-known "sieve of Eratosthenes," may be used to any limit to separate prime from composite numbers by writing additional columns of numbers and striking out all the multiples except the first multiple of those primes whose squares are included within the limit.

	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

50. A Greek mathematician named Eratosthenes, who was born 275 B.C., devised this plan of finding prime numbers. Instead of marking out the composites, he cut them out. Can you see why the table of primes that was left was called "Eratosthenes' sieve"?

51. How many and what prime numbers in the 2d ten numbers? In the 10th ten? In the 5th ten?

52. Give the primes less than 100 whose units' digit is 1. 3. 7. 9.

53. Find the sum of all the primes in the 3d ten. In the 6th ten. In the 4th ten. In the 7th ten.

54. Name all the prime numbers less than 100 whose tens' digit is 2. 4. 1. 3. 5. 7. 9. 6. 8.

NOTE TO TEACHER. This adaptation of "Eratosthenes' sieve" may be made helpful in studying composite numbers, divisors, and multiples.

Let the columns of numbers be written on the board in large figures. Instead of striking out multiples, draw a circle or square around each of them, using crayon of the same color to inclose the multiples of a particular number. If, for instance, the multiples of 2 are inclosed in blue, those of 3 in red, those of 5 in green, those of 7 in yellow, 30 shows itself at once by its motley framing as a multiple of 2, 3, and 5; 42 by its slightly different framing as a multiple of 2, 3, and 7; 66 as a multiple of 2, 3, and 11.

The following questions are based upon a diagram of that kind and refer to numbers less than 101.

55. Point out the multiples of 3 whose units' digit is 2. 3. 5. 6. 7. 8. 9. 4.

56. Point out the multiples of 7 whose units' digits is 1. 2. 4. 9.

57. What figure ends the expression of all the multiples of 5 that are odd numbers? Even numbers?

58. Point out all the numbers that are multiples of 2 and also of 5, beginning with the least.

59. Show the common multiples of 2 and 3, beginning with the least common multiple.

60. Show the common multiples of 2 and 7, beginning with the least common multiple.

61. Beginning with the least common multiple, show all the common multiples of 3 and 7. Of 3 and 5. Of 5 and 7. Of 2, 3, and 5. Of 2, 3, and 7. Of 2, 5, and 7.

62. Class Exercise. The teacher or a pupil pointing to a number in the diagram, members of the class tell of what numbers it is a common multiple, and whether or not it is their least common multiple.

PRIME FACTORS

63. Those factors of a number that are prime, are called **Prime Factors.** What are the prime factors of 4? Of 6? Of 8?

64. Find the prime factors of 42.

$$\begin{array}{r|l} 2 & 42 \\ 3 & 21 \\ \hline & 7 \end{array}$$

To find the prime factors of a number divide it by the smallest prime number of which it is a multiple. Then divide the quotient by the smallest prime number of which it is a multiple. Continue dividing until the quotient is prime. In this case 42 divided by 2 gives a quotient of 21, 21 divided by 3 gives the prime quotient 7. Hence the prime factors of 42 are 2, 3, and 7.

65. Find the prime factors of:

10	15	20	24	27	32	35	39	44	48
12	16	21	25	28	33	36	40	45	49
14	18	22	26	30	34	38	42	46	50

66. Find the prime factors of all the even numbers greater than 49 and less than 59. Of all the composite odd numbers between those limits.

67. Find the prime factors of all the even numbers between 59 and 69. Of all the composite odd numbers.

68. Find the prime factors of all the multiples of 5 between 69 and 91. Of all the multiples of 3 between those limits. Of all the composite numbers between 90 and 101.

69. Resolve into prime factors:

a 336	*c* 1225	*e* 639	*g* 3105	*i* 1470	*k* 1296
b 3456	*d* 2214	*f* 560	*h* 888	*j* 810	*l* 1488

70. Class Exercises. —— may give to the class a number that is the product of several small prime numbers, and the class may find its prime factors.

71. Separate the first 100 numbers into two lists, one of prime numbers, the other of composite numbers. Write opposite each composite number the prime factors of which it is composed.

Suggestion to Teacher. Class drill upon these lists should be given frequently until their contents are learned.

LEAST COMMON MULTIPLE

72. Forty contains how many more fives than eights?

73. Class Exercise. —— may name a number which is a common multiple of two or more numbers. The class may give the numbers and tell how many times their common multiple contains each of them.

74. The **Least Common Multiple** of two or more prime numbers is their product; the next common multiple is twice their product; the next is three times their product. What is the next? Write the first six common multiples of 2 and 5, and underscore the least.

75. Write the first four common multiples of 3 and 5. Give the least common multiple and find how many times 3 and 5 are each contained in it.

76. Write the first three common multiples of 2 and 7. Find how many times 2 and 7 are each contained in their least common multiple.

77. Find the least common multiple of 2, 3, and 5. How many threes does it contain? How many twos? How many fives?

78. Find the least common multiple of 2, 3, and 7. How many sevens in it? Twos? Threes?

79. The abbreviation for least common multiple is l.c.m. Find the l.c.m. of 2, 5, and 7. How many fives in it? Sevens? Twos?

80. Find the l.c.m. of 5 and 9. Use the following method of finding the l.c.m. mentally: Think of the multiples of the larger number in order until one is found which is a multiple of the smaller. For instance, in finding the l.c.m. of 5 and 9, think of the multiples 9, 18, 27, 36, until the first that contains 5 is reached.

81. Find the l.c.m. of:

a	10 and 3	10 and 5	10 and 8	10 and 12	10 and 15
b	6 and 9	4 and 9	7 and 9	11 and 9	8 and 9
c	8 and 3	8 and 5	8 and 6	8 and 10	8 and 11

82. Find the l.c.m. of 6, 5, and 3. Can a number be a multiple of 6, without being also a multiple of 3?

83. Find the l. c. m. of:

a	6, 2, and 3	10, 5, and 2	20, 10, and 5	7, 5, and 6
b	10, 5, and 6	8, 4, and 12	5, 10, and 15	8, 4, and 3
c	2, 3, 4, and 6	3, 4, 8, and 6	4, 8, and 7	3, 6, and 8

84. 12 is the l. c. m. of 2, 3, 6, and 4. Make similar statements about the numbers 18, 20, 24, 25, 30, 36, 35, and 48.

85. Which of the first 12 multiples of 3 are common multiples of 12 and 3?

86. Can you find the *greatest* common multiple of 3 and 4? Of any two other numbers? Explain.

SUGGESTION TO TEACHER. Some of the pupils may discover that the search for the greatest common multiple leads into infinity.

87. Draw a line 18 in. long, and show how many times a 3-in. line can be laid off upon it. How many times can a 6-in. line be laid off upon it?

88. How long is the shortest line that can be laid off into 2-in. lines or 7-in. lines? 3-in. lines or 7-in. lines? 7-in. lines or 5-in. lines? 5-in. lines or 11-in. lines?

89. How large is the smallest number that can be divided into groups of 2 and of 7? Into groups of 3 and of 7? Into groups of 7 and of 5? Into groups of 5 and of 11?

90. CLASS EXERCISE. —— may name a composite number, and others may show the different groups into which it may be separated.

91. A company of children were playing games. At first they played games which required them to be divided into groups of three. Afterwards they played in groups of four. Every child played all the time. What is the least number of children there could have been in the company?

SUGGESTION TO TEACHER. Illustrate, by grouping children, for the benefit of those pupils who cannot imagine clearly.

92. How many roses must a girl have, to be able to divide them into bunches of 3 roses or bunches of 5 roses? How many bunches may she have of 3 roses? Of 5 roses?

93. A teacher has just enough pupils to divide into groups of 7 pupils or groups of 4 pupils. How many pupils has she? How many groups of each kind can she have?

94. I have just enough books to be arranged on a number of shelves, 12 books on a shelf, or by using more shelves, 9 books on each shelf. How many books have I? How many shelves would be needed under the first arrangement? Under the second?

95. What is the least number of gallons that can be exactly measured by either of two casks, one holding 6 gal., the other 8 gal.? How many times can the smaller cask be filled by them? The larger cask?

96. What is the smallest sum of money that can be entirely spent in buying books at 15¢ apiece, or in buying books at 9¢ apiece? How many of each kind of books could be bought?

97. How long is the shortest piece of ribbon that can be cut without remainder into lengths of 2 yd., 3 yd., or 5 yd. each? How many lengths of each kind could be made?

98. What is the least number of bananas that a mother can exactly divide between her 2 sons, or among her 4 daughters, or among all her children? How many bananas would each child receive in each case?

NOTE TO TEACHER. Written methods of finding the l. c. m. and g. c. d. are useful, because convenient, but the reasons for the processes are beyond the comprehension of ordinary children in the grade for which this work is designed. Hence the methods should be presented as convenient rules that have been discovered by mathematicians. The reasons for these rules should be learned later.

99. By the following rule find the l. c. m. of 8 and 10:

To find the least common multiple of several numbers —

Arrange the numbers in a horizontal line, leaving out all numbers that are factors of any of the other numbers. Find the smallest prime number that will exactly divide any two of them, and divide by it all the numbers of which it is a factor, placing the quotients and undivided numbers below. Continue this process until no prime number will divide two of the numbers in the last horizontal line. Find the product of the divisors, quotients, and undivided numbers.

100. Find the l. c. m. of 12, 16, and 18.

SOLUTION.

2	12, 16, 18
2	6, 8, 9
3	3, 4, 9
	1, 4, 3

$2 \times 2 \times 3 \times 4 \times 3 = 144$ l. c. m.

101. Find by the written method the l. c. m. of:

a 25, 60, 72, 35 *c* 63, 12, 84, 72 *e* 54, 81, 14, 63

b 24, 16, 15, 20 *d* 16, 12, 24 *f* 15, 9, 6, 5

102. Find l. c. m. of 5, 6, 18, 15.

3)	5, 6, 18, 15
	6, 5

$3 \times 6 \times 5 = 90$. *Ans.*

Why may the 5 and 6 be crossed out and not considered in finding the l. c. m. of 5, 6, 18 and 15?

103. Find l. c. m. of:

a 1, 2, 3, 4, 5, 6, 7, 8, 9 *d* 4, 8, 12, 24, 48, 84

b 8, 12, 16, 24, 36, 48 *e* 5, 10, 15, 20, 30, 40

c 4, 3, 6, 7, 8, 16, 9, *f* 7, 28, 35, 14, 70

104. CLASS EXERCISE. —— may name four composite numbers, and the class may find their l. c. m.

105. CLASS EXERCISE. —— may name three prime numbers, and the class may find their l. c. m.

106. Numbers that are multiples of any given number are said to be divisible by that number. Is 7 divisible by 3? Name several numbers that are divisible by 10.

DIVISIBILITY OF NUMBERS

107. Illustrate the following principle:

PRINCIPLE 1. *A number that ends in* 2, 4, 6, 8, *or* 0 *is divisible by* 2.

108. Tell without dividing which of the following numbers are not divisible by 2, and what the remainder is in each case: 8906. 2127. 2139. 2111. 2145. 1898.

109. Name in order the first fourteen multiples of 5.

110. When a multiple of 5 is expressed in figures, what figures may represent the units' digit?

PRINCIPLE 2. *A number that ends in* 5 *or* 0 *is divisible by* 5.

111. Without dividing, select from the following the numbers that are not divisible by 5, and tell what the remainder is in each case: 75. 120. 81. 22500. 393. 920.

112. When a number is divisible by 2 and by 5, with what figure must its expression end? By what other number is it divisible?

113. Among all the prime numbers less than 100, can you find any the sum of whose digits is 9?

114. Write a number of two places the sum of whose digits is 9. Find how many times 9 is contained in that number.

115. Write a number of three places the sum of whose digits is 9, and find how many times 9 is contained in that number.

116. Write a number of four places the sum of whose digits is 9, and find how many times that number contains 9.

117. Write a number the sum of whose digits is 18, and see whether it contains 9 exactly.

118. Write numbers the sum of whose digits is 9 or some multiple of 9, and divide those numbers by 9 until you see the truth of the following principle:

PRINCIPLE 3. *Any number is divisible by* 9 *if the sum of its digits is a multiple of* 9.

119. Tell without dividing which of the following numbers are not divisible by 9, and give the remainder in each case: 2025. 105. 117. 112. 2114. 189. 207. 1026. 4154.

120. Write several large numbers of which 9 is a divisor.

121. Write ten multiples of 3 no one of which is less than 36. Find the sum of the digits of each of them, and see if that sum is a multiple of 3.

PRINCIPLE 4. *A number is divisible by* 3 *if the sum of its digits is a multiple of* 3.

122. Tell at sight which of the following numbers are not divisible by 3, and give the remainder in each case: 213. 411. 6951. 343. 1125.

123. Can you find a multiple of 9 that is not a multiple of 3? Name a multiple of 3 that is not a multiple of 9.

124. Would it be possible for a number to be a multiple of 10 and not a multiple of 2 and 5? Explain.

125. Choose numbers ending in 0, and show what factors they have besides 2, 5, and 10.

126. Can a number be divisible by 6 without being divisible by 3 and by 2?

127. Write an even number the sum of whose digits is divisible by 3. As the number is divisible by 3 and by 2, it is divisible by 6. Write three other numbers divisible by 6.

128. Write three numbers each of which is divisible by 3 and by 5. Find how many times each of them contains 15.

129. Write three numbers ending in 0 the sum of whose digits is divisible by 3. Find how many times each of them contains 30.

130. Write three numbers divisible by 2 and by 9. Divide each of them by 18.

131. Write three numbers divisible by 5 and by 9. What number between 40 and 50 is a factor of each of them? How can you tell?

132. Count by 4's to 100.

133. Add some multiples of 4 to 100, and see if the numbers thus obtained are divisible by 4.

134. Add to 100 some numbers that are not multiples of 4, and see if the resulting numbers are divisible by 4. Explain.

PRINCIPLE 5. *A number is divisible by* 4 *if the number expressed by its two right-hand figures is divisible by* 4.

135. Tell without dividing the whole number which of the following numbers are not divisible by 4, and give the remainder in each case: 127. 244. 365. 782. 728. 496. 338. 2672.

SUGGESTION TO TEACHER. Lead pupils to see that as 100 is a multiple of 4, any number of hundreds is a multiple of 4, and if there is added to any number of hundreds a number which is an aggregation of fours, the result will be a still greater aggregation of fours.

136. Write a number of four places. Let the number expressed by the two right-hand digits be a multiple of 4. Let the sum of the digits of the whole number be a multiple of 3. Find how many times 12 is contained in the number.

137. Write three numbers divisible by 4 and by 5. What multiple of 10 besides 10 is a factor of each of them? Explain.

138. Write three numbers divisible by 4 and by 9. What number between 30 and 40 is a factor of each of them? Explain.

139. How many eights in 1000?

140. To 1000 add 3 eights or 24. How many eights in the number thus formed?

141. Add to 1000 a number which is not a multiple of 8. Is the sum divisible by 8? Explain.

PRINCIPLE 6. *A number is divisible by* 8 *if the number expressed by its three right-hand figures is divisible by* 8.

142. Tell without dividing the whole number which of the following numbers are not divisible by 8, and give remainders: 3640. 5728. 9076. 4126. 5345. 1724. 8638. 1124. 10008.

143. Make some numbers which are divisible by 8 and by 5, and tell how you make them. With what figure do they end? What two multiples of 10 besides 10 are contained in each of them?

144. Tell how to compose numbers that are divisible by 8, and also by 9 and hence by 72.

COMMON DIVISORS

145. A number which is a factor of each of two or more numbers is called their **Common Divisor.** Illustrate.

146. Turn to the diagram on page 67 and find all the numbers in it of which 11 is a common divisor.

147. Name all the numbers less than 100 of which 8 is a common divisor. Give all the numbers less than 100 that have as a common divisor: 9. 10. 12. 6.

148. What common divisor have all even numbers?

149. Give a common divisor of 14, 21, 28, 35, 42, 49, 56, and 63.

150. Name three multiples of 11, and give a common divisor of them.

151. Make a list of sets of numbers that have one or more common divisors, and write the **Greatest Common Divisor** of each set.

SUGGESTION FOR CLASS EXERCISE. Let a pupil name two or more numbers that have a common divisor, and let the class discover the divisor.

152. What divisor is common to the 7th even number and the 11th odd number?

153. 2 is a common divisor of 10 and of 20. Is it the greatest common divisor of these two numbers?

What is the greatest common divisor of 24 and 36?

154. Give at sight the greatest common divisor of:

a 10, 20, and 40	*g* 70, 80, and 90
b 15, 30, and 45	*h* 60, 72, and 84
c 18, 27, and 45	*i* 63, 72, and 90
d 16 and 24	*j* 28, 32, 40, and 44
e 50, 75, and 100	*k* 15 and 25
f 25, 30, and 35	*l* 12, 18, and 30

155. A candy manufacturer filled some boxes with chocolates, and some others of the same size with bonbons. There were 24 lb. of chocolates and 28 lb. of bonbons. What is the largest number of pounds each box can contain?

156. A boy wishes to divide two ropes, one 42 ft. long, the other 56 ft., into pieces of equal length, each as long as possible. How long will each piece be after this division, and how many pieces will there be?

157. Mr. Allen has three strips of land. The first contains 10 acres, the next 12 acres, the next 14 acres. He wishes to lay them off into the largest possible equal lots. How many acres will there be in each lot, and into how many lots can each piece be divided?

158. The abbreviation for greatest common divisor is g. c. d. What is the g. c. d. of 35 and 65?

The g. c. d. of two numbers may be easily found by the following process of continued division. It is to be used with numbers which are so large that their divisors cannot be readily found by inspection.

159. By the following rule find the g. c. d. of 8 and 10:

To find the greatest common divisor of two numbers —

Divide the greater number by the less. If there is a remainder, use it as a divisor of the preceding divisor, and continue until there is no remainder. The last divisor is the greatest common divisor.

160. Find by continued division the g. c. d. of 49 and 168.

```
49)168(3
  147
   21)49(2
     42
      7)21(3
```

Using 49 as a divisor of 168, the quotient is 3 and remainder 21. Using 21 as a divisor of 49, the quotient is 2 and remainder 7. Using 7 as a divisor of 21, the division is exact, hence 7 is the last divisor, or the g. c. d. of 49 and 168.

161. Find by continued division the g. c. d. of the following:

a 24 and 132	*i* 77 and 847	*q* 198 and 252
b 36 and 120	*j* 18 and 243	*r* 176 and 242
c 35 and 105	*k* 96 and 224	*s* 361 and 431
d 49 and 140	*l* 85 and 187	*t* 288 and 536
e 64 and 480	*m* 125 and 175	*u* 84 and 154
f 72 and 252	*n* 105 and 195	*v* 189 and 405
g 30 and 735	*o* 135 and 245	*w* 960 and 204
h 44 and 242	*p* 795 and 1105	*x* 236 and 576

162. Class Exercise. Let —— compose two large numbers having a common divisor, and let the class find this common divisor by continued division.

POWERS AND ROOTS

163. The product obtained by multiplying a number by itself one or more times is called a **Power** of that number. Illustrate.

164. The product of two equal factors is the **Square** of each factor. 2×2 is expressed 2^2, and read 2 square, or 2 to the second power. Find values of: 1^2. 30^2. 50^2. 120^2. 15^2. 20^2.

165. Give quickly in order the first 12 numbers that are perfect squares. Learn them.

166. 9^2 equals how many times 3^2? 16^2 how many times 8^2? 8^2 equals how many times 2^2?

167. What two perfect squares less than 100 have 6 for their units' digit? 9? 4? 1?

168. Can you find a perfect square less than 100 whose tens' digit is 9? 7? 5? 3?

169. How much is $\frac{1}{10}$ of 8^2? Of 11^2?

170. Multiply 6^2 by the first prime number after 31.

171. The product of three equal factors is called the **Cube** of each factor. $2 \times 2 \times 2$ is expressed 2^3, and read 2 cube, or 2 to the third power. Find values of: 2^3. 3^3. 4^3. 5^3. 1^3.

172. Continue the following table through 12^3. Learn the table.

$$1^3 = 1$$
$$2^3 = 8$$

173. Name a perfect cube whose units' digit is: 1. 2. 3. 4. 5. 6. 7. 8. 9. 0.

174. Multiply the cube of 8 by .07. By .125.

175. Find 6% of: 4^3. 9^3. 7^3. 11^3. 50^3. 12^3. 60^3.

176. $\frac{8^3}{4^3} = ?$ $\frac{9^3}{3^3} = ?$ $\frac{12^3}{6^3} = ?$ $\frac{10^3}{5^3} = ?$ $\frac{12^3}{4^3} = ?$

SUGGESTION TO TEACHER. By the following work lead pupils to discover the relations between a solid that is a cube and the third power of the number that measures one of its dimensions. Inch cubes should be used in this work until pupils are able to image the solids clearly without them.

177. How many cubic inches does a 2-inch cube contain? A 3-inch cube? A 4-inch cube? A 5-inch cube? A 6-inch cube? A 7-inch cube? An 8-inch cube? A 9-inch cube?

178. One of the boys may draw a square yard on the floor in one corner of the room. How many cubic blocks 1 ft. in dimensions would cover the square yard?

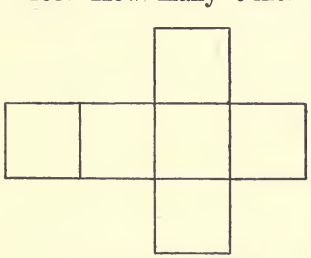
179. One of the girls may show how high she thinks the blocks must be piled to make a cubic yard. Another member of the class may measure with the yard stick and see how nearly right she is.

180. How many cubic feet in the lowest layer of blocks? How many layers would it take to make a cubic yard? How many cubic feet in a cubic yard?

181. How many layers of inch cubes would be required to cover a square foot? How many layers of the inch cubes to make a cubic foot? How many inch cubes in a cubic foot?

182. Into how many 2-inch cubes can a 4-inch cube be divided?

183. How many 6-inch cubes can be packed into a box whose inside dimensions are each 1 ft.?



6 Square Centimeters
FIG. 2.

184. Copy Fig. 2 on paper or pasteboard, making each square 1 sq. cm. Cut out the copy and fold and fasten it so that it will inclose a cubic centimeter. How many such cubes would be required to cover a square decimeter? How many layers of them would be required to make a cubic decimeter?

185. A cubic decimeter is called a **Liter.** How many cubic centimeters does it contain?

186. Copy Fig. 2 on paper or pasteboard, making each square 1 sq. dm. Cut, fold, and fasten to make a cubic decimeter or a liter.

SUGGESTION TO TEACHER. The most perfect liters and cubic centimeters made by the children should be kept as a part of the school apparatus.

187. How many square centimeters in all the surfaces of a liter?

188. In the metric system the liter is the measure that corresponds very nearly to the quart in the English liquid measure. How much will 12 liters of oil cost at 65 cents per liter?

189. Place a cubic centimeter upon a cube that holds a liter. How many cubic centimeters in the figure thus formed? How many square centimeters in all its surfaces?

SUGGESTION TO TEACHER. It should be explained that a liter is a unit of measure, and not a fixed form.

190. How many liters will be contained in a box that is 3 dm. long, 2 dm. wide, and 5 dm. high?

191. About how many liters of wheat can be put into a peck measure? (A bushel equals about 35 liters.)

192. $2 \times 2 \times 2 \times 2$ is 2^4, which is read 2 to the fourth power. Raise to the fourth power each of the first five numbers.

193. Raise to the fifth power each of the first three odd numbers.

194. Raise to the sixth power each of the first two even numbers.

195. Raise 10 to the seventh power.

196. Give the number whose prime factors are 2, 2, and 3. 2, 3, and 3. 2, 5, and 5.

197. Find values of x.

a $x = 3^2 \times 5$	e $x = 2^5 \times 3$	i $x = 7^2 \times 13$
b $x = 2^2 \times 7$	f $x = 2^6 \times 5$	j $x = 2^3 \times 5 \times 11^2$
c $x = 2^3 \times 3^4$	g $x = 3^3 \times 11$	k $x = 2^5 \times 3 \times 5^2$
d $x = 2^4 \times 3^3$	h $x = 5^3 \times 11$	l $x = 3^4 \times 5^2 \times 7$

198. Each of the two equal factors that compose a perfect square is called a **Square Root** of that number. Give the square root of 9. 25. 16.

199. $\sqrt{}$ is used as the sign of square root. $\sqrt{16} = ?$ $\sqrt{81} = ?$ $\sqrt{144} = ?$ $\sqrt{121} = ?$ $\sqrt{4} = ?$ $\sqrt{64} = ?$ $\sqrt{100} = ?$

200. Divide .63 by $\sqrt{49}$. By $\sqrt{9}$. By $\sqrt{81}$.

201. Divide .36 by $\sqrt{9}$. By $\sqrt{16}$. By $\sqrt{4}$. By $\sqrt{81}$.

202. Divide 55.44 by $\sqrt{81}$. By $\sqrt{36}$. By $\sqrt{121}$. By $\sqrt{49}$. By $\sqrt{64}$. By $\sqrt{16}$. By $\sqrt{9}$. By $\sqrt{144}$.

203. How long is one side of a square whose area is 9 sq. ft.? 49 sq. ft.? 100 sq. ft.? 81 sq. ft.? 25 sq. ft.?

204. How much is 50% of $\sqrt{100}$? 25% of $\sqrt{64}$? 75% of $\sqrt{64}$? 25% of $\sqrt{144}$?

205. Multiply $\sqrt{144}$ by .3. By .05. By .007.

206. Find 6% of $\sqrt{144}$. Of $\sqrt{64}$. Of $\sqrt{81}$.

207. How much is 3 times $\sqrt{9}$? 4 times $\sqrt{9}$?

208. "$5\sqrt{9}$" is read "5 times the square root of 9." Find the value of the expression.

209. $7\sqrt{4} = ?$ $6\sqrt{25} = ?$ $8\sqrt{36} = ?$ $2\sqrt{49} = ?$ $4\sqrt{25} = ?$

210. $3\sqrt{16} = ?$ $4\sqrt{100} = ?$ $2\sqrt{81} = ?$ $6\sqrt{64} = ?$ $3\sqrt{144} = ?$

211. How long is the perimeter of a square containing 49 sq. in.?

212. At $1.25 per rod, how much will it cost to fence a square lot containing 25 sq. rd.? Represent.

SUGGESTION TO TEACHER. For oral "quick work" exercises similar to the following are useful: "Think of the 3d multiple of 6, subtract 2, take the square root, add 1, square, add 5, take $\frac{1}{3}$, take $\frac{1}{10}$, add 1, square, add 5, take square root." Allow children to lead the work, letting them prepare their numbers beforehand to read to the class, until they are able to extemporize.

213. Each of the three equal factors that compose a number that is a perfect cube is called a **Cube Root** of that number. Give the cube root of: 8. 512. 64. 1000. 729. 1331. 1728. 125. 216. 343.

214. Give the cube root of a perfect cube whose units' digit is: 1. 2. 3. 4. 5. 6. 7. 8. 9. 0.

215. How long is one side of a cube that contains 8 cu. in.? 27 cu. in.? 1728 cu. in.? 64 cu. in.? 729 cu. in.? 1000 cu. in.? 512 cu. in.? 216 cu. in.? 125 cu. in.? 343 cu. in.?

216. $\sqrt[3]{}$ is read "The cube root of." How much is $\sqrt[3]{64}$? $\sqrt[3]{343}$? $\sqrt[3]{729}$? $\sqrt[3]{1728}$?

217. Multiply the $\sqrt[3]{8}$ by the first prime number after 40

218. Multiply $\sqrt[3]{125}$ by .001. $\sqrt[3]{512}$ by .75.

219. Find 50% of $\sqrt[3]{64}$. Of $\sqrt[3]{1728}$.

220. $\sqrt[3]{729}-\sqrt[3]{216}=?$ $\sqrt[3]{1000}\div\sqrt[3]{125}=?$ $\sqrt[3]{1331}\times\sqrt[3]{343}=?$

MISCELLANEOUS EXERCISES

1. Write in decimal form and add: 1 ten-thousandth, $\frac{3}{10000}$, 1 hundred-thousandth, $\frac{111}{10000}$, $\frac{41}{1000}$, 1 millionth, $\frac{23}{1000000}$, $\frac{9}{100000}$.

2. If you have a string a foot long and cut one inch from each end, how long is the string that is left?

3. When a line 3.4 ft. long is cut from each end of a line that is 1 rd. or 16.5 ft. long, how long is the line that is left?

4. The diagonal of a certain schoolroom is 35.1 ft. John makes a mark on the diagonal 7 ft. from one corner, and James makes a mark 9 ft. from the opposite corner. If each boy stands at the mark he has drawn, how far apart are they?

SUGGESTION TO TEACHER. In most classes there are some pupils who fail to visualize. Select two of these to take the parts of John and James in illustrating this and similar problems.

5. Find the sum of 81.375 and the prime number nearest to 24.

6. Find the difference between 21.84 and the largest prime that can be expressed by two digits.

7. When a decimal of 3 places is multiplied by an integer, how many decimal places should be pointed off in the product? Illustrate.

8. When an integer is multiplied by a decimal of 2 places, how many decimal places should be pointed off in the product? Illustrate.

9. How many decimal places in the square of .007?

10. $8.283 \div 3 = ?$ $45.6 \div 12 = ?$ $.286 \div 22 = ?$

11. Tell how you divide a decimal by a decimal.

12. $.12 \div .4 = ?$ $.15 \div .005 = ?$ $.75 \div .5 = ?$ $.84 \div .12 = ?$

13. When one decimal is divided by another decimal of the same denomination, how many decimal places are there in the quotient?

14. At \$.0 per pound, how many pounds of sugar can be bought for \$.45? For \$.75? \$1.25? \$2.50? \$8?

15. H riet has some money in the bank, the interest of which is \$1.30 every year. How long must the money stay in the bank that the interest may be \$5.85?

16. $1.728 \div .0012 = ?$

17. Give the prime factors of the first odd composite number after 81.

18. The largest prime factor of 66 is how many times the smallest prime factor of 66?

19. Find the l. c. m. of 3, 8, 4, 9, 6, 12.

20. Find the g. c. d. of 44 and 66. Of 128 and 144.

21. Divide 7235.2 by the l. c. m. of 4 and 7.

22. Divide 4.725 by the g. c. d. of 45 and 105.

23. Find by cancellation the value of x:

a $\quad \frac{7 \times 8 \times 4}{14 \times 32} = x$

b $\quad \frac{3 \times 7 \times 9}{21 \times 18 \times 5} = x$

c $\quad \frac{64 \times 21}{42 \times 8 \times 8} = x$

d $\quad \frac{25 \times 21}{35 \times 30} = x$

e $\quad \frac{48 \times 63}{36 \times 24 \times 18} = x$

f $\quad \frac{16 \times 25 \times 36}{200 \times 18 \times 6} = x$

g $\quad \frac{49 \times 63}{21 \times 84} = x$

h $\quad \frac{48}{24 \times 36} = x$

i $\quad \frac{56}{21 \times 16} = x$

24. What is one of the two equal factors of 121?

25. Name a perfect square whose units' digit is 9, and give its square root.

26. Give quickly the first 12 numbers that are perfect cubes.

27. How long is one edge of a cube that contains 1000 cu. in.? 1728 cu. in.?

28. Give one of the three equal factors of 216. Of 729.

29. Find the difference between 0^2 and 1^2. 1^2 and 2^2. 2^2 and 3^2. 3^2 and 4^2. 4^2 and 5^2. 5^2 and 6^2. 6^2 and 7^2. 7^2 and 8^2. 8^2 and 9^2.

30. Write these differences in a column and tell whether they are even numbers or odd numbers.

31. Find the sum of the first 7 odd numbers. Compare that sum with the square of 7.

32. Compare the sum of the first 8 odd numbers with the square of 8. The sum of the first 5 odd numbers with the square of 5. Of the first 9 odd numbers with the square of 9.

33. Find the sum of the first 5 even numbers. Subtract the square of 5 from that sum.

34. Find how much the sum of the first 7 even numbers exceeds the square of 7.

35. Take Ex. 34, substituting other numbers for 7.

36. Ella's record on an arithmetic test was 75%. What fractional part of her work was right and what part wrong?

37. Mr. Hudson had $8000 in bank and took out 20% of it. How much did he take out? How much had he left?

38. Edward buys oranges at the rate of 4 for 25¢, which is just one half of what he receives for them. What is the selling price of each?

39. A milkman's horse ran away with a wagon containing 4 gal. of milk, and 25% of it was spilled. How many quarts of milk were spilled? If the milk was worth 6¢ per quart, what money value was lost?

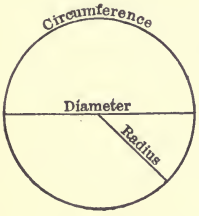


FIG. 3

40. From a liter of oil 13% was spilled. How many cubic centimeters of oil remained?

41. Draw a circle. What is a radius? Diameter? Circumference? The radius of a circle equals what part of the diameter?

42. How long is the diameter of a circle whose radius is 5 in.? $3\frac{1}{2}$ in.? 7.5 in.?

43. What is the diameter of the largest circle that can be cut from a piece of paper 3 in. square?

44. The surface passed over in 1 hr. by the minute hand of a clock is what figure? The minute hand of a clock in a tower is $2\frac{1}{2}$ ft. long. How long is the diameter of the circle it passes over every hour?

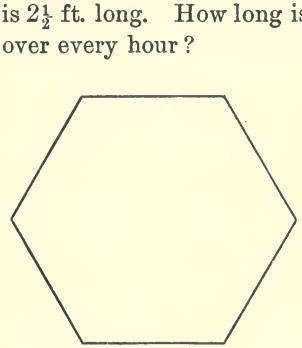


Fig. 4.

45. A plane figure bounded by six straight lines is called a **Hexagon.** When the sides are all equal, and the angles are all equal, as in Fig. 4, the hexagon is called a **Regular Hexagon.** What kind of angles has a regular hexagon?

46. If each side of a regular hexagon is 6.75 in. long, how long is the perimeter of the hexagon?

47. Draw a hexagon that is not regular.

Suggestion to Teacher. Show the following method of drawing a regular hexagon: Draw a circle with a radius of any convenient length. Beginning at any point of the circumference, lay off the radius as a chord six times consecutively. Erase the circle.

48. Draw a regular hexagon whose sides are each 3 in. long. How long is the perimeter? How does the side of a regular hexagon compare with the radius of the circle in which it is inscribed?

49. By drawing diagonals the regular hexagon may be divided into 6 equilateral triangles. Draw them, and find how long each diagonal is. How long is the perimeter of each equilateral triangle?

50. If the perimeter of the hexagon were 32.4 in., how long would the perimeter of each equilateral triangle be?

51. How many triangles in 50% of the hexagon?

52. A plane figure bounded by four equal straight lines, and having no right angles, is called a **Rhombus.** What kind of angles has a rhombus?

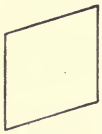


FIG. 5.

53. Draw a rhombus by the following method:

Draw the line AB of any convenient length. With AB as a base construct an isosceles triangle CAB, making AC greater than $\frac{3}{4}$ of AB. With AB as a base construct an isosceles triangle ADB, making AD equal to AC. Erase the construction line AB. (A construction line is a line forming no part of a figure, but used simply to help in its construction.)

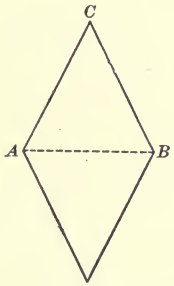


FIG. 6.

54. Construct a rhombus each of whose sides is 5 in.

55. Mr. Jones laid out a flower bed in the shape of a rhombus, each side of which was 4.75 ft. long. How long was the entire edge of the flower bed?

56. If the entire edge had been 28.8 ft. long, how long would have been one side of the flower bed?

57. Draw a rhombus and the long diagonal of the rhombus. Into what kind of triangles is the rhombus divided?

58. If the side of a rhombus is 7.5 in., and its longer diagonal is 10.875 in., how long is the perimeter of one of the triangles into which the long diagonal cuts the rhombus?

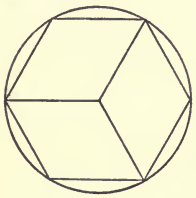


FIG. 7.

59. Draw a circle and inscribe a hexagon. Join the vertex of each alternate angle with the center of the circle. Into what kind of figures is the hexagon divided? How long would the perimeter of each of the figures be, if the radius of the circle were 8 cm.? 12 cm.?

60. Each rhombus is what fractional part of the hexagon?

61. Finish the following course of reasoning:

Since the whole of anything equals 100% of it,

$\frac{1}{3} = 33\frac{1}{3}\%$ of it.

$\frac{2}{3} = —\%$ of it.

62. Write in each rhombus the per cent which it is of the hexagon. Shade one rhombus and tell what per cent of the hexagon is unshaded.

63. How much is $33\frac{1}{3}\%$ of 12? 21? 24? 30? 45? 48?

64. How much is $66\frac{2}{3}\%$ of 15? 27? 18? 36? 33? 6?

65. $33\frac{1}{3}\%$ of a school of 48 pupils are boys. How many girls are there?

66. How much is $33\frac{1}{3}\%$ more than \$15? \$300? \$600?

67. How much is $66\frac{2}{3}\%$ more than \$900? \$1200? \$1800?

68. \$3000 − $66\frac{2}{3}\%$ of 3000 = ? 2100 − $66\frac{2}{3}\%$ of 2100 = ?

69. Each of the equal sides of an isosceles triangle is $33\frac{1}{3}\%$ longer than the base, which is 15 in. long. How long is the perimeter of the triangle?

70. A merchant found that some of his goods were shopworn and marked them at a reduction of 25% of their cost. How were goods marked that cost 12¢? 20¢? 40¢? \$1.00? \$1.60? \$10.00?

71. Find the selling price of goods marked at the following prices, which are to be reduced in price $33\frac{1}{3}\%$ on account of being out of style. Cloaks at \$7.50, bead trimming at \$1.50 per yard, lace ruffling \$.57 per yard.

72. A grocer bought goods at the following prices. For how much must they be sold to gain $33\frac{1}{3}\%$? 25%?

a Tomatoes @ 12¢ per pound.

b Raisins @ 6¢ per pound.

c Molasses @ 36¢ a gallon.

d Oranges @ 24¢ a dozen.

e Bananas @ 18¢ a dozen.

f Potatoes @ 30¢ a bushel.

73. At the end of a season a merchant decided to reduce prices $33\frac{1}{3}\%$ on all of the following goods whose prices were over \$1.00 and to reduce them $66\frac{2}{3}\%$ on all those whose prices were less than a dollar. Find the new selling prices.

a Lace @ \$1.80 per yard.
b Ribbon @ \$.75 per yard.
c Calico @ \$.06 per yard.
d Silk @ \$2.70 per yard.
e Velvet @ \$1.68 per yard.
f Alpaca @ \$.60 per yard.

74. William is 15 years old. His age is $33\frac{1}{3}\%$ of his father's age. How old is his father?

75. Mr. Gage had \$396.66 in a bank and took out $33\frac{1}{3}\%$ of it. How much remained in the bank?

76. Mrs. Wallace lent Mr. Brown \$1200 until the interest amounted to $66\frac{2}{3}\%$ of the principal. How much was the interest? How much did Mr. Brown then owe, including principal and interest?

77. Write the following fractions in a column and opposite to each its value in %: $\frac{1}{2}$. $\frac{1}{3}$. $\frac{2}{3}$. $\frac{1}{4}$. $\frac{3}{4}$. $\frac{1}{5}$. $\frac{2}{5}$. $\frac{3}{5}$. $\frac{4}{5}$.

78. How many minutes in $33\frac{1}{3}\%$ of an hour? In $66\frac{2}{3}\%$? 25%? 20%? 40%? 50%?

79. How many hours in 50% of the time from 9 A.M. Monday to 9 A.M. Tuesday? In $33\frac{1}{3}\%$ of it? In 75%? 20%?

80. How many square centimeters in 80% of a square decimeter? In 25%? In $33\frac{1}{3}\%$?

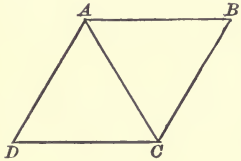


FIG. 8.

81. Draw the equilateral triangle *ADC*, one of whose sides represents **3 in.** With *AC* as a base line construct another equilateral triangle *ACB*. Erase *AC*. What kind of a figure is *ABCD*? How long is its perimeter?

82. With either side of the rhombus as a base line, construct another equilateral triangle. Erase the base line. What kind of a figure have you drawn? How long is its perimeter?

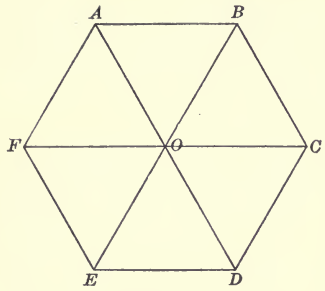


FIG. 9.

83. Continue adding equilateral triangles until you have a regular hexagon.

84. Complete the following reasoning. Since the whole of anything $= 100\%$,

$\frac{1}{6}$ of it $= 16\frac{2}{3}\%$

$\frac{5}{6}$ of it $= —\%$

85. On your copy of **Fig. 9** write in each equilateral triangle the % which it is of the hexagon.

86. If $16\frac{2}{3}\%$ of the hexagon were shaded, what % of it would be unshaded?

87. What % of the hexagon is the figure *AOCB*? *AODCB*? *ABCDEFO*? *CDEFAO*?

88. How long is the perimeter of the six-pointed star represented by Fig. 10 if each side is 3.5 in.? What kind of angles are those whose vertices are at the points of the star?

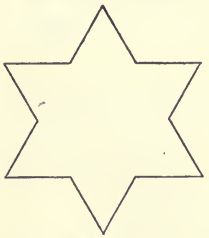


FIG. 10.

89. Make a six-pointed star.

Draw a regular hexagon, and construct an equilateral triangle upon each of its sides. Erase the sides of the original hexagon. The star may also be made by prolonging the sides of the hexagon until they meet.

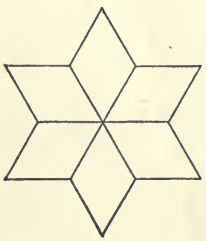


FIG. 11.

90. Divide your star into 6 equal rhombuses. Write in each rhombus the % which it is of the star.

91. Put a letter at the center and one at each angle of your copy of Fig. 11 and tell what figure is $33\frac{1}{3}\%$ of it. $83\frac{1}{3}\%$. 50%. $66\frac{2}{3}\%$.

92. What is $16\frac{2}{3}\%$ of 12? Of 24? Of 72? Of 84? Of 120? Of 144?

93. What is $83\frac{1}{3}\%$ of 18? Of 30? Of 48? Of 66? Of 144?

94. To make a profit of $16\frac{2}{3}\%$ for what price must goods be sold that cost 6¢? 18¢? 15¢? 30¢? 54¢? 60¢?

95. What must be the selling price of the same goods to allow a profit of $83\frac{1}{3}\%$?

96. Select from the following list the per cents which are most easily used by reducing them to common fractions in their lowest terms, and give the equivalent fractions:

$33\frac{1}{3}\%$ 11% $16\frac{2}{3}\%$ 25% $83\frac{1}{3}\%$ 50% 17%
20% 3% 40% 75% 9% $66\frac{2}{3}\%$

97. James had a dollar and lost 17 cents. What per cent of his money was left?

98. Mary has only a dollar. Can she lose 101% of it? Explain.

99. 20 equals what part of 30? Express it in per cent.

100. Class Exercise. —— may give a number, and the class may give $33\frac{1}{3}\%$ of it. $16\frac{2}{3}\%$. $66\frac{2}{3}\%$. $83\frac{1}{3}\%$.

101. Class Exercise. —— may give a number, and the class may give the number of which his number is $16\frac{2}{3}\%$. $33\frac{1}{3}\%$. 25%.

102. Draw a right triangle whose base is 3 in. and perpendicular 4 in. If your drawing is correct, the hypotenuse will be 5 in. Each side of the triangle equals what part of its perimeter?

103. Draw a right triangle whose base is 6 in. and perpendicular 8 in. Its hypotenuse is just twice as long as the hypotenuse of the triangle given in Ex. 102. Each of its sides equals what part of its perimeter?

104. Draw a square 3 in. in dimensions. If you drew a larger square, having each of its sides 1 in. from the corresponding side of the first square, how long would its perimeter be?

105. Separate the following into two lists, one of odd numbers, the other of even numbers. How many are there of each?

874; MDCCCLXXXVIII; the square of 7; the fifth multiple of 4; the product of 7 and 8; the quotient of 84 divided by 2; the difference between 81 and 18; the sum of 85 and 37; the largest number that can be expressed by two figures; the largest factor of 12 except itself; the number that is 5 greater than 212; the largest number that can be expressed by three figures; the smallest number that can be expressed by three figures; the number that means a dozen; the number that tells how many days in May; the integer between 17,345 and 17,347; one of the equal factors of 25; the factor that helps 7 to make 77; the square root of 100; the number that shows how many quarts in a peck; the denominator of the fraction $\frac{3}{14}$; the greatest common divisor of 6 and 8; the remainder after dividing 25 by 11; the smallest multiple of 7 that will contain 5; the least common multiple of 4 and 7; the number that is just half way between 30 and 50; the smallest prime number greater than 25; the largest prime number less than 25; the numerator of the fraction $\frac{22}{81}$; the number that tells how many square inches in a square foot; the number that is just as much less than 15 as it is greater than 11; the average of 19, 20, and 21; the first composite number; the number that shows how many pounds in a ton; the number that shows how many cubic inches in a cubic foot; the number that shows how many sides a pentagon has; the largest prime number that can be written with two figures; the smallest prime number that can be written with three figures; the quotient of 13.14 divided by .06; 50% of 862; the largest prime factor of 102; the number that shows how many millimeters in a meter.

CHAPTER III

RATIO

SUGGESTION TO TEACHER. Review ratio as given in Hornbrook's "Primary Arithmetic." See notes on pp. 117 and 118 and tables on pp. 145, 160, 174, 183, of that book.

1. A 3-inch line equals what part of a 4-inch line, or what is the ratio of a 3-inch line to a 4-inch line?

2. What is the ratio of a pint to a quart? Of a quart to a gallon? Of an inch to a foot? Of a foot to a yard? Of 2 ft. to a yard? Of an ounce to a pound? Of 8 oz. to a pound? Of 15 oz. to a pound?

3. CLASS EXERCISE. —— may name a number less than 100, and the class may give its ratio to 100.

4. 6 is how many times 3, or what is the ratio of 6 to 3?

5. What is the ratio of a yard to a foot? Of a foot to an inch? Of a foot to 3 in.? Of a foot to 6 in.? Of a foot to 7 in.? 9 in.? 11 in.?

6. CLASS EXERCISE. —— may name some number greater than 10, and the class may give its ratio to 10.

7. The ratio of two numbers is the quotient of the first of those numbers divided by the second. Thus the ratio of 10 to 5 is $10 \div 5$, or 2. The ratio of 7 to 5 is $7 \div 5$, or $1\frac{2}{5}$. What is the ratio of 4 to 5?

8. Draw a rectangle 4 in. long and 1 in. wide. A rectangle 3 in. long and 1 in. wide equals how many fourths of the first rectangle? A rectangle 8 in. long and 1 in. wide equals how many fourths of the first rectangle? What do $\frac{8}{4}$ equal?

The ratios indicated by "parts" and "times" are really of the same kind. They both express the quotient of one quantity divided by another of the same kind.

9. What is the ratio of a second to a minute? Of a year to a month?

10. Build from inch cubes or draw right prisms like the following:

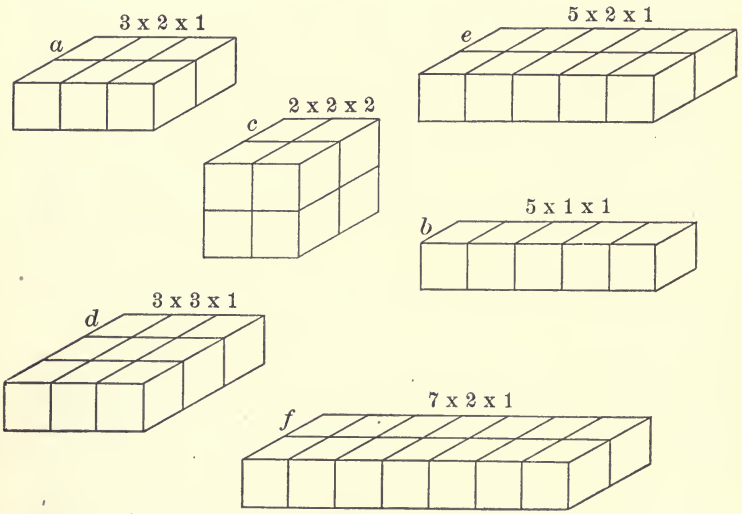


FIG. 1.

11. Find ratios of:

a to b	a to c	b to e	f to d	d to c
b to c	b to d	e to d	c to f	f to a
c to b	c to d	b to f	c to e	a to e

12. Mr. Jones works every day from 8 until 12 o'clock, and from 1 until 5 o'clock. At 9 o'clock in the morning, what is the ratio of the work he has done to the work he still has to

do that day? What is the ratio of the work he has done to his whole day's work?

13. At ten o'clock, what is the ratio of the work he has done to a day's work? At 12 o'clock? At 1 o'clock? At 3 o'clock? At 5 o'clock?

14. What is the ratio of a rod to a mile?

15. If your home is a mile from the schoolhouse, how many rods must you travel each school day of two sessions, if you go home at noon?

16. Joseph rode a mile on his bicycle. When he had ridden a rod, what was the ratio of the distance he had ridden to that which he afterward rode?

17. Ella walked to the home of her cousin, who lived a mile away. What was the ratio of the distance she had walked to the remaining distance after she had gone 16 rd.? 32 rd.? 80 rd.? 120 rd.?

18. What is the ratio of an ounce to a pound?

19. Margaret had half a pound of candy and gave away all of it except one ounce. What was the ratio of what she had left to what she had at first?

20. What is the ratio of a pound to a ton?

21. Just after a ton of hay was weighed in market, a horse ate one pound of it. What was the ratio of what he ate to what was left?

22. Ratio is expressed by a colon. Give ratios of: 15 : 3. 3 : 15. 16 : 2. 2 : 16. 3 : 18. 18 : 3. 5 : 20.

23. Give quickly the ratio of 2 to each of the first ten multiples of 2. Give the ratio of the 2d multiple of 2 to each of the first ten multiples of 2. Do the same with the 3d multiple of 2. With the 4th, 5th, 6th, 7th, 8th, 9th, and 10th.

24. What is the ratio of the 2d multiple of any number to its 3d multiple? Of its 2d multiple to its 4th? Illustrate.

25. What is the ratio of 2471 to 17?

26. What is the ratio of 1.422 to 1.8?

27. The ratio of 3 to 6 is $\frac{1}{2}$; the ratio of 6 to 3 is 2. These two ratios between the numbers 3 and 6 are called reciprocal ratios. Give the reciprocal ratios between the following numbers: 2 and 3. 3 and 5. 8 and 4. 9 and 12. 18 and 20.

28. 19 equals how many twentieths of 20? 20 equals how many nineteenths of 19?

29. John is 8 yr. old, and his sister Mary is 16 yr. old. What is the ratio of Mary's age to John's? Of John's age to Mary's age?

30. When is the ratio of one number to another number an integer? Illustrate.



FIG. 2.

31. The line AB represents a distance of 35 mi. divided into 5 equal parts. How much is the distance AE? CB? AF? EB?

32. Find the ratio of AF to AB. CF to CB. AB to AD. AB to DB. AD to CF. AB to CF.

33. What is the ratio of the first composite number after 19 to the first composite number after 29? Of the first odd number after 20 to the first odd number after 5? Of the first prime number after 7 to the first even number after 20?

34. Draw a 2-inch square and a 4-inch square and find the ratio of each square to the other.

35. Find the reciprocal ratios of a 3-inch square and a 4-inch square. Of an 8-inch square and a 6-inch square.

36. What is the ratio of $3\sqrt{4}$ to $2\sqrt{25}$? $2\sqrt{9} : 6\sqrt{16} = ?$

37. $2\sqrt{49} : 3\sqrt{121} = ?$ $3\sqrt{36} : 2\sqrt{144} = ?$

38. $5\sqrt{16} : 4\sqrt{9} = ?$ $2\sqrt{81} : 3\sqrt{64} = ?$

39. $3\sqrt{100} : 5\sqrt{16} = ?$ $2\sqrt{25} : 5\sqrt{36} = ?$

40. Build with cubes and find the following ratios: An inch cube to a 3-inch cube. An inch cube to a 4-inch cube. A 2-inch cube to a 3-inch cube.

41. If a 5-inch cube is cut into inch cubes, what is the ratio of one of the small cubes to the large cube?

42. What is the ratio of a cube an inch in dimensions to a 6-inch cube?

43. $3^3 : 4^3 = ?$ $40^3 : 60^3 = ?$ $11^3 : 9^3 = ?$ $6^3 : 20^3 = ?$

44. Give quickly the cube root of: 27. 64. 8. 216. 512. 729. 343. 1000. 125.

45. $\sqrt[3]{125} : \sqrt[3]{1000} = ?$ $\sqrt[3]{729} : \sqrt[3]{216} = ?$ $\sqrt[3]{1728} : \sqrt[3]{512} = ?$

46. Image the following figures and tell the ratio of one side of each figure to its perimeter. A square. A regular hexagon. A rhombus. A regular pentagon. Express the ratios in %.

47. What is the ratio of the perimeter of a square yard to the perimeter of a square foot? Of the perimeter of a square centimeter to the perimeter of a square decimeter?

48. What number is that whose ratio to 8 is $\frac{3}{4}$? Or what is $\frac{3}{4}$ of 8?

49. How much is $\frac{2}{3}$ of 12? $\frac{5}{7}$ of 49? $\frac{3}{5}$ of 15? $\frac{4}{7}$ of 21?

SUGGESTION TO TEACHER. In order to insure correct reasoning on the part of pupils, they should occasionally be required to explain the steps by which they arrive at results, as: since $\frac{1}{7}$ of 21 is 3, $\frac{4}{7}$ of 21 are 4 times 3 or 12. After this is thoroughly understood, the habit of mental cancellation should be encouraged. For instance, in finding $\frac{4}{7}$ of 21, children may be led to visualize the expression and mentally to cancel the terms.

50. Find values:

$\frac{5}{6}$ of 36 $\frac{7}{8}$ of 56 $\frac{7}{9}$ of 63 $\frac{8}{11}$ of 77 $\frac{7}{9}$ of 72 $\frac{8}{9}$ of 36

51. If a boy earns \$ 77 in 11 wk., how much would he earn in 3 wk.? 5 wk.? 9 wk.?

52. When 9 yd. of calico cost 72¢, what is the cost of 2 yd.? 5 yd.? 7 yd.? 8 yd.?

53. Goods that cost 8¢ a yd. are sold for $\frac{3}{4}$ of their cost. What is the selling price?

54. Give quickly the selling price of goods:

a Bought at \$ 0.12 and sold at $\frac{5}{6}$ of the cost.

b Bought at \$ 0.18 and sold at $\frac{1}{6}$ more than cost.

c Bought at \$ 0.20 and sold at $\frac{1}{4}$ more than cost.

d Bought at \$ 0.40 and sold at $\frac{1}{4}$ more than cost.

e Bought at \$ 0.50 and sold at $\frac{1}{5}$ more than cost.

f Bought at \$ 0.60 and sold at $\frac{1}{4}$ more than cost.

g Bought at \$ 0.80 and sold at $\frac{1}{4}$ less than cost.

h Bought at \$ 1.00 and sold at $\frac{3}{4}$ of the cost.

i Bought at \$ 0.40 and sold at $\frac{3}{4}$ of the cost.

SUGGESTION TO TEACHER. Before the following work is taken up, pupils should be drilled in finding reciprocal ratios of pairs of numbers until they readily see the truth of the first statement in the solution of Ex. 55.

55. 6 is $\frac{3}{4}$ of what number?

SOLUTION. 6 is $\frac{3}{4}$ of the number that is $\frac{1}{4}$ of 6. $\frac{1}{3}$ of 6 is 2. $\frac{1}{4}$ of 6 are 8.

56. 8 is $\frac{4}{5}$ of? $\frac{2}{3}$ of? $\frac{4}{7}$ of? $\frac{4}{9}$ of?

57. 12 is $\frac{6}{5}$ of? $\frac{4}{3}$ of? $\frac{3}{2}$ of?

58. Find the values of x.

a $10 = \frac{2}{3}$ of x. $\frac{5}{7}$ of x. $\frac{2}{3}$ of x. $\frac{5}{8}$ of x. $\frac{5}{8}$ of x.

b $12 = \frac{2}{3}$ of x. $\frac{3}{4}$ of x. $\frac{6}{7}$ of x. $\frac{4}{9}$ of x. $\frac{3}{7}$ of x. $\frac{2}{11}$ of x.

c $15 = \frac{3}{4}$ of x. $\frac{3}{7}$ of x. $\frac{3}{8}$ of x. $\frac{5}{8}$ of x. $\frac{5}{4}$ of x. $\frac{5}{9}$ of x.

d $18 = \frac{2}{7}$ of x. $\frac{3}{10}$ of x. $\frac{6}{11}$ of x. $\frac{9}{13}$ of x. $\frac{2}{5}$ of x. $\frac{9}{7}$ of x.

e $20 = \frac{2}{3}$ of x. $\frac{5}{7}$ of x. $\frac{10}{11}$ of x. $\frac{10}{7}$ of x. $\frac{10}{9}$ of x. $\frac{10}{3}$ of x.

f $5 = \frac{2}{3}$ of x. $10 = \frac{4}{5}$ of x. $7 = \frac{2}{3}$ of x. $9 = \frac{4}{5}$ of x.

59. Anna's age is $\frac{2}{3}$ of Mary's age. What is the ratio of Mary's age to Anna's age. If Anna is 12 years old, how old is Mary?

60. James has 24 marbles. He has $\frac{2}{3}$ as many as John. How many marbles has John?

61. Land in one part of a certain county in Illinois is worth \$20 an acre, which is only $\frac{4}{5}$ of the price of land in another part of the county. What is the price of the better land? What is the value of the land owned by Mr. Baxter, who has 40 acres of each kind?

62. Mr. Walker sold gingham at 8¢ a yard, which was $\frac{4}{5}$ of what it cost him. How much did it cost?

63. Find the cost of goods:

a Sold at \$0.09, which was $\frac{3}{2}$ of the cost.

b Sold at \$0.20, which was $\frac{5}{4}$ of the cost.

c Sold at \$1.98, which was $\frac{2}{3}$ of the cost.

d Sold at \$2.97, which was $\frac{3}{4}$ of the cost.

e Sold at \$1.47, which was $\frac{7}{5}$ of the cost.

64. 24 marbles will cost how many times as much as 12 marbles of the same kind?

65. What is the cost of 24 marbles when 12 marbles cost \$.25? \$.08? \$.60?

66. What is the ratio of the price of 10 hats to the price of 1 hat? To the price of 2 hats? 5 hats? 7 hats?

67. What will be the cost of 10 hats when 5 hats cost \$3? \$7? \$9? \$7.50?

Use ratio. 10 hats cost how many times as much as 5 hats?

68. What will be the cost of 10 hats when 2 hats are worth \$3? \$4? \$5.25? \$7.65?

69. If 3 articles of the same kind cost \$.17, how much will 12 such articles cost? 18 articles? 21 articles? 30 articles?

70. If 5 things cost \$19, how much will 15 things of the same kind cost?

71. Take Ex. 70, substituting another number for 5 and for 15 some multiple of the number that you have substituted.

72. If 10 acres of land are sold for \$375, how much would 80 acres cost at the same rate? 60 acres? 100 acres?

73. Find the cost of 48 oranges when 5¢ are paid for 6 oranges. For 8 oranges. For 4 oranges. For 12 oranges.

74. If 15 marbles are sold for 9¢, how much do 5 marbles cost? 3 marbles? 20 marbles?

MISCELLANEOUS EXERCISES

1. Add the square of 7.9 to 7.9.

2. Subtract the cube of 1.3 from 10.

3. Divide 14^2 by .007.

4. A regular hexagon is inscribed in a circle whose radius is 8 in. How long is the perimeter of the hexagon? Represent.

5. How long is one side of a regular hexagon whose perimeter is 5.4 cm.?

6. How long is one side of a regular pentagon whose perimeter is 8.45 cm.?

7. Is it correct to say that an inch line is $\frac{1}{4}$ of a 4-inch line? If the inch line were in Boston and the 4-inch line in New York, would the shorter line be a part of the longer? What part of the longer line would the shorter line equal?

8. What is the ratio of an hour to a day? Of a week to a day?

9. Give the ratio of 21 to each of the first 12 multiples of 7.

10. What is the ratio of a square whose side is 3 ft. to a rectangle 9 ft. by 8 ft.?

11. Find the prime factors of 546. Of 495.

12. Find the ratio of the largest prime factor of 35 to the largest prime factor of 39. Of the largest prime factor of 49 to the largest prime factor of 15. Of the smallest prime factor of 49 to the smallest prime factor of 15.

13. $60^3 : 30^3 = ?$ $6^3 : 5^3 = ?$ $7^3 : 5^3 = ?$ $8^3 : 12^3 = ?$

14. $7\sqrt{4} : 3\sqrt{49} = ?$ $4\sqrt{25} : 2\sqrt{100} = ?$

15. How long is the shortest line that can be divided into either 8-inch lines or 10-inch lines?

16. Divide the l. c. m. of 2 and 5 by the l. c. m. of 3 and 5.

17. Divide the g. c. d. of 36 and 45 by the g. c. d. of 12 and 3.

18. Which power of 6 is 216?

19. Which power of 2 is 16? 64? 256?

20. Which power of 10 is the denominator of the decimal .11? .0125? .6? .345? .00004? .000009?

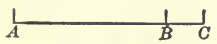


FIG. 3.

21. AB represents a distance of 320 rd. or 1 mi., which is just $\frac{4}{5}$ of the distance from A to C. How far is it from A to C? From B to C?

22. Mary had some money in a toy bank. She took out 35¢, which was $\frac{5}{7}$ of it. How much was left?

23. Lizzie spent 15¢ for a singing book, which was $\frac{3}{4}$ of the price of her arithmetic. The price of the arithmetic was $\frac{2}{9}$ of the price of her story book. How much did they all cost?

24. On Monday John rode on his bicycle 21 mi., which was $\frac{3}{8}$ of the distance he rode during the rest of the week. How far did he ride during the whole week?

25. A boy had 12 agates, for which he paid 60¢. He sold them for 6¢ apiece. How much did he gain on each?

26. Mr. Cooper paid $64, which was 50% of what he owed. How much does he still owe?

27. When a man hires a house to live in, he is said to pay rent for it. When he hires money to use, he is said to pay interest for it. If you have deposited $100 in a bank that pays 4% interest, how much interest will you receive each year?

28. How much would you receive each year if you had $100 at 6%? 3%? 7%?

29. If a man borrows $25 and pays .06 of $25 as interest for 1 yr., how much interest does he pay?

30. At 6% what is the interest of $14 for a yr.? Of $16? $30? $40? $60?

31. At 8% what is the interest each year of $7? $11?

32. At 7% what is the interest each year of $9? $12?

33. At 8% what is the interest of $12 for a year? For 2 yr.? 6 yr.? 8 yr.? 12 yr.?

34. At 8% what is the interest of $7 for a year? For $\frac{1}{2}$ yr.? For $1\frac{1}{2}$ yr.? $2\frac{1}{2}$ yr.? $3\frac{1}{2}$ yr.? $1\frac{1}{4}$ yr.? $3\frac{1}{4}$ yr.? $7\frac{1}{4}$ yr.?

35. At 6% what is the interest of $8 for a yr.? For $\frac{1}{3}$ yr.? For $3\frac{1}{3}$ yr.? $4\frac{1}{3}$ yr.? $5\frac{1}{4}$ yr.? $6\frac{1}{4}$ yr.?

36. Class Exercise. —— may tell how many dollars he would like to have at interest at 6%, and the class may tell how much interest he would have each year from it. How much in 2 yr. 3 yr. $4\frac{1}{2}$ yr. $5\frac{1}{4}$ yr. $6\frac{1}{3}$ yr.

37. Mrs. Ware lent \$800 @ 4% interest, \$900 @ 3%, and \$2500 @ 5%. How much interest did she receive each year from those loans?

38. What is the ratio of a 5-inch square to an oblong 13 in. long and 5 in. wide?

39. Draw on paper a rectangle 4 in. long and 3 in. wide. Draw a diagonal. Cut out the rectangle and divide it along the diagonal. Into what kind of triangles is a rectangle divided by a diagonal? What is the area of the rectangle? Of each triangle?

40. Show the truth of the following statement:

The area of a right triangle equals one half the area of a rectangle which has the same base and altitude.

41. What is the area of a right triangle whose base is 8 in. and altitude 5 in.?

42. Give directions for finding the area of a right triangle.

43. Find the area of a right triangle whose base is 8 cm. and altitude 9 cm. Base 27 in., altitude 13 in.

44. Find the area of a right triangle whose base is 12 in. and whose altitude is 25% of the base.

45. Can you bisect a rectangle and place the two parts so as to form an isosceles triangle? Represent.

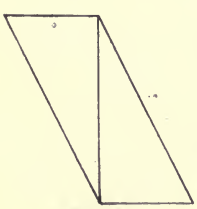


FIG. 4.

46. Bisect a rectangle and place the parts as in Fig. 4. Show two horizontal parallel lines; two oblique parallel lines.

47. A four-sided plane figure whose opposite sides are parallel, and whose angles are not right angles, is called a **Rhomboid.** Draw a rhomboid.



FIG. 5.

48. How long is the perimeter of a rhomboid whose short sides are each 7 in. and long sides each 10 in.?

49. How long is the perimeter of a rhomboid two of whose sides are each 1.75 ft. long, and the other two each 2.5 ft. long?

50. Represent and find perimeters of rhomboids having:

A long side, 18 in.; short side, $33\frac{1}{3}\%$ of a long side.

A short side, 12 in.; long side, 25% longer than a short side.

A short side, 1 ft. 4 in.; long side, 50% longer than a short side.

51. What per cent of the angles of a rhomboid are obtuse?

52. Find a rhombus in your book, and see whether it agrees with the definition of a rhomboid.

A rhombus differs from other rhomboids in having all its sides equal.

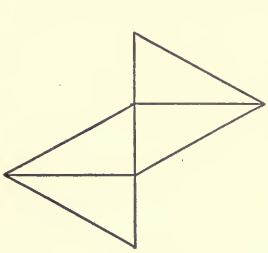


FIG. 6.

53. Draw and cut out two equal equilateral triangles. Cut them in two, and arrange the right triangles thus formed as in Fig. 6. How long would the perimeter of Fig. 6 be if each side of the equilateral triangles were 6 cm. long? 16 cm.? 25 cm.?

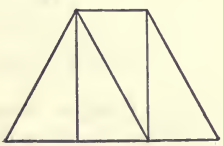


FIG. 7.

54. Place the four triangles as in Fig. 7. What kind of a figure is formed? How long is its perimeter if each side of the original triangles is 10 in. long? 11 in. long?

55. By changing the position of one triangle, change the figure into a rhombus. Find length of perimeter if each side of the original triangles is 7 in. long.

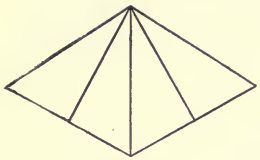


FIG. 8.

56. Place the four triangles as in Fig. 8, and name the figure. How long would the perimeter of Fig. 8 be if each side of the two equilateral triangles were 6 in. long? 1 ft. 3 in. long?

57. Place the four triangles so as to form a rectangle.

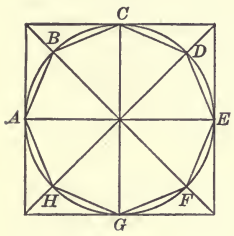


FIG. 9.

58. Draw a 2-inch square. Draw its diagonals. With the point where the diagonals meet as a center and a radius of 1 in., draw a circle. At how many points do the sides of the square touch the circle? At how many points do the diagonals cut the circumference? Draw lines *AB*, *BC*, etc., between these points as in Fig. 9.

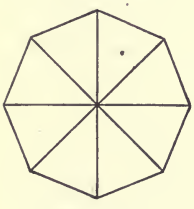


FIG. 10.

59. Erase all but the part shown in Fig. 10. Such a figure is called an **Octagon**. How many sides has an octagon? How long would the perimeter of your octagon be if each side were 7 in.? 9 in.?

60. Fold the octagon in various ways, and see whether the angles are all equal, and whether the sides are all equal. If they are, what kind of an octagon is it?

61. What kind of angles has a regular octagon?

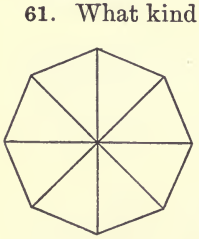


FIG. 11.

62. Draw diagonals of the octagon as in Fig. 11. Into how many isosceles triangles is the octagon divided? Each triangle is what part of the octagon?

63. If the area of the octagon were 60.48 sq. in., what would be the area of one of the isosceles triangles?

CHAPTER IV

FRACTIONS

1. Draw a line an inch long and divide it into halves and quarters. How many halves of an inch are there in an inch? How many fourths of an inch? How many eighths? How many thousandths of an inch? How many millionths?

A **Fraction** is an expression of one or more of the equal parts into which a unit is divided.

2. In the expression $\frac{3}{4}$, 4, the denominator of the fraction, shows that some unit is considered as separated into 4 equal parts; 3, the numerator, shows how many of those parts are expressed. What is meant by the expression $\frac{5}{8}$ of an inch?

3. In the fraction $\frac{13}{16}$ which number is the denominator? What does the 16 show? What name is given to the number above the line?

4. What is meant by $\frac{7}{8}$ of an inch? $\frac{3}{4}$ of an apple?

5. Make a mental picture of what each of the following expressions represents, and tell how much each lacks of a unit of its own kind: $\frac{5}{6}$ of a pie; $\frac{3}{4}$ of an apple; $\frac{7}{12}$ of a foot; $\frac{8}{9}$ of a square yard; $\frac{4}{5}$ of a regular pentagon; $\frac{1}{6}$ of a regular hexagon; $\frac{7}{8}$ of a 2-inch cube.

6. A fraction whose numerator is less than its denominator is called a **Proper Fraction.** Give some proper fractions and tell how much each lacks of being equal to the whole of which it is a part.

7. A fraction whose numerator is equal to or greater than its denominator is called an **Improper Fraction.** Give some improper fractions and tell how much each exceeds one unit.

8. Separate the following fractions into two lists, one of proper fractions, the other of improper fractions:

$\frac{5}{6}$ $\frac{11}{11}$ $\frac{9}{8}$ $\frac{8}{9}$ $\frac{7}{10}$ $\frac{21}{20}$

$\frac{25}{28}$ $\frac{21}{25}$.3 $\frac{50}{25}$ $\frac{50}{100}$ 101%

9. Write a proper fraction whose terms are 5 and 7. Write an improper fraction with the same numbers as terms.

10. A number that consists of an integer and a fraction is called a **Mixed Number**; as $3\frac{1}{2}$. How many halves of a circle in $3\frac{1}{2}$ equal circles? Illustrate. Does the following explanation seem to you to be true?

As there are 2 halves in 1 whole, in 3 wholes there are 3 times 2 halves, or 6 halves, 6 halves + 1 half = 7 halves.

11. Change $2\frac{3}{4}$ to an improper fraction and explain the process.

12. Change to equivalent improper fractions:

$7\frac{1}{2}$ $3\frac{1}{3}$ $8\frac{1}{3}$ $16\frac{2}{3}$ $5\frac{5}{9}$ $7\frac{7}{9}$ $5\frac{6}{7}$ $20\frac{1}{4}$

$8\frac{4}{7}$ $15\frac{2}{7}$ $21\frac{3}{4}$ $6\frac{8}{9}$ $5\frac{4}{5}$ $1\frac{3}{7}$ $5\frac{7}{8}$ $11\frac{5}{7}$

13. Tell how a mixed number is changed into an equivalent improper fraction.

14. Change to equivalent improper fractions:

$8\frac{1}{9}$ $4\frac{9}{21}$ $7\frac{3}{19}$ $9\frac{1}{51}$ $7\frac{18}{19}$ $5\frac{13}{14}$ $12\frac{3}{5}$

$7\frac{7}{12}$ $9\frac{7}{11}$ $8\frac{15}{16}$ $5\frac{5}{7}$ $4\frac{3}{5}$ $5\frac{2}{3}$ $4\frac{5}{11}$

15. Write a mixed number whose fractional part is $\frac{3}{4}$. Change it to an equivalent improper fraction.

16. Write a mixed number whose integral part is 7. Change it to an equivalent improper fraction.

17. Class Exercise. —— may give a mixed number, and the class may reduce it to an improper fraction.

18. How many wholes are there in $\frac{6}{2}$? $\frac{8}{2}$? $\frac{10}{2}$?

SUGGESTION TO TEACHER. Lead pupils to express in their own way the evident fact that since it takes 2 halves to make a whole, there will be as many wholes in any number of halves as there are groups of 2 in that number.

19. To reduce a fraction is to change its form without changing its value. Reduce the following improper fractions to mixed numbers:

$$\frac{8}{7} \quad \frac{17}{3} \quad \frac{21}{5} \quad \frac{18}{8} \quad \frac{20}{15} \quad \frac{19}{18} \quad \frac{48}{21} \quad \frac{176}{25} \quad \frac{198}{14}$$

20. Give directions for reducing an improper fraction to a whole or a mixed number.

21. Reduce to integral or mixed numbers:

$$\frac{9836}{8} \quad \frac{61381}{37} \quad \frac{2134}{67} \quad \frac{413}{413} \quad \frac{697}{225} \quad \frac{381}{25}$$

22. CLASS EXERCISE. —— may give an improper fraction, and the class may change it to a mixed number or to an integer.

23. A fraction is an expression of division. $\frac{14}{7}$ equals how many units? In the expression $\frac{14}{7}$, which number is the dividend? Which is the divisor? What is the quotient? Show the same with regard to the expression $\frac{10}{5}$. With $\frac{21}{3}$. With $\frac{15}{5}$.

SUGGESTION TO TEACHER. Lead pupils to see that proper fractions also express division. $\frac{1}{3}$ indicates that one unit is divided by 3. Let lines be drawn and divided to illustrate such facts as that $\frac{1}{3}$ of 2 yd. or of 6 ft. equals $\frac{2}{3}$ of a yd., that $\frac{3}{4}$ of a ft. or 9 in. equals $\frac{1}{4}$ of 3 ft.

24. Draw a line 3 in. long. Divide each inch into fourths and show that $\frac{3}{4}$ of 1 in. equals $\frac{1}{4}$ of 3 in.

25. When the numerator and denominator of a fraction are made to change places, the process is called inverting the fraction, as $\frac{2}{3}$ inverted is $\frac{3}{2}$. The fraction resulting from this inversion of a fraction is called the **Reciprocal** of the original fraction. Thus $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$. What is the reciprocal of the fraction $\frac{2}{3}$? $\frac{5}{7}$? $\frac{9}{11}$? $\frac{12}{7}$?

26. Which is greater, $\frac{3}{5}$ or its reciprocal? $\frac{8}{7}$ or its reciprocal?

27. The ratio of Mr. A's money to Mr. B's money is $\frac{3}{5}$. What is the ratio of Mr. B's money to Mr. A's money? If Mr. A's money is \$ 8, how much has Mr. B?

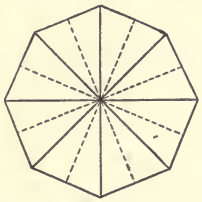


FIG. 1.

28. Draw a regular octagon. Divide it by diagonals into 8 equal isosceles triangles. Divide each isosceles triangle into equal right triangles as in Fig. 1. Each isosceles triangle equals what part of the octagon? Each right triangle equals what part of an isosceles triangle? Of the octagon? $\frac{1}{2}$ of $\frac{1}{8} = ?$

SUGGESTION TO TEACHER. Let a large copy of Fig. 1 be drawn upon the board as a basis for the following exercises.

29. Find from the figure the values of the following:

$\frac{1}{2}$ of $\frac{1}{2}$	$\frac{1}{2}$ of $\frac{1}{4}$	$\frac{1}{4}$ of $\frac{1}{4}$	$\frac{1}{2}$ of $\frac{3}{4}$
$\frac{1}{2}$ of $\frac{1}{8}$	$\frac{1}{2}$ of $\frac{3}{8}$	$\frac{1}{2}$ of $\frac{5}{8}$	$\frac{1}{2}$ of $\frac{7}{8}$

30. A fraction of a fraction is called a **Compound Fraction**. What is the value of the compound fraction $\frac{1}{2}$ of $\frac{1}{4}$ when expressed in simple form?

31. How many inches equal $\frac{1}{3}$ of $\frac{1}{4}$ of a foot?

32. See if the following reasoning is true:

Since $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$,

$\frac{1}{3}$ of $\frac{4}{5} = 4$ times $\frac{1}{15}$ or $\frac{4}{15}$,

$\frac{2}{3}$ of $\frac{4}{5} = 2$ times $\frac{4}{15}$ or $\frac{8}{15}$.

33. The following rule is founded upon the same reasoning:

To find the simple form of the value of a compound fraction —

Find the product of the numerators for the numerator of the simple fraction and the product of the denominators for its denominator. Cancel if possible.

By the same reasoning find the value of $\frac{3}{7}$ of $\frac{5}{8}$.

34. Simplify:

a $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{6}$

b $\frac{4}{9}$ of $\frac{9}{11}$ of $\frac{33}{45}$

c $\frac{7}{8}$ of $\frac{5}{9}$ of 72

d $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{3}{5}$ of $\frac{15}{22}$

e $\frac{5}{8}$ of $\frac{16}{25}$

f $\frac{7}{9}$ of $\frac{27}{28}$

g $\frac{6}{11}$ of $\frac{55}{72}$

h $\frac{4}{7}$ of $\frac{21}{32}$ of $\frac{4}{9}$

i $\frac{5}{9}$ of $\frac{3}{4}$ of $\frac{18}{25}$

j $\frac{6}{7}$ of $\frac{3}{8}$ of $\frac{14}{27}$

k $\frac{5}{9}$ of $\frac{3}{25}$ of $2\frac{1}{2}$

l $\frac{6}{7}$ of $\frac{21}{32}$ of 8

m $\frac{5}{11}$ of $\frac{22}{25}$ of $7\frac{1}{2}$

n $\frac{3}{5}$ of $\frac{7}{10}$ of $6\frac{1}{4}$

o $\frac{4}{5}$ of $\frac{7}{8}$ of $8\frac{1}{3}$

35. How many square centimeters in $\frac{9}{10}$ of $\frac{9}{10}$ of a square decimeter?

36. Mr. King owned $\frac{1}{2}$ of a farm and sold $\frac{3}{5}$ of his share. What part of the farm did he sell? If there were 200 acres in the farm, how many acres did he sell and how many had he left?

37. Six boys divided a number of marbles equally among themselves. Edward Wells, one of the boys, gave $\frac{1}{3}$ of his share to his younger brother. What part of the marbles did Edward keep? If there were 54 marbles, how many did he keep?

38. Mr. Hubbard owned $\frac{5}{8}$ of a mine and sold $\frac{2}{3}$ of his share. If the mine was worth $80,000, how much did he receive?

39. How many cubic feet in $\frac{2}{3}$ of $\frac{4}{5}$ of a cubic yard? In $\frac{4}{3}$ of $\frac{3}{4}$ of a cubic yard?

40. How many minutes in $\frac{3}{5}$ of $\frac{2}{3}$ of an hour? In $\frac{4}{7}$ of $\frac{7}{8}$ of an hour?

41. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{8}{7}$ is how much less than 1 unit? Than 2 units?

42. $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{28}{9}$ is how much more than 1 unit? How much less than 3 units? Than 8 units?

43. Find, from the octagon on page 111, the values of x in the following equations:

$\frac{1}{2} = \frac{x}{16}$ $\frac{1}{2} = \frac{x}{8}$ $\frac{1}{2} = \frac{x}{4}$ $\frac{1}{4} = \frac{x}{16}$ $\frac{1}{4} = \frac{x}{8}$ $\frac{2}{4} = \frac{x}{16}$

$\frac{3}{4} = \frac{x}{16}$ $\frac{3}{4} = \frac{x}{8}$ $\frac{1}{8} = \frac{x}{16}$ $\frac{3}{8} = \frac{x}{16}$ $\frac{5}{8} = \frac{x}{16}$ $\frac{7}{8} = \frac{x}{16}$

44. In changing $\frac{1}{2}$ to 16ths, by what number are the numerator and the denominator multiplied? How do you find it?

45. Change $\frac{1}{2}$ to a fraction of equal value, having a larger denominator, and show the truth of the following principle:

PRINCIPLE 1. *Multiplying both terms of a fraction by the same number does not change the value of the fraction.*

46. Change $\frac{1}{3}$ to 6ths. $\frac{2}{3}$ to 6ths.

47. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to 12ths.

48. Change $\frac{3}{4}$, $\frac{4}{5}$, $\frac{3}{10}$ to 20ths. $\frac{1}{3}$, $\frac{5}{6}$, $\frac{2}{9}$ to 18ths.

49. To 24ths change $\frac{5}{12}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$.

50. Write 10 fractions whose value is $\frac{1}{2}$, but whose denominators are all different. What is the ratio of the numerator of each fraction to its denominator?

SUGGESTION TO TEACHER. Develop the idea that the value of a fraction does not depend upon the magnitude of the numbers by which it is expressed, but upon their ratio.

51. Write 7 fractions whose value is $\frac{1}{3}$, and whose denominators are all different. What is the ratio of the numerator of each fraction to its denominator?

52. Tell how you change fractions to equivalent fractions expressed in higher terms; that is, by larger numbers.

53. Change $\frac{1}{2}$ to hundredths and write it as per cent.

54. Change to hundredths and write as per cent:

$\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{1}{8}$

55. When $\frac{6}{18}$ is expressed as $\frac{1}{3}$, what has been done to each term of the fraction $\frac{6}{18}$?

56. Choose some fraction whose value is $\frac{1}{4}$, although expressed differently, and show the truth of the following principle:

PRINCIPLE 2. *Dividing both terms of a fraction by the same number does not change the value of the fraction.*

57. What number will divide both terms of the fraction $\frac{6}{10}$? What fraction results from the division?

58. In this way reduce to lowest terms: $\frac{10}{15}$. $\frac{5}{15}$. $\frac{9}{15}$. $\frac{12}{15}$.

59. Reduce $\frac{24}{48}$ to lower terms, but not to its lowest terms.

60. What is meant by the phrase, "reducing a fraction to lowest terms"?

61. In reducing a fraction to its lowest terms, by which of the common divisors of both terms is it best to divide them? Why?

62. Reduce to lowest terms and tell what common divisor you use: $\frac{15}{20}$. $\frac{35}{40}$. $\frac{55}{60}$. $\frac{14}{21}$. $\frac{18}{27}$. $\frac{6}{24}$. $\frac{18}{24}$.

63. Reduce to lowest terms:

$\frac{42}{49}$	$\frac{54}{63}$	$\frac{16}{24}$	$\frac{32}{40}$	$\frac{66}{77}$	$\frac{35}{77}$	$\frac{11}{22}$
$\frac{22}{88}$	$\frac{33}{44}$	$\frac{6}{33}$	$\frac{14}{44}$	$\frac{2}{8}$	$\frac{3}{24}$	$\frac{10}{32}$
$\frac{5}{40}$	$\frac{56}{84}$	$\frac{54}{99}$	$\frac{27}{36}$	$\frac{18}{45}$	$\frac{36}{63}$	$\frac{63}{72}$

64. Tell how a fraction is reduced to its lowest terms.

65. Find by continued division the greatest common divisor of 147 and 196, and reduce the fraction $\frac{147}{196}$ to lowest terms.

66. Reduce to lowest terms:

$\frac{357}{527}$	$\frac{264}{312}$	$\frac{492}{779}$	$\frac{418}{874}$	$\frac{854}{1769}$
$\frac{36}{120}$	$\frac{35}{105}$	$\frac{49}{140}$	$\frac{64}{480}$	$\frac{72}{252}$
$\frac{30}{785}$	$\frac{44}{242}$	$\frac{77}{847}$	$\frac{18}{243}$	$\frac{96}{224}$

67. How many inches in $\frac{125}{150}$ ft.? In $\frac{63}{84}$ ft.? In $\frac{440}{880}$ ft.?

68. As John was studying fractions one evening, his uncle said to him, "If you will go on an errand for me I will give you $\frac{2222}{8888}$ of a dollar." John did so, and received the money. How much was it?

69. If John's uncle had given him $\frac{22222222}{44444444}$ of a dollar, how many cents would he have received?

70. What is the use of reducing fractions to lower terms?

71. CLASS EXERCISE. ——may give a fraction which the class may reduce to lowest terms.

72. Can you reduce $\frac{8}{9}$ to lower terms? Give reason for your "Yes" or "No."

73. Write a fraction whose numerator and denominator are both prime numbers. Can you reduce it to lower terms? Give reason for your answer.

74. Numbers that have no common factor are said to be prime to each other, although they may be composite numbers. $\frac{8}{9}$ is a fraction whose terms are prime to each other. Are the terms prime numbers or composite numbers?

75. Write 5 different fractions, each having for its terms composite numbers which are prime to each other.

76. Write a fraction with 7 for a numerator and the square of 7 for the denominator, and reduce it to lowest terms.

77. Write a fraction with the square of 8 for the numerator and the cube of 8 for the denominator, and reduce it to lowest terms.

78. A fraction whose denominator is 10, or any other power of 10, may be expressed as a decimal or as a common fraction. **Express $\frac{5}{10}$ as a decimal. .50 as a common fraction.**

79. Express in decimal form and in common form each of the following: Thirteen hundredths; one hundred sixty-seven thousandths; two thousand six hundred seven ten-thousandths; forty-three thousandths; six hundred fifty-one hundred-thousandths; forty one millionths.

80. Write as common fractions and give in lowest terms: .5. .25. .125. 99%. 75%. .072. .064. 5%. 40%. 80%. 90%.

81. Class Exercise. —— may give a decimal fraction, and the class may express it as a common fraction in its lowest terms.

ADDITION AND SUBTRACTION OF FRACTIONS

82. A man dying left 3 sevenths of his property to his wife, 3 sevenths to his children, and the remainder to a library. What share of it did the library receive?

83. 1 ninth + 4 ninths + 2 ninths = ?

84. How are fractions added when they have the same denominator?

85. Fractions that have the same denominator are called **Similar Fractions.** Write three similar fractions and find their sum.

86. Give four proper fractions that express 13ths and find their sum.

87. From $\frac{6}{17}$ take $\frac{2}{17}$. $\frac{8}{11} - \frac{3}{11} = ?$

88. Find from the octagon on page 111 the values of x:

$\frac{1}{2} + \frac{3}{16} = \frac{x}{16}$ $\quad \frac{1}{4} - \frac{3}{16} = \frac{x}{16}$ $\quad \frac{3}{4} - \frac{5}{16} = \frac{x}{16}$

$\frac{1}{8} + \frac{5}{16} = \frac{x}{16}$ $\quad \frac{3}{8} - \frac{5}{16} = \frac{x}{16}$ $\quad \frac{5}{8} - \frac{5}{16} = \frac{x}{16}$

89. $\frac{8}{8} - \frac{11}{16} = ?$ $\quad \frac{16}{16} - \frac{1}{8} = ?$ $\quad \frac{4}{16} - \frac{1}{8} = ?$

90. $\frac{1}{2} + \frac{7}{16} = ?$ $\quad \frac{1}{2} - \frac{1}{4} = ?$ $\quad \frac{8}{4} - \frac{8}{16} = ?$

91. $\frac{1}{2} + \frac{7}{16}$ of the octagon needs how much to complete the figure?

92. In adding $\frac{1}{2}$ and $\frac{1}{4}$, how must $\frac{1}{2}$ be changed? Why?

93. By what number must each term of the fraction $\frac{1}{4}$ be multiplied to change the fraction to 12ths? How did you find that number?

94. Can the terms of the fraction $\frac{1}{4}$ be multiplied by an integer that will change it to 11ths?

95. Give five different numbers that can be used as denominators for fractions whose value is $\frac{3}{4}$.

SUGGESTION TO TEACHER. Lead pupil to see that in changing a fraction to an equivalent fraction having a larger denominator the new denominator must be a multiple of the old.

96. Change $\frac{4}{5}$ to 15ths by the following rule:

To change a fraction to an equivalent fraction having a larger denominator —

Multiply both terms of the fraction by the quotient obtained by dividing the new denominator by the old denominator.

97. Change $\frac{2}{3}$ to an equivalent fraction having the denominator 21. 27. 24. 33. 36. How many 10ths equal $\frac{3}{5}$? How many 30ths? 20ths? 35ths? 50ths?

98. Copy the following equations, substituting for x its value:

$$\frac{1}{2} = \frac{x}{42} \qquad \frac{5}{7} = \frac{x}{42} \qquad \frac{5}{21} = \frac{x}{42} \qquad \frac{2}{3} = \frac{x}{42} \qquad \frac{5}{6} = \frac{x}{42}$$

99. Find the least number that is a multiple of 4 and 7. Change $\frac{3}{4}$ and $\frac{5}{7}$ to equivalent fractions having that number for their denominators. Add those fractions.

100. Change $\frac{5}{8}$ and $\frac{2}{5}$ to equivalent fractions having for their denominators the least common multiple of 8 and 5. Add those fractions.

101. Change $\frac{5}{6}$ and $\frac{4}{7}$ to equivalent fractions having the least integral denominator that they can both use. Add the fractions.

102. Change $\frac{1}{6}$ and $\frac{1}{5}$ to equivalent fractions having for their denominator the least common multiple of their present denominators. Then add them.

103. The least common denominator that two or more fractions may have is the least common multiple of their denominators. Express the following fractions with their least common denominator: $\frac{3}{4}$. $\frac{5}{6}$. $\frac{7}{8}$. $\frac{1}{12}$. $\frac{1}{2}$. $\frac{2}{3}$.

104. Change $\frac{1}{3}$ and $\frac{1}{4}$ to equivalent fractions having their least common denominator, and explain your method of changing them.

105. Express with least common denominator and find values: $\frac{1}{2}+\frac{3}{8}$. $\frac{3}{4}+\frac{5}{6}$. $\frac{5}{7}+\frac{9}{14}$. $\frac{1}{8}+\frac{7}{12}$.

106. Find values:

a $\frac{1}{2}-\frac{1}{4}$	*c* $\frac{1}{4}-\frac{1}{8}$	*e* $\frac{1}{2}-\frac{1}{7}$	*g* $\frac{1}{8}-\frac{1}{9}$
b $\frac{1}{5}-\frac{1}{10}$	*d* $\frac{3}{8}-\frac{1}{3}$	*f* $\frac{5}{7}-\frac{1}{2}$	*h* $\frac{3}{4}-\frac{3}{8}$

Add:

	a	*b*	*c*	*d*
107.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$	$\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$	$\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
108.	$\frac{1}{2}, \frac{1}{4}, \frac{1}{7}$	$\frac{1}{2}, \frac{3}{4}, \frac{5}{7}$	$\frac{1}{2}, \frac{1}{5}, \frac{1}{7}$	$\frac{1}{2}, \frac{2}{5}, \frac{3}{7}$
109.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$	$\frac{1}{2}, \frac{2}{3}, \frac{2}{5}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{11}$	$\frac{1}{2}, \frac{2}{3}, \frac{10}{11}$
110.	$\frac{1}{2}, \frac{4}{5}, \frac{9}{10}$	$\frac{2}{3}, \frac{3}{5}, \frac{7}{10}$	$\frac{1}{2}, \frac{3}{5}, \frac{7}{8}$	$\frac{2}{3}, \frac{4}{5}, \frac{5}{8}$
111.	$\frac{3}{4}, \frac{3}{5}, \frac{3}{8}$	$\frac{1}{4}, \frac{1}{5}, \frac{1}{10}$	$\frac{3}{4}, \frac{4}{5}, \frac{9}{10}$	$\frac{3}{4}, \frac{5}{6}, \frac{6}{7}$
112.	$\frac{1}{3}, \frac{1}{7}, \frac{1}{9}$	$\frac{2}{3}, \frac{4}{7}, \frac{7}{9}$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{9}$	$\frac{3}{4}, \frac{5}{8}, \frac{7}{9}$
113.	$\frac{1}{3}, \frac{1}{7}, \frac{1}{12}$	$\frac{2}{3}, \frac{5}{7}, \frac{7}{12}$	$\frac{3}{4}, \frac{2}{7}, \frac{11}{12}$	$\frac{1}{4}, \frac{1}{9}, \frac{1}{12}$
114.	$\frac{3}{4}, \frac{6}{7}, \frac{5}{12}$	$\frac{1}{5}, \frac{1}{6}, \frac{1}{9}$	$\frac{3}{5}, \frac{5}{6}, \frac{2}{9}$	$\frac{3}{5}, \frac{7}{8}, \frac{3}{10}$
115.	$\frac{7}{12}-\frac{7}{15}=?$	$\frac{8}{15}-\frac{3}{20}=?$	$\frac{17}{20}-\frac{11}{30}=?$	$\frac{9}{40}-\frac{7}{50}=?$

SUGGESTION TO TEACHER. For additional practice let the class subtract the second fraction from the sum of the first and third in Examples 107–114.

116. How much greater than $\frac{1}{15}$ is $\frac{1}{12}$? $\frac{1}{10}$? $\frac{1}{9}$? $\frac{1}{6}$? $\frac{1}{5}$?

117. $\frac{7}{12}$ is how much more than $\frac{7}{18}$? $\frac{7}{20}$? $\frac{7}{24}$? $\frac{7}{28}$? $\frac{7}{30}$?

118. Class Exercise. —— may give two fractions, and the class may find their sum or their difference as may be directed.

119. Draw a line AB and mark two points in it, C and D. If AC represents $\frac{1}{2}$ a mile, CD $\frac{1}{3}$ of a mile, and DB $\frac{5}{11}$ of a mile, what distance is represented by AB?

120. John spent $\frac{1}{4}$ of his money for a top, $\frac{1}{3}$ of it for a ball, and $\frac{1}{6}$ of it for candy. What part of it had he left?

121. How much money did John spend for each article and how much had he left if he had at first 12¢? 48¢? 24¢?

122. $\frac{3}{7} + \frac{5}{14}$ is how much more than $\frac{3}{7} - \frac{5}{14}$?

123. The sum of $\frac{2}{3}$ and $\frac{5}{6}$ is how much more than their difference?

124. Express in lowest terms the ratio of 15 to 20 and of 16 to 30. Find their sum. Their difference.

125. Find the sum of $\frac{5}{7}$ and its reciprocal. Of $\frac{3}{8}$ and its reciprocal.

126. Find the sum of $\frac{4}{9}$ and its reciprocal. Find the difference between $\frac{3}{5}$ and its reciprocal.

127. What is the square of 10? The 5th power? 3d power?

128. Which power of 10 is the denominator of the decimal .01? .0015? .003? .00008? .000009?

129. Add $\frac{7}{10}$, $\frac{9}{1000}$, $\frac{11}{100}$. Add $\frac{17}{100}$, $\frac{3}{10}$, $\frac{21}{1000}$.

130. Add:

a	b	c	d
.07	.165	.06	.485
.018	.2145	.016	.6
.5	.31	.07	.21

131. When several fractions have denominators that are powers of 10, is it easier to add them as common fractions or as decimals? Why?

132. Write in decimal form and add:

$\frac{3}{100}$ $\frac{4}{1000}$ $\frac{25}{10000}$ $\frac{6}{10}$ $\frac{105}{10000}$

133. Write in decimal form and find values:

$\frac{3}{10} - \frac{18}{100}$ $\frac{75}{1000} - \frac{2}{100}$ $\frac{18}{1000} - \frac{3}{10000}$ $\frac{41}{100000} - \frac{17}{1000000}$

134. Which is greater, 3 or 3.00? 7 or 7.000?

135. Change $\frac{3}{4}$ to a decimal.

3 reduced to 100ths equals 3.00. As 3 equals 3.00, $3 \div 4$ equals $3.00 \div 4$. $3.00 \div 4$ equals .75. Hence $\frac{3}{4} = .75$.

136. Change $\frac{5}{8}$ to a decimal by the following rule. Explain.

To reduce a common fraction to a decimal —

Annex ciphers to the numerator and divide by the denominator.

137. Change to decimals and add:

$\frac{1}{4}$ $\frac{3}{8}$ $\frac{4}{5}$ $\frac{7}{16}$ $\frac{11}{25}$ $\frac{25}{32}$ $\frac{7}{8}$ $\frac{15}{16}$ $\frac{7}{25}$

138. Change $\frac{1}{3}$ to a decimal, stopping at thousandths.

Common fractions cannot be reduced to exact decimals, if when reduced to their lowest terms their denominators contain any factors other than 2 and 5.

139. Reduce the following fractions to decimals, not carrying the work beyond ten-thousandths:

$\frac{2}{3}$ $\frac{3}{7}$ $\frac{8}{9}$ $\frac{3}{5}$ $\frac{14}{25}$ $\frac{11}{14}$ $\frac{7}{9}$ $\frac{5}{6}$ $\frac{17}{25}$

$\frac{117}{125}$ $\frac{61}{72}$ $\frac{83}{96}$ $\frac{3}{23}$ $\frac{7}{20}$ $\frac{5}{14}$ $\frac{8}{21}$ $\frac{9}{40}$ $\frac{11}{50}$

140. Reduce the following to decimals of not more than 3 places. If the division is not exact, make the remainder the numerator of a common fraction:

$\frac{71}{125}$ $\frac{30}{49}$ $\frac{8}{11}$ $\frac{2}{9}$ $\frac{2}{8}$ $\frac{1}{28}$ $\frac{3}{50}$ $\frac{5}{8}$ $\frac{7}{20}$

141. Tell at sight which of the following fractions can be reduced to exact decimals:

$\frac{7}{15}$ $\frac{4}{7}$ $\frac{6}{25}$ $\frac{2}{9}$ $\frac{4}{11}$ $\frac{14}{28}$ $\frac{9}{27}$ $\frac{20}{24}$ $\frac{15}{40}$

Reduce the fractions to decimals of not more than three places.

142. Divide 2 by each number larger than itself that is expressed by one digit. Express the quotient as a decimal of not more than three places.

143. Divide three by each number between 10 and 20, and express the quotient as a decimal of not more than three places.

144. Change to decimals, stopping at hundredths:

$\frac{1}{8}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{9}{8}$ $\frac{1}{8}$

145. How many 100ths or per cent equal $\frac{1}{3}$? $\frac{2}{3}$? $\frac{4}{3}$? $\frac{5}{3}$? $\frac{7}{3}$?

146. How many per cent equal $\frac{1}{12}$? $\frac{5}{12}$? $\frac{7}{12}$? $\frac{11}{12}$?

147. How many per cent equal $\frac{1}{16}$? $\frac{3}{16}$? $\frac{5}{16}$? $\frac{7}{16}$? $\frac{9}{16}$? $\frac{11}{16}$?

148. What per cent of anything is $\frac{1}{7}$ of it? $\frac{2}{7}$? $\frac{2}{11}$? $\frac{5}{11}$?

149. Add $2\frac{2}{3}$ and $7\frac{1}{2}$:

$2\frac{2}{3}$
$7\frac{1}{2}$ $\frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ or $1\frac{1}{6}$, which added to the sum of 2 and 7 = $10\frac{1}{6}$.
$10\frac{1}{6}$

150. Add:

a	*b*	*c*	*d*	*e*
$7\frac{5}{6}$	$10\frac{3}{5}$	$8\frac{2}{3}$	$6\frac{2}{9}$	$2\frac{3}{8}$
$3\frac{3}{4}$	$6\frac{4}{7}$	$3\frac{1}{12}$	$8\frac{3}{4}$	$6\frac{7}{16}$
$8\frac{5}{9}$	$5\frac{7}{15}$	$4\frac{3}{4}$	$7\frac{5}{12}$	$8\frac{3}{4}$

151. Tell how the following mixed numbers are added:

a	*b*	*c*	*d*
$8\frac{1}{3}$	$16\frac{2}{3}$	$66\frac{2}{3}$	$83\frac{1}{3}$
$12\frac{1}{2}$	$18\frac{3}{4}$	$8\frac{1}{3}$	$62\frac{1}{2}$
$6\frac{1}{4}$	$6\frac{1}{4}$	$12\frac{1}{2}$	$16\frac{2}{3}$

152. Add:

$91\frac{2}{3}$ $41\frac{2}{3}$ $93\frac{3}{4}$ $68\frac{3}{4}$ $33\frac{1}{3}$ $66\frac{2}{3}$ $56\frac{1}{4}$

153. Class Exercise. —— may give three mixed numbers, and the class may find their sum.

154. A farmer used $77\frac{1}{2}$ acres of land for wheat, $40\frac{7}{8}$ acres for corn, $1\frac{5}{8}$ acres for vegetables, $29\frac{1}{2}$ acres for pasturage, and $10\frac{1}{4}$ acres for an orchard. How many acres were in the farm?

155. Mr. White has 3 fences on his farm; one is $168\frac{2}{11}$ rd. long, another $456\frac{4}{11}$ rd. long, and another $328\frac{5}{11}$ rd. long. How many rods of fencing has he in all? How much did his fence cost at 75¢ a rod?

156. After selling $3\frac{5}{8}$ acres, a farmer had left $123\frac{3}{4}$ acres. How much land had he at first?

157. How long is it from half past eight A.M. to noon? From quarter before nine to half past eleven? From quarter past two to quarter to six? From half past ten to quarter past one? From a quarter of eleven to half past three?

158. From 14
take $7\frac{1}{2}$
$6\frac{1}{2}$

We subtract $\frac{1}{2}$ from 1 of the units of the minuend. The remainder is $\frac{1}{2}$. As 1 unit has been subtracted from the 4 units 3 units are left. 7 from 13 leaves 6.

		a	*b*	*c*	*d*	*e*	*f*
159.	From	8	9	18	25	16	20
	take	$6\frac{1}{2}$	$5\frac{3}{4}$	$4\frac{6}{7}$	$3\frac{8}{9}$	$11\frac{1}{6}$	$4\frac{1}{2}$

160. Mary's aunt sent her 6 yd. of cashmere for a dress. $5\frac{3}{8}$ yd. were used. How many yards were left?

161. In a jumping match Thomas jumped 3 ft. and his brother jumped $2\frac{5}{12}$ ft. How much farther did Thomas jump than his brother?

162. Make problems in which a mixed number is subtracted from an integer.

163. Subtract:

a	*b*	*c*	*d*	*e*	*f*	*g*
$3\frac{5}{7}$	$6\frac{8}{11}$	$8\frac{5}{9}$	$8\frac{7}{12}$	$4\frac{7}{8}$	$5\frac{5}{6}$	$4\frac{5}{9}$
$2\frac{3}{7}$	$3\frac{5}{11}$	$2\frac{1}{3}$	$2\frac{1}{6}$	$3\frac{1}{4}$	$2\frac{1}{3}$	$3\frac{1}{3}$

164. A piece of string $3\frac{1}{6}$ ft. long was cut from a piece $37\frac{1}{12}$ ft. long. How much was left? $5\frac{1}{4}$ ft. were cut from the remainder. How much then remained?

165. From $4\frac{1}{3}$ take $\frac{2}{3}$.

$4\frac{1}{3}$
$\frac{2}{3}$
$3\frac{2}{3}$

As $\frac{2}{3}$ cannot be subtracted from $\frac{1}{3}$, we subtract $\frac{2}{3}$ from $1\frac{1}{3}$. This leaves only 3 units in the minuend.

166. Find difference:

a	*b*	*c*	*d*	*e*	*f*	*g*
$8\frac{1}{7}$	$6\frac{1}{8}$	$8\frac{1}{9}$	$6\frac{1}{4}$	$8\frac{2}{5}$	$6\frac{2}{7}$	$7\frac{1}{8}$
$\frac{5}{7}$	$\frac{2}{8}$	$\frac{7}{9}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{4}{7}$	$\frac{3}{8}$

167. Find difference:

a	*b*	*c*	*d*	*e*	*f*	*g*
$4\frac{1}{5}$	$8\frac{1}{8}$	$7\frac{1}{9}$	$3\frac{4}{9}$	$6\frac{1}{7}$	$11\frac{5}{11}$	$9\frac{1}{12}$
$3\frac{3}{5}$	$2\frac{3}{8}$	$4\frac{7}{9}$	$1\frac{5}{9}$	$4\frac{6}{7}$	$2\frac{7}{11}$	$6\frac{7}{12}$
h	*i*	*j*	*k*	*l*	*m*	*n*
$8\frac{2}{7}$	$9\frac{1}{5}$	$7\frac{3}{5}$	$8\frac{1}{8}$	$4\frac{3}{8}$	$6\frac{1}{4}$	$8\frac{4}{13}$
$3\frac{5}{7}$	$2\frac{4}{5}$	$4\frac{4}{5}$	$2\frac{5}{6}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$3\frac{7}{13}$

168. Write two mixed numbers whose fractional parts have the same denominator. Let the mixed number whose integral part is the larger have the smaller fractional part. Find difference between the mixed numbers.

169. Find difference:

a	*b*	*c*	*d*	*e*	*f*	*g*
$3\frac{1}{3}$ or $\frac{2}{6}$	$7\frac{1}{5}$	$8\frac{1}{3}$	$6\frac{2}{7}$	$9\frac{3}{8}$	$8\frac{6}{11}$	$12\frac{3}{5}$
$1\frac{1}{6}$	$2\frac{1}{10}$	$2\frac{2}{9}$	$3\frac{1}{14}$	$4\frac{3}{16}$	$5\frac{1}{7}$	$9\frac{1}{4}$

170. Write two mixed numbers the fractional parts of which have different denominators. Let the mixed number that has the larger integral part have also the larger fractional part. Find their difference.

171. Class Exercise. —— may give two mixed numbers like those described in Ex. 170, and the class may find their difference.

172. Find difference:

a	*b*	*c*	*d*	*e*	*f*	*g*
$17\frac{5}{7}$	$5\frac{1}{4}$	$7\frac{1}{7}$	$6\frac{1}{6}$	$8\frac{1}{3}$	$5\frac{1}{6}$	$8\frac{1}{5}$
$4\frac{6}{7}$	$2\frac{11}{14}$	$3\frac{3}{14}$	$2\frac{1}{3}$	$2\frac{7}{9}$	$3\frac{7}{12}$	$6\frac{7}{10}$
h	*i*	*j*	*k*	*l*	*m*	*n*
$9\frac{1}{4}$	$6\frac{4}{15}$	$5\frac{7}{18}$	$8\frac{1}{10}$	$6\frac{1}{5}$	$7\frac{1}{8}$	$4\frac{3}{8}$
$2\frac{7}{8}$	$2\frac{3}{5}$	$2\frac{7}{9}$	$4\frac{7}{20}$	$4\frac{5}{12}$	$2\frac{1}{7}$	$2\frac{3}{7}$

173. Write two mixed numbers whose fractional parts have different denominators. Let the mixed number whose integral part is the greater have the smaller fractional part. Find their difference.

174. If a line is $3\frac{1}{2}$ in. long, how many inches must be added to make it $5\frac{3}{4}$ in. long? Represent.

175. $3\frac{5}{7}$ + what = $4\frac{6}{7}$? $5\frac{2}{7}$? $6\frac{4}{7}$? $9\frac{5}{7}$?

176. Find values of x:

a $10\frac{1}{2} - 7\frac{1}{2} = x.$
b $6 - 4\frac{1}{5} = x.$
c $7 - 3\frac{1}{2} = x.$
d $13\frac{1}{3} - 4\frac{1}{3} = x.$
e $9 - 3\frac{1}{4} = x.$
f $10\frac{1}{3} - 5\frac{5}{9} = x.$
g $7\frac{1}{2} - 2\frac{1}{2} = x.$
h $12 - 7\frac{3}{5} = x.$

177. A weighs $148\frac{7}{8}$ lb., B $157\frac{3}{4}$ lb., C $161\frac{1}{2}$ lb., D $175\frac{1}{8}$ lb. How much do they all weigh?

178. What is the difference between the weights of A and C? A and B? A and D? B and D? B and C? C and D?

179. Mr. Otis rode $23\frac{5}{8}$ miles on Monday, $11\frac{5}{6}$ miles on Tuesday. On Wednesday he rode as far as on Monday and Tuesday. How far did he ride in the three days?

180. Mr. Carr rode on his bicycle to a city $91\frac{3}{4}$ miles distant. The first day he rode $16\frac{1}{8}$ miles; the next day he rode $3\frac{1}{8}$ miles more than on the first day. On the third day he rode $2\frac{1}{2}$ miles more than on the second day. How far did he ride in those three days? How many miles more did he ride before he reached the city?

181. Mr. Grey planted $75\frac{1}{2}$ acres in wheat, $45\frac{3}{4}$ acres in corn, and $7\frac{7}{8}$ acres in oats. How many acres of grain did he cultivate?

182. In a township containing $23{,}039\frac{1}{16}$ acres, the roads occupy $345\frac{3}{5}$ acres, and the rest is divided into farms. How many acres in the farms of that township?

183. A stove burned $180\frac{3}{4}$ lb. of coal in one week, $175\frac{7}{8}$ lb. in another week, and $205\frac{1}{4}$ lb. in another week. How many lb. did it burn in the three weeks?

184. A, B, and C own a mine. A owns $\frac{5}{12}$ of it, B owns $\frac{2}{5}$ of it, and C owns the rest. How much does C own?

185. If the mine is worth $\$ 248{,}400$, what is the value of each man's share?

186. A has $75\frac{5}{8}$ acres of land, B has $13\frac{3}{8}$ more acres than A and $4\frac{5}{8}$ acres less than C. How many acres has B? C? A and B? A and C? B and C?

187. A farmer has a field in the form of a trapezoid. One of the parallel sides is $71\frac{4}{11}$ rd. long, the other is $68\frac{5}{8}$ rd. long. Of the non-parallel sides, one is $53\frac{5}{8}$ rd. and the other is $54\frac{1}{3}$ rd. Represent and find length of perimeter of the trapezoid.

MULTIPLICATION OF FRACTIONS

188. Multiply $\frac{1}{3}$ by $\frac{1}{2}$.

To multiply any number by $\frac{1}{2}$ is to take $\frac{1}{2}$ of it.

189. By $\frac{1}{2}$ multiply:

$\frac{3}{5}$ $\frac{2}{7}$ $\frac{3}{4}$ $\frac{6}{11}$ $\frac{5}{6}$ $\frac{8}{9}$ $\frac{5}{7}$

190. Make a rule for multiplying a fraction by a fraction.

Multiply:

	a	*b*	*c*	*d*
191.	$\frac{4}{5} \times \frac{7}{8}$	$\frac{21}{16} \times \frac{8}{35}$	$\frac{15}{28} \times \frac{49}{30}$	$\frac{6}{11} \times \frac{55}{72}$
192.	$\frac{45}{64} \times \frac{16}{27}$	$\frac{50}{77} \times \frac{11}{25}$	$\frac{18}{35} \times \frac{7}{9}$	$\frac{27}{32} \times \frac{8}{9}$
193.	$\frac{49}{60} \times \frac{5}{7}$	$\frac{18}{25} \times \frac{25}{36}$	$\frac{5}{11} \times \frac{22}{25}$	$\frac{33}{50} \times \frac{25}{44}$
194.	$\frac{25}{48} \times \frac{16}{15}$	$\frac{30}{35} \times \frac{7}{15}$	$\frac{25}{42} \times \frac{21}{50}$	$\frac{36}{65} \times \frac{26}{27}$

195. Class Exercise. —— may give two fractions and the class may find their product.

196. Class Exercise. —— may give a proper fraction and an improper fraction and the class may find their product.

197. Class Exercise. —— may give three fractions of such a kind that cancellation may be used in finding their product and the class may find the product, canceling wherever possible.

198. Multiply $\frac{4}{7}$ by itself.

199. Square:

$\frac{2}{3}$ $\frac{4}{5}$ $\frac{5}{7}$ $\frac{5}{9}$ $\frac{10}{11}$ $\frac{12}{13}$ $\frac{8}{7}$ $\frac{15}{16}$ $\frac{12}{20}$ $\frac{17}{15}$ $\frac{25}{50}$ $\frac{41}{55}$

200. What part of a square inch is a rectangle that is $\frac{1}{2}$ of an inch long and $\frac{1}{2}$ of an inch wide? Draw a figure and prove your work.

201. What part of a square yard is a rectangle $\frac{1}{3}$ of a yard square? How many square feet in it?

202. Multiply $\frac{2}{3}$ by the square of $\frac{2}{3}$.

203. Cube:

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{7}$ $\frac{1}{8}$ $\frac{2}{5}$ $\frac{3}{8}$ $\frac{5}{9}$ $\frac{7}{10}$ $\frac{4}{11}$ $\frac{7}{12}$

204. Draw a rectangle whose length is $\frac{1}{2}$ of a foot and width $\frac{1}{3}$ of a foot. What fraction of a square foot is its area? Prove by reducing the fractions of a foot to inches.

205. What fraction of a square foot is a rectangle $\frac{5}{6}$ of a foot long and $\frac{1}{2}$ of a foot wide? How many square inches in it?

206. How long is the perimeter of a rectangle $\frac{5}{6}$ of a foot long and $\frac{1}{2}$ of a foot wide? Give the area of the rectangle in fractions of a square foot, and also in square inches.

207. How many square feet in a square $\frac{3}{4}$ of a foot in dimensions? How many square inches?

208. How many square feet in a square $\frac{5}{6}$ of a foot in dimensions? How many square inches?

209. Add the product of $\frac{1}{7} \times \frac{3}{2}$ to the product of $\frac{5}{6} \times \frac{3}{7}$.

210. Subtract the product of $\frac{3}{8} \times \frac{4}{9}$ from the product of $\frac{9}{8} \times \frac{2}{3}$.

211. The product of several numbers is called their continued product. What is the continued product of 3, 5, and 7? Of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{8}$? Of $\frac{7}{8}$, $\frac{4}{21}$, and $\frac{3}{5}$? Of $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{4}{7}$?

212. Multiply $\frac{11}{15} \times \frac{3}{1}$.

Observe that the denominator of the fraction $\frac{3}{1}$ may be omitted without changing the result.

Multiply:

	a	*b*	*c*	*d*	*e*
213.	$\frac{3}{8} \times 24$	$\frac{4}{7} \times 35$	$\frac{5}{9} \times 27$	$\frac{7}{8} \times 64$	$\frac{2}{11} \times 33$
214.	$\frac{7}{10} \times 100$	$\frac{4}{5} \times 18$	$\frac{6}{7} \times 21$	$\frac{7}{12} \times 18$	$\frac{4}{9} \times 24$
215.	$36 \times \frac{5}{12}$	$48 \times \frac{7}{8}$	$32 \times \frac{3}{14}$	$39 \times \frac{2}{13}$	$60 \times \frac{11}{28}$
216.	$77 \times \frac{4}{11}$	$42 \times \frac{2}{3}$	$63 \times \frac{2}{27}$	$54 \times \frac{8}{45}$	$65 \times \frac{3}{13}$

217. Multiplying the numerator of a fraction by an integer has what effect upon the value of the fraction? Illustrate.

218. Dividing the denominator of a fraction by an integer has what effect upon the value of the fraction? Illustrate.

219. Give either of two ways by which a fraction may be multiplied by an integer.

220. Multiply 12 by a proper fraction. Is the product greater or less than 12?

221. Multiply 12 by an improper fraction. Is the product greater or less than 12?

222. When will the product of an integer and a fraction be greater than the integer?

223. Multiply 64 by $3\frac{1}{4}$.

$$\begin{array}{l} 64 \\ 3\frac{1}{4} \\ \hline 192 = \text{the product of 64 and 3.} \\ 16 = \text{the product of 64 and } \frac{1}{4}. \\ \hline 208 = \text{the product of 64 and } 3\frac{1}{4}. \end{array}$$

Multiply:

	a	*b*	*c*	*d*
224.	55	164	164	125
	$4\frac{1}{5}$	$2\frac{1}{8}$	$5\frac{2}{3}$	$5\frac{3}{5}$
225.	72	81	208	144
	$7\frac{1}{9}$	$13\frac{1}{4}$	$3\frac{1}{4}$	$6\frac{7}{12}$
226.	81	115	64	343
	$6\frac{1}{9}$	$10\frac{1}{3}$	$6\frac{3}{8}$	$4\frac{3}{7}$
227.	66	201	192	512
	$5\frac{1}{6}$	$11\frac{1}{7}$	$4\frac{3}{4}$	$7\frac{5}{8}$
228.	172	981	111	169
	$10\frac{1}{4}$	$2\frac{2}{3}$	$10\frac{1}{8}$	$8\frac{1}{13}$

	a	*b*	*c*	*d*
229.	385	243	78	408
	$7\frac{1}{5}$	$8\frac{2}{9}$	$12\frac{5}{6}$	$11\frac{3}{4}$

230. Multiply an integer by a mixed number and explain your method.

231. At 60¢ a yard, what is the cost of $7\frac{1}{2}$ yd. of silk? $8\frac{1}{3}$ yd.? $10\frac{3}{4}$ yd.? $12\frac{2}{5}$ yd.? $14\frac{1}{8}$ yd.? $15\frac{3}{8}$ yd.? $18\frac{7}{8}$ yd.?

232. 160 square rods equal 1 acre. How many square rods in $\frac{3}{4}$ of an acre? In $\frac{5}{8}$ A.? $\frac{9}{10}$ A.? $\frac{15}{16}$ A.? $\frac{3}{32}$ A.? $\frac{4}{5}$ A.?

233. Mr. Hill has a lot 40 rd. long and 3 rd. wide. What fractional part of an acre is it?

234. How many square rods in $2\frac{3}{4}$ A.? $5\frac{5}{8}$ A.? $7\frac{3}{10}$ A.? $4\frac{7}{20}$ A.?

235. Multiply $16\frac{2}{3}$ by 6.

$16\frac{2}{3}$
6

96 = the product of 16 and 6
4 = the product of $\frac{2}{3}$ and 6

100 = the product of $16\frac{2}{3}$ and 6

Multiply:

	a	*b*	*c*	*d*
236.	$15\frac{1}{4}$	$92\frac{1}{2}$	$39\frac{3}{4}$	$122\frac{2}{5}$
	8	24	12	22
237.	$124\frac{3}{8}$	$72\frac{2}{3}$	$164\frac{2}{4}$	$109\frac{1}{3}$
	16	15	4	27
238.	$24\frac{1}{3}$	$119\frac{3}{5}$	$81\frac{1}{8}$	$98\frac{2}{5}$
	9	15	16	5

239. Tell how a mixed number is multiplied by an integer.

240. At $18\frac{3}{4}¢$ per yard, what is the cost of 4 yd. of gingham? 7 yd.? 8 yd.? 10 yd.? 12 yd.?

241. How long is the perimeter of an equilateral triangle, a side of which is $4\frac{7}{12}$ ft. long?

242. If each side is $4\frac{9}{12}$ ft. long, how long is the perimeter of a rhombus? Of a regular octagon? Of a regular hexagon? Of a regular pentagon?

243. At $33\frac{1}{3}¢$ per yard, what is the cost of 8 yd. of dress goods? 9 yd.? 12 yd.? 14 yd.? 15 yd.? 18 yd.?

244. Multiply $2\frac{1}{2}$ by $3\frac{1}{4}$.

Before small mixed numbers are multiplied together, they should be reduced to improper fractions.

$$\frac{5}{2} \times \frac{13}{4} = \frac{65}{8} = 8\frac{1}{8}$$

Multiply:

245.

a	$2\frac{1}{2}$ by $3\frac{1}{4}$	*d*	$5\frac{5}{9}$ by $12\frac{1}{2}$
b	$3\frac{1}{3}$ by $6\frac{1}{4}$	*e*	$18\frac{3}{4}$ by $11\frac{1}{9}$
c	$4\frac{4}{9}$ by $6\frac{1}{4}$	*f*	$7\frac{1}{2}$ by $66\frac{2}{3}$

246. Square:

$2\frac{1}{2}$ $1\frac{1}{9}$ $3\frac{1}{3}$ $2\frac{1}{7}$ $6\frac{1}{4}$ $1\frac{2}{3}$ $1\frac{5}{7}$ $2\frac{3}{4}$

247. Cube:

$1\frac{1}{2}$ $1\frac{1}{3}$ $3\frac{1}{3}$ $3\frac{1}{2}$ $4\frac{1}{2}$ $5\frac{1}{2}$ $3\frac{2}{3}$ $2\frac{1}{4}$

248. Class Exercise. —— may give two small mixed numbers, and the class may find their product.

249. Class Exercise. —— may give a small mixed number, and the class may find its square.

250. Draw on the floor a square $5\frac{1}{2}$ yd. in dimensions. Each side is 1 rd. long. Find the number of square yards in a square rod.

251. How many feet long is a rod? Find the number of square feet in a square rod.

252. Each of the short sides of a rectangle is $7\frac{5}{8}$ in. The long sides are $9\frac{2}{5}$ in. each. Find the area of the rectangle.

253. One side of a rectangle is $11\frac{6}{7}$ in., and each of its adjacent sides is $3\frac{1}{2}$ in. shorter. Find the area of the rectangle.

254. Draw on paper or pasteboard a circle whose radius is $3\frac{1}{2}$ in. Cut it out. By measuring the circumference with a tape measure, it will be found to be nearly 22 in. $\frac{22}{7}$ is considered the ratio of the circumference of a circle to its diameter. What is the ratio of a diameter to the circumference?

255. Find the circumference of a circle whose diameter is 7 in. 21 in. 14 in.

256. How long is the circumference of a circle whose radius is 21 in.? $3\frac{1}{2}$ in.? $10\frac{1}{2}$ in.? $17\frac{1}{2}$ in.?

257. How long is the circumference of the largest circle that can be cut from a piece of paper $4\frac{2}{3}$ in. square?

258. A round flower bed 14 ft. across has a border of pinks, set 6 in. apart. How many pinks in $\frac{1}{11}$ of the border? Represent.

259. Mrs. Smith's wash bench is 4 ft. long and $1\frac{3}{4}$ ft. wide. A tub is set upon it in such a way that the lowest hoop of the tub touches the front edge and also the back edge of the bench without extending over either edge. What is the circumference of the hoop?

260. A round tin pail with straight sides is 8 in. across and 10 in. high. How long is the diameter of the largest plate that can be placed on the bottom of it? The circumference?

261. If a ball is cut into two equal parts by one cut, what is the shape of the flat surface of each part?

262. A round apple 4 in. in diameter was cut into halves. One of the halves was laid with its flat side down upon a plate, in such a way that no part of the cut surface of the apple extended beyond the plate. Find the diameter and the circumference of the smallest plate that could be used for that purpose.

263. A part of a circumference is called an **Arc**. Draw a circle and divide its circumference into several arcs.

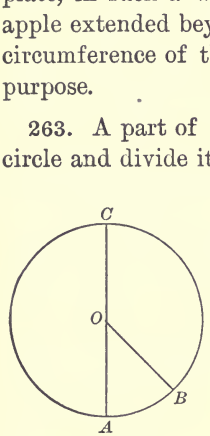


Fig. 2.

264. If the radius OA (Fig. 2) is 3 in., how long is the diameter AC? How long is the circumference?

265. What part of the circumference is the arc AC? How long is it?

266. The arc AB is $\frac{1}{8}$ of the circumference. How long is it?

267. What part of the circumference is the arc BC? How long is it?

268. If the radius of a circle is 5 in., how long is the diameter? The circumference? An arc which is $\frac{7}{11}$ of the circumference? $\frac{14}{25}$ of the circumference?

269. Find the length of an arc which is $\frac{7}{10}$ of the circumference of a circle whose radius is 5 in. $12\frac{1}{2}$ in. $37\frac{1}{2}$ in.

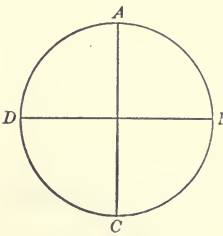


Fig. 3.

270. The circumference $ABCD$ is divided into how many equal arcs? If the diameter of the circle $ABCD$ is 7 in., how long is the arc AB? The arc ABC? The arc BCD? The arc $ABCD$?

271. An arc which is $\frac{1}{4}$ of a circumference is called a **Quadrant**. If the diameter of the circle $ABCD$ were 28 cm., how long would a quadrant be?

272. Multiply $124\frac{2}{3}$ by $6\frac{1}{2}$.

$124\frac{2}{3}$
$6\frac{1}{2}$

744 = the product of 124 by 6.
62 = the product of 124 by $\frac{1}{2}$.
4 = the product of $\frac{2}{3}$ by 6.
$\frac{1}{3}$ = the product of $\frac{2}{3}$ by $\frac{1}{2}$.

$810\frac{1}{3}$ = the product of $124\frac{2}{3}$ by $6\frac{1}{2}$.

273. This method of multiplying mixed numbers together is useful when the numbers are large. Can you see why?

Multiply:

	a	*b*	*c*	*d*
274.	$441\frac{7}{8}$	$344\frac{16}{21}$	$288\frac{3}{7}$	$456\frac{3}{8}$
	$16\frac{2}{7}$	$21\frac{3}{4}$	$14\frac{5}{8}$	$16\frac{5}{6}$
	a	*b*	*c*	*d*
275.	$819\frac{4}{5}$	$64\frac{2}{3}$	$816\frac{4}{7}$	$432\frac{2}{7}$
	$15\frac{5}{9}$	$9\frac{3}{4}$	$28\frac{1}{6}$	$42\frac{1}{12}$

276. Square $64\frac{3}{4}$. $32\frac{5}{8}$. $24\frac{5}{6}$. $22\frac{2}{11}$. $36\frac{7}{9}$.

277. Cube $12\frac{1}{4}$. $16\frac{1}{8}$. $15\frac{1}{3}$. $14\frac{1}{7}$.

278. At the rate of $17\frac{3}{5}$ mi. per hour, how far will a steamboat go between nine o'clock Monday morning and half past ten on Tuesday morning?

Find the cost:

279. Of $5\frac{1}{2}$ yd. of cloth at $\$ 4\frac{3}{5}$ a yard.

280. Of $7\frac{1}{5}$ A. of land at $\$ 24\frac{1}{4}$ an acre.

281. Of $7\frac{3}{5}$ T. of hay at $\$ 21\frac{1}{3}$ a ton.

282. Of $10\frac{1}{4}$ yd. of ribbon at $\$.23$ a yard.

283. Multiply $\frac{1}{10}$ by $\frac{1}{10}$ and write the product in decimal form.

284. Find the product of $\frac{17}{10}$ and $\frac{81}{100}$ and express it as a mixed number.

285. Express $\frac{17}{10}$ and $\frac{81}{100}$ in decimal form and find their product.

286. If asked for the product of two fractions having for denominators some power of 10, would you find it easier to multiply them as common fractions or as decimals? Why?

287. Give a rule for pointing off the product of two decimals.

288. Let $a = .04$, $b = .02$, $c = .005$, $d = .0007$, $e = .00002$, $f = .3$. Find the value of ab, ac, ad, ae, af, bc, bd, be, bf, cd, cf.

DIVISION OF FRACTIONS

289. How many times is 1 fourth of an inch contained in 3 fourths of an inch? $\frac{1}{6}$ of anything in $\frac{5}{6}$ of it? $\frac{3}{4} \div \frac{1}{4} = ?$ $\frac{5}{6} \div \frac{1}{6} = ?$

290. 6 sevenths $\div$ 2 sevenths $= ?$ $\frac{8}{9} \div \frac{2}{9} = ?$ 10 elevenths $\div$ 5 elevenths $= ?$ $\frac{12}{13} \div \frac{6}{13} = ?$

291. $\frac{8}{13} \div \frac{3}{13} = ?$

$\frac{8}{13}$ contain $\frac{3}{13}$ as many times as 8 units of any kind contains 3 units of the same kind. $8 \div 3 = \frac{8}{3}$ or $2\frac{2}{3}$.

292. What are similar fractions? Illustrate.

293. Divide $\frac{12}{17}$ by a similar fraction.

294. Class Exercise. —— may give two similar fractions, and the class may divide the greater by the less.

295. How is a fraction divided by a similar fraction?

296. Use $\frac{8}{9}$ as a dividend and $\frac{4}{9}$ as a divisor.

297. Multiply $\frac{3}{4}$ by its reciprocal.

298. Multiply several fractions by their reciprocals and compare the results.

299. How many times is $\frac{3}{4}$ of anything contained in $\frac{3}{4}$ of the same thing? $\frac{7}{8}$ in $\frac{7}{8}$? $\frac{5}{6}$ in $\frac{5}{6}$?

300. Which is greater, $\frac{3}{4} \div \frac{3}{4}$ or $\frac{3}{4} \times \frac{4}{3}$? $\frac{7}{8} \div \frac{7}{8}$ or $\frac{7}{8} \times \frac{8}{7}$?

301. $\frac{6}{7}$ divided by $\frac{2}{7}$ = ? $\frac{6}{7}$ multiplied by the reciprocal of $\frac{2}{7}$ = ?

302. Think of two similar fractions. Divide the larger fraction by the smaller. Compare the result with the result obtained by multiplying the larger fraction by the reciprocal of the smaller. Think of two fractions that are not similar. Reduce them to similar fractions and make the same comparison. Continue this until you see the reason for the following rule:

To divide a fraction by a fraction —

Multiply the dividend by the reciprocal of the divisor.

303. By this method divide $\frac{8}{11}$ by $\frac{2}{11}$.

Divide:	*a*	*b*	*c*	*d*
304.	$\frac{7}{16}$ by $\frac{3}{4}$	$\frac{8}{21}$ by $\frac{2}{7}$	$\frac{9}{16}$ by $\frac{3}{8}$	$\frac{7}{8}$ by $\frac{3}{5}$
305.	$\frac{8}{21}$ by $\frac{4}{7}$	$\frac{16}{25}$ by $\frac{4}{5}$	$\frac{18}{25}$ by $\frac{9}{10}$	$\frac{1}{4}$ by $\frac{3}{8}$
306.	$\frac{7}{12}$ by $\frac{3}{4}$	$\frac{5}{9}$ by $\frac{2}{3}$	$\frac{18}{25}$ by $\frac{4}{5}$	$\frac{21}{40}$ by $\frac{7}{10}$
307.	$\frac{35}{88}$ by $\frac{7}{11}$	$\frac{63}{75}$ by $\frac{9}{25}$	$\frac{25}{81}$ by $\frac{5}{9}$	$\frac{16}{25}$ by $\frac{4}{35}$
308.	$\frac{35}{72}$ by $\frac{35}{63}$	$\frac{15}{28}$ by $\frac{5}{7}$	$\frac{14}{25}$ by $\frac{7}{10}$	$\frac{11}{12}$ by $\frac{22}{27}$
309.	$\frac{8}{9}$ by $\frac{4}{27}$	$\frac{15}{28}$ by $\frac{5}{21}$	$\frac{42}{55}$ by $\frac{7}{33}$	$\frac{18}{35}$ by $\frac{9}{10}$
310.	$\frac{17}{27}$ by $\frac{2}{3}$	$\frac{45}{88}$ by $\frac{9}{22}$	$\frac{54}{77}$ by $\frac{18}{15}$	$\frac{21}{88}$ by $\frac{7}{22}$

311. How do you find the ratio of one number to another? Find the ratio of $\frac{35}{48}$ to $\frac{3}{16}$. To $\frac{5}{12}$. To $\frac{7}{24}$.

Find ratios:

	a	*b*	*c*	*d*
312.	$\frac{5}{12} : \frac{3}{4}$	$\frac{25}{42} : \frac{5}{7}$	$\frac{22}{45} : \frac{11}{18}$	$\frac{11}{20} : \frac{22}{25}$
313.	$\frac{9}{25} : \frac{3}{5}$	$\frac{19}{25} : \frac{2}{15}$	$\frac{9}{16} : \frac{3}{4}$	$\frac{14}{27} : \frac{7}{9}$
314.	$\frac{25}{66} : \frac{5}{11}$	$\frac{14}{15} : \frac{7}{20}$	$\frac{5}{7} : \frac{5}{9}$	$\frac{16}{21} : \frac{4}{7}$

315. Multiply the numerator of a fraction by an integer. Is the value of the fraction increased or decreased?

316. Find by trial how the value of a fraction is changed by multiplying its denominator. By dividing its numerator. By dividing its denominator.

317. Which gives the greater quotient, 16 divided by 8, or 16 multiplied by $\frac{1}{8}$?

318. $\frac{16}{21} \div 8 = ?$

We may write 8 as $\frac{8}{1}$. The question is then $\frac{16}{21} \div \frac{8}{1}$ = what? This is solved by the rule for dividing one fraction by another.

Find values of x:

	a	b	c
319.	$\frac{6}{7} \div 2 = x$	$\frac{15}{17} \div 3 = x$	$\frac{6}{17} \div 12 = x$
320.	$\frac{7}{8} \div 7 = x$	$\frac{8}{11} \div 16 = x$	$\frac{9}{20} \div 6 = x$
321.	$\frac{8}{9} \div 4 = x$	$\frac{2}{13} \div 8 = x$	$\frac{22}{25} \div 11 = x$

322. Divide 28 by $\frac{4}{9}$.

Consider 28 as $\frac{28}{1}$.

323. Divide 24 by: $\frac{3}{7}$. $\frac{8}{9}$. $\frac{6}{11}$. $\frac{2}{3}$. $\frac{4}{5}$. $\frac{12}{17}$. $\frac{5}{6}$.

324. Divide 41 by: $\frac{2}{3}$. $\frac{3}{4}$. $\frac{4}{5}$. $\frac{6}{7}$. $\frac{8}{9}$. $\frac{12}{13}$. $\frac{16}{17}$.

Divide after reducing mixed numbers to improper fractions:

	a	b	c	d
325.	$3\frac{1}{2} \div 2\frac{1}{2}$	$7\frac{1}{2} \div 18\frac{3}{4}$	$8\frac{1}{3} \div 1\frac{2}{3}$	$5\frac{5}{9} \div 16\frac{2}{3}$
326.	$2\frac{1}{3} \div 3\frac{1}{3}$	$13\frac{1}{3} \div 4\frac{4}{9}$	$6\frac{2}{3} \div 7\frac{1}{2}$	$41\frac{2}{3} \div 62\frac{1}{2}$
327.	$3\frac{1}{2} \div 6\frac{1}{4}$	$12\frac{1}{2} \div 5\frac{5}{9}$	$37\frac{1}{2} \div 18\frac{3}{4}$	$87\frac{1}{2} \div 3\frac{1}{8}$

328. Class Exercise. —— may give a small mixed number for a dividend and another small mixed number for a divisor, of such a kind that cancellation may be used in finding the quotient. The class may find their quotient.

329. At a picnic $5\frac{1}{2}$ pies were divided equally among 44 persons. What part of a pie did each receive?

330. Mr. Tod has $13\frac{1}{3}$ acres devoted to celery, which is just four times as much as his brother has. How many acres of celery has his brother?

331. The top of a newel post is an octagon whose perimeter is $1\frac{1}{12}$ ft. How long is one side of the octagon?

332. The circumference of a circle is $9\frac{3}{7}$ in. How long is its diameter? Its radius? Represent.

333. Find the length of the diameter of a circle, the circumference of which is $11\frac{4}{7}$ in.

334. A quadrant of a circle is $4\frac{5}{7}$ in. How long is the circumference? Diameter? Radius?

335. For 75¢, how many yards of lace can be bought at $2\frac{1}{2}$ ¢ per yard? At $3\frac{1}{3}$ ¢? At $6\frac{1}{4}$ ¢? At $8\frac{1}{3}$ ¢? At $12\frac{1}{2}$ ¢? At $16\frac{2}{3}$ ¢? At $37\frac{1}{2}$ ¢? At $62\frac{1}{2}$ ¢? At $66\frac{2}{3}$ ¢? At $83\frac{1}{3}$ ¢? At $87\frac{1}{2}$ ¢?

336. If a coat costs $12\frac{1}{2}$ dollars, how many coats may be bought for $62\frac{1}{2}$ dollars?

337. If $1\frac{3}{4}$ yd. of cloth are required for a coat, how many coats can be made from $87\frac{1}{2}$ yd.?

338. If Jerry walks $2\frac{1}{2}$ mi. an hour, in how many hours will he walk $7\frac{1}{2}$ mi.? $1\frac{1}{2}$ mi.? 1 mi.? $\frac{1}{2}$ mi.? $\frac{1}{4}$ mi.? $\frac{1}{8}$ mi.?

339. What number divided by 3 will give 5 for a quotient? What fraction divided by 3 will give $\frac{5}{8}$ for a quotient? Prove.

340. What mixed number multiplied by 3 will give $16\frac{1}{2}$? $5\frac{4}{7}$? $22\frac{1}{5}$? $18\frac{1}{2}$? $6\frac{3}{7}$?

341. A fraction that has a fraction in one or both of its terms is called a **Complex Fraction**; as, $\frac{\frac{1}{2}}{\frac{3}{4}}$, $\frac{2\frac{1}{2}}{7\frac{1}{2}}$, $\frac{7}{2\frac{1}{2}}$. Write a complex fraction whose numerator is a mixed number, and denominator a whole number.

$\frac{\frac{1}{2}}{\frac{3}{4}}$ is read "$\frac{1}{2}$ divided by $\frac{3}{4}$."

342. Write and read complex fractions as follows:

a Numerator an integer, denominator a simple fraction.

b Numerator a simple fraction, denominator a mixed number.

c Numerator an integer, denominator a mixed number.

d Numerator and denominator both simple fractions. Both mixed numbers.

343. A complex fraction, like other fractions, is merely an expression of the division of the numerator by the denominator, and it is reduced to a simple fraction by performing that division; as,

$$\frac{\frac{5}{6}}{\frac{1}{2}} = \frac{5}{6} \div \frac{1}{2}, \text{ or } \frac{5}{\cancel{6}} \times \frac{\cancel{2}}{1} = \frac{5}{3}.$$
$$\quad 3$$

Reduce to their simplest form the following complex fractions:

a	*b*	*c*	*d*	*e*	*f*
$\frac{4\frac{2}{7}}{3\frac{1}{8}}$	$\frac{1\frac{1}{3}}{4\frac{4}{9}}$	$\frac{3\frac{1}{2}}{7}$	$\frac{1\frac{2}{3}}{5\frac{5}{9}}$	$\frac{3\frac{5}{7}}{13}$	$\frac{17}{8\frac{1}{2}}$

344. Simplify:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
$\frac{8\frac{1}{3}}{33\frac{1}{3}}$	$\frac{12\frac{1}{2}}{83\frac{1}{3}}$	$\frac{16\frac{2}{3}}{87\frac{1}{2}}$	$\frac{41\frac{2}{3}}{91\frac{2}{3}}$	$\frac{2\frac{1}{2}}{20}$	$\frac{3\frac{1}{3}}{30}$	$\frac{6\frac{1}{4}}{50}$	$\frac{7\frac{1}{2}}{60}$

345. Class Exercise. —— may give the hardest complex fraction that he wrote in Ex. 342, and the class may reduce it to a simple fraction.

346. The product of two numbers is 15. One of them is 3. What is the other? How is it found?

347. The product of two fractions is $\frac{9}{35}$. One of them is $\frac{3}{7}$. What is the other?

348. Class Exercise. —— may give the product of two fractions and one of the fractions. The class may find the other fraction.

349. Divide:

$\frac{1}{100} \div \frac{1}{10}$ $\frac{1}{1000} \div \frac{1}{10}$ $\frac{1}{10000} \div \frac{1}{100}$ $\frac{8}{1000} \div \frac{4}{100}$

350. Reduce the same fractions to decimal form and find the quotients. Which is the easier way of dividing in this case? Why?

351. Let $a=.4$, $b=.08$, $c=.032$, $d=.0016$. Find values of:

$$\frac{a}{b} \quad \frac{a}{c} \quad \frac{a}{d} \quad \frac{b}{a} \quad \frac{b}{c} \quad \frac{b}{d}$$

$$\frac{c}{a} \quad \frac{c}{b} \quad \frac{c}{d} \quad \frac{d}{a} \quad \frac{d}{b} \quad \frac{d}{c}$$

352. Change $\frac{3}{5}$ and $1\frac{3}{25}$ to decimals and divide the greater by the less.

353. Change to decimals and divide: $\frac{5}{8}$ by $\frac{1}{4}$. $\frac{7}{8}$ by $\frac{3}{4}$.

354. If 4.5 yd. of silk cost \$ 6.75, how much will 1 yd. cost? 3.7 yd.? 6.75 yd.?

355. Mr. K bought a lot in Washington, D.C., for \$ 4500, paying \$1.875 per square foot. How many square feet in the lot?

356. If 1.7 yd. of cloth is used to make a coat, how many coats can be made from 81.6 yd.?

357. How much cloth at \$.75 a yard can be bought for \$ 45.75?

358. At the rate of 8.25 mi. an hour, in how many hours will a stage run 125 mi.?

359. If a barrel of beef costs \$ 14.25, how many barrels can be bought for \$ 798?

360. Traveling 215.6 mi. a day, in how many days will a steamer go 1000 mi.?

361. If a dollar gains 5¢ interest each year, in how many years will it gain another dollar?

362. In how many years will $1.00 double itself at 4%? 8%? 10%?

363. One third of John's money is 7¢. How much has he?

364. Thomas spent $\frac{2}{3}$ of his money and had 5¢ left. How much had he at first? Explain.

365. How much money has a boy who can spend $\frac{3}{4}$ of his money and have left 7¢? 9¢? 2¢? 12¢? $1.00? $8.00?

366. How much money has a boy who after spending 3¢ will have left $\frac{4}{5}$ of his money? $\frac{7}{8}$? $\frac{9}{10}$? $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$?

367. Three fourths of John's money is 15¢. How much is $\frac{1}{4}$ of it? How much is the whole?

368. $6 = \frac{3}{5}$ of what number?

SOLUTION BY ANALYSIS. As 3 fifths of the number = 6
1 fifth of the number = 2
5 fifths of the number = 10

369. 8 is $\frac{4}{5}$ of what number? Analyze as above.

370. Analyze. 12 is $\frac{2}{5}$ of what number? $\frac{4}{7}$ of what?

371. Find values of x. Analyze.

a $\frac{2}{11}$ of $x = 14$	e $\frac{3}{7}$ of $x = 15$	i $\frac{6}{7}$ of $x = 18$
b $\frac{2}{3}$ of $x = 10$	f $\frac{8}{9}$ of $x = 16$	j $\frac{10}{9}$ of $x = 20$
c $\frac{5}{6}$ of $x = 25$	g $\frac{4}{9}$ of $x = 28$	k $\frac{7}{6}$ of $x = 28$
d $\frac{6}{7}$ of $x = 24$	h $\frac{3}{4}$ of $x = 30$	l $\frac{5}{4}$ of $x = 75$

372. How long is a line $\frac{3}{4}$ of which is 9 in. long?

373. A man rode 16 mi. on Monday, which was $\frac{2}{3}$ of the distance he rode on Tuesday. How far did he ride on Tuesday?

374. $\frac{3}{5}$ of John's money is 21¢. How much will he have left if he gives away $.05?

375. $\frac{2}{3}$ of James's money is $20. How much will he have left if he gives away $\frac{1}{5}$ of his money?

376. How much money must a boy have that he may lose $\frac{1}{6}$ of it and have 10 ¢ left? 30 ¢? 40 ¢? 60 ¢?

377. How much money must a man have so that after gaining $\frac{1}{6}$ as much, he may have $700? $210? $441? $7.70?

378. Mary has 12 ¢. Her money equals $\frac{3}{10}$ of Florence's money. How much has Florence?

379. William has 8 marbles. He has $\frac{2}{5}$ as many marbles as James has. How many marbles has James?

380. Alice has 14 ¢, which is just $\frac{2}{9}$ of the money she needs to buy her geography. What is the price of the geography?

381. Make problems in which a certain number is a fractional part of the number which is to be found.

382. John gave away $\frac{2}{3}$ of his marbles and then had 30 marbles left. How many had he at first?

383. Susie gave $\frac{3}{5}$ of her money to her sister, and found that she had 16 cents left. How much had she at first?

384. Harry gave away $\frac{3}{8}$ of his pigeons and sold $\frac{3}{8}$ of them. He had 15 pigeons left. How many had he at first?

385. In a storm, a ship's crew threw overboard 30 bbl. flour, which was $\frac{3}{11}$ of the whole cargo. How much was the whole?

386. A man owned $\frac{4}{5}$ of a mine. He sold $\frac{1}{2}$ of his share for $6000. How much was the whole mine worth?

387. Mr. Buchanan sold $\frac{1}{8}$ of his share of a store for $2000. What was his share worth? His share was $\frac{3}{7}$ of the whole value. What was the whole value?

388. Owning $\frac{1}{2}$ of a quarry, Mr. Harris sold $\frac{6}{7}$ of his share for $6000. What was the value of the quarry?

389. Mr. Madison owned $\frac{1}{4}$ of an Indiana gas well. He sold $\frac{3}{5}$ of his share for \$1500. What was the value of the whole well?

390. $\frac{1}{12}$ of the pins in a cushion were crooked, and there were 66 straight pins. How many were there in all, and how many were crooked?

391. If $8\frac{3}{4}$ yd. of tape cost \$.70, how much will 1 yd. cost? $9\frac{1}{2}$ yd.?

392. If $16\frac{2}{3}$ yd. of rope cost 100 cents, how much will 1 yd. cost? $23\frac{3}{4}$ yd.?

393. If 6 bu. of seed cost \$15, how much will $19\frac{3}{5}$ bu. cost?

394. If $\frac{3}{8}$ of a quart of seed cost \$.18, how much will 1 pk. cost?

MISCELLANEOUS EXERCISES

1. Divide .0096 by .12. By .008. By 24. By 3.2. By .16.

2. Divide .000048 by .012. By .4. By .0024. By 2.4.

3. Divide .144 by .04. By 48. By 1.6. By .0003.

4. What is the ratio of 889.44 to .102? To .105? To .108?

5. Multiply 7^2 by the 4th prime. 9^2 by the 6th prime.

6. $\frac{3}{2}$ of $\frac{7}{15}$ of $\frac{5}{12}$ of 24 hr. is how much less than a day?

7. 24 sheets of paper make a quire. How many sheets in $\frac{3}{8}$ of $\frac{5}{6}$ of $\frac{4}{5}$ of a quire?

8. How many sheets in $\frac{7}{12}$ of $\frac{4}{7}$ of $\frac{3}{8}$ of a quire? In $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{6}{35}$ of a quire?

9. Mrs. Smith is $\frac{3}{4}$ as old as Mr. Smith, who is 48 yr. old. Their daughter Alice is $\frac{4}{9}$ as old as her mother. How old is Alice?

10. Add $\frac{1}{15}$, $\frac{1}{25}$, $\frac{1}{45}$, $\frac{1}{10}$. Add $\frac{1}{54}$, $\frac{1}{81}$, $\frac{1}{45}$, $\frac{1}{18}$.

11. From $\frac{1}{81}$ take $\frac{1}{108}$. From $\frac{1}{54}$ take $\frac{1}{63}$.

12. Multiply the first prime number after 9 by $\frac{2}{3}$.

13. Multiply the largest prime factor of 330 by $2\frac{3}{7}$.

14. Multiply $64\frac{1}{5}$ by the largest prime factor of 390.

15. Multiply $8\frac{4}{5}$ by the l. c. m. of 5, 6, and 10.

16. Multiply the l. c. m. of 8, 6, 9, and 12 by $3\frac{1}{24}$.

17. Multiply the g. c. d. of 36, 48, and 60 by $3\frac{1}{2}$.

Let $a = \frac{2}{7}$; $b = 2\frac{1}{2}$; $c = 10\frac{1}{2}$; $d = 5\frac{1}{2}$; $e = \frac{4}{7}$. Find values of:

18. $a \times b$ or ab $\quad ac \quad ad \quad ae \quad bc \quad bd \quad cd \quad ce \quad de$

19. $a + c \quad a + e \quad a + d \quad b + c \quad b + \quad d + e$

20. $c - a \quad d - a \quad e - a \quad c - b \quad c - d \quad d - b$

Let $a = \frac{5}{6}$; $b = 3\frac{1}{3}$; $c = 1\frac{7}{20}$; $d = 2\frac{2}{3}$; $e = \frac{1}{6}$. Find values of:

21. $\frac{a}{b} \quad \frac{a}{c} \quad \frac{a}{d} \quad \frac{a}{e} \quad \frac{b}{e} \quad \frac{b}{d} \quad \frac{b}{a} \quad \frac{c}{b} \quad \frac{d}{e}$

22. $\frac{5}{6}$ of Anna's money is \$.50. How much will she have after giving away 7¢?

23. Of what number is 21 three fourths? $\frac{3}{8}$? $\frac{3}{7}$? $\frac{3}{10}$?

24. Of what number is 16 four sevenths? $\frac{4}{9}$? $\frac{4}{5}$? $\frac{4}{11}$?

25. If $\frac{4}{15}$ of the price of a house is \$400, what is the price of the house? How much will five such houses cost?

26. If 1 apple costs $\frac{2}{3}$¢, how much will 4 doz. apples cost?

27. If $\frac{3}{4}$ of the price of an orange is 3¢, how much will a dozen oranges cost? 6 doz.?

28. At $16\frac{2}{3}$¢ a yard, how many yards of ribbon can be bought for \$1? \$2? \$3? \$5?

29. 12 doz. make a gross. When buttons are bought for 25¢ a gross, what is the cost of 1 button?

30. If 500 pins cost 10¢, how much will 1 pin cost?

31. If a gross of pencils cost 50¢, how much will 1 pencil cost?

32. Mrs. Norton paid 5¢ for a box of toothpicks, in which there were 2000 toothpicks. How many did she get for a cent? What was the cost of 1 toothpick?

33. She paid a nickel for a box of matches. What was the price of each match if there were 500 matches in a box?

34. A gross of glass vials cost 48¢. How much did 1 vial cost?

35. Find the average receipts of a peanut stand for 6 days. Monday, $1.37; Tuesday, $2.11; Wednesday, $1.87; Thursday, $1.04; Friday, $1.75; Saturday, $3.10.

36. If the average cost of keeping up the stand was $1.25 per day, what were the owner's average gains per day?

37. Suppose a pie to be exactly round, and $10\frac{1}{2}$ in. in diameter. If it were cut into 6 equal pieces, how long would the curved edge of each piece be?

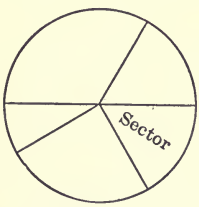


FIG. 4.

38. The surface which is bounded by an arc and two radii is called a **Sector**. Show five sectors in Fig. 4.

You may remember the figure of a sector more easily if you recall the way in which pies, waffles, and round cakes are usually cut.

39. How long is the perimeter of a sector of a circle whose radius is 4 in., if the arc of the sector is $5\frac{3}{7}$ in.? Represent.

40. Draw a circle whose radius is $3\frac{1}{2}$ in. Divide it into 4 equal sectors. Write the word "sector" in each. Write upon each line of the perimeter of a sector the length of the line and find the length of the perimeter of a sector.

41. What kind of an angle is the angle of a sector which is $\frac{1}{4}$ of a circle? Less than $\frac{1}{4}$? Greater than $\frac{1}{4}$?

42. Draw a circle and apply its radius six times as a chord. What regular polygon have you drawn? Each arc thus cut off is what part of the circumference? If the radius is $2\frac{5}{8}$ in., how long is the circumference? Each arc?

43. Draw lines from the ends of each arc to the center of the circle. What are these lines called? What kind of angles do they make?

44. Erase the chords. Find length of the perimeter of each sector, supposing the radius of the circle to be $2\frac{5}{8}$ in. $5\frac{1}{4}$ in. $10\frac{1}{2}$ in.

45. Erase radii so as to leave the circle divided into three equal sectors. Find length of perimeter of each sector, assuming the radius to be $\frac{7}{8}$ in. $\frac{9}{11}$ in. $1\frac{3}{11}$ in.

46. If the radius of a circle is $2\frac{4}{5}$ in., how long is the perimeter of a sector which is $\frac{1}{6}$ of the circle?

47. Find the perimeter of the sector which remains when a sector that is $\frac{1}{6}$ of a circle is subtracted from the circle.

48. If a strip of paper 5 in. long were curled around so that its edge inclosed a circle, how long would the circumference of that circle be?

49. The circumference of the wheel of a toy wagon is 20 in. How far does the wagon run when the wheel turns around once? 3 times?

SUGGESTION. Let pupils roll a coin, button, or other circular objects, as a help in realizing the conditions of these problems.

50. How far will a hoop $2\frac{1}{2}$ ft. in circumference run in turning 7 times? 9 times?

51. How many times will a wheel 2 ft. in circumference revolve in running 8 ft.?

52. How many times will a wheel 6 ft. in circumference revolve in running 12 ft.? 8 yd.?

53. A mile is 5280 ft. If the front wheels of a wagon are each 6 ft. in circumference, and the hind wheels are 8 ft., how many times will each wheel revolve in running a mile?

54. Draw a rhomboid whose long sides are each double a short side. How long would its perimeter be if each short side were $8\frac{1}{3}$ in.? $12\frac{1}{2}$ in.?

55. How many square feet in a lot 30 ft. wide, and 150 ft. deep? If the owner uses $\frac{1}{4}$ of the lot for a house, and $\frac{1}{4}$ for a chicken yard, how many square feet remain?

56. A house is 48 ft. long, and the distance from the ridge-pole to the eaves on each side is 23 ft. How many shingles will be required to cover it if 9 shingles are required to cover a square foot?

57. John is $8\frac{7}{15}$ yr. old, and his sister is $6\frac{7}{12}$ yr. old. What is their average age?

58. A grocer bought 3 cheeses, one weighing $32\frac{7}{8}$ lb., another $28\frac{3}{4}$ lb., another $41\frac{3}{8}$ lb. How many pounds were there in all?

59. Which is greater, and how much, $\frac{8}{9} \times \frac{3}{4}$, or $\frac{8}{9} \div \frac{3}{4}$?

60. What is the area of a rectangle $3\frac{3}{4}$ ft. long and $1\frac{3}{5}$ ft. wide?

61. What is the area of a right triangle whose base is $4\frac{1}{5}$ in., and altitude $3\frac{1}{5}$ in.?

62. What is the use of reducing fractions to a least common denominator?

63. Write the fraction that expresses the ratio of the first composite number after 18 to the first composite number after 30, and reduce the fraction to its lowest terms.

64. When Arthur was a year old his father placed \$ 50 in the bank as money to be used in sending him to college. He put \$ 50 in the bank on every birthday until, at the age

of 18, Arthur was ready for college. How much money had been placed in the bank for him?

65. Arthur's expenses at college for the first year were $218.75; for the second year, $310.50; for the third year, $365.25. How much of the amount was left at the end of the third year?

66. Arthur's expenses for the last year were $410.90. He received $465.67 as interest. How much was left of the money when he had finished his college course?

67. A man bought a lot for $2000, built a house upon it for $2500, and sold the property so as to gain $100 on his investment. For how much did he sell it?

68. Charles bought a ball for $.08, and sold it for $.12. The gain equaled what part of the cost? What per cent?

69. A man's salary is $2400 a year. He saves $\frac{1}{6}$ of it one year, $\frac{1}{3}$ of it the next year, and $\frac{3}{8}$ of it the next year. How much has he saved at the end of the third year?

70. A gentleman had 1200 books in his library, and gave away $\frac{2}{5}$ of them. He lost $\frac{1}{6}$ of the remainder. How many books were left?

71. He added 200 more volumes to the library, and then gave away $\frac{3}{8}$ of it. How many had he left?

72. The base of an isosceles triangle is 3 ft. The ratio of one of the equal sides to the base is $\frac{5}{3}$. How long is the perimeter of the triangle? Represent.

73. How long is the perimeter of an isosceles triangle whose base is 14 in. and each of whose equal sides is 5 in. longer than the base?

74. How long is the perimeter of an isosceles triangle whose base is 21 in. and each of whose equal sides is $33\frac{1}{3}\%$ longer than the base?

75. The perimeter of a certain isosceles triangle is 25 in. and one of the equal sides is 9 in. How long is the base?

76. The base of an isosceles triangle is 11 in. and the perimeter 35 in. How long is each of the equal sides?

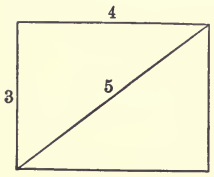


FIG. 5.

77. Draw a rectangle 4 in. long and 3 in. wide. Draw a diagonal of it. Into what kind of figures does a diagonal divide a rectangle? If the angles of your figure are exact right angles, and if your lines are exactly drawn, the diagonal will be just 5 in. long.

SUGGESTION TO TEACHER. Let pupils find by trial that if 3 in. be measured off upon one of the lines about a right angle and 4 in. upon the other line, the joining line will be 5 in. long.

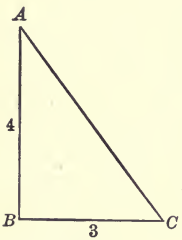


FIG. 6.

78. In the right triangle ABC how long is the hypotenuse AC if the numbers represent inches?

79. If BC and AB were each twice as long as they are, AC would be twice as long as it is. If AB is 8 in. and BC 6 in., how long is AC? Prove by measuring.

80. If the perpendicular sides of a right triangle are in the ratio of 3 to 4, the ratio of the hypotenuse to the less side is $\frac{5}{3}$, and the ratio of the hypotenuse to the other side is $\frac{5}{4}$. In a right triangle whose base is 30 and altitude 40 how long is the hypotenuse? Represent.

81. How long is the hypotenuse of a right triangle whose perpendicular sides are 9 in. and 12 in.? 21 and 28? 15 and 20? 33 and 44?

82. The first steamship crossed the Atlantic Ocean in MDCCCXIX. For how many years has it been possible for Americans to go to Europe in a steamship?

83. Imagine a block of ice 1 yd. in dimensions. How many square feet are there in all the surfaces?

84. Imagine the same figure with one cubic foot cut out of one corner of it. How many square feet in all its surfaces?

85. Imagine a cubic yard of ice, and suppose a cubic foot of it to be cut from the middle of one side. How many square feet in all the surfaces of the solid that is left?

86. How many square feet in all the surfaces of the solid that would be left, if the cubic foot were put back in its place and the cubic foot above it were taken away?

87. If a box 1 yd. in dimensions were packed $\frac{2}{3}$ full of groceries, how many cubic feet of space would be left?

88. What part of a cubic yard is a cube which is $\frac{2}{3}$ of a yard in dimensions? How many cubic feet are there in it?

89. A coal dealer bought 1246 tons of coal at $\$4\frac{7}{8}$ a ton, and sold it for $\$6\frac{7}{8}$ a ton. What was his gain on each ton? On the whole?

90. A man bought $\$88\frac{1}{5}$ worth of furniture, paying in weekly installments of $\$14\frac{7}{10}$ each. In how many weeks did he pay for the furniture?

91. A grocer bought strawberries at the rate of 4 boxes for a quarter, and sold them at the rate of 3 boxes for a quarter. How much did he gain on each box? On a dozen boxes? On a gross of boxes?

92. Mr. Jones worked $\frac{5}{6}$ of a day on Monday, $\frac{3}{4}$ of a day on Tuesday, and a whole day on Wednesday, on Thursday, and on Friday. On Saturday he worked $\frac{1}{2}$ a day. At $\$3$ per day, how much did he earn in the week?

CHAPTER V

DENOMINATE NUMBERS

1. How many feet equal a yard? How many pints equal a quart? How many ounces equal a pound? How was it decided in these cases how many units of a certain denomination should make one of the next higher denomination?

SUGGESTION TO TEACHER. Let facts concerning the origin of our systems of measuring be obtained from encyclopedias and other sources of information and brought to the class. Pupils should understand that the value of a unit in terms of lower denominations is an arbitrary value, varying in different kinds of measurements.

2. Numbers that show measurements whose values are settled by custom or law are called **Denominate Numbers,** as 5 bushels, 2 hours, 1 dollar. Denominate numbers that consist of more than one denomination are called **Compound Denominate Numbers.** Write a compound denominate number whose largest denomination is bushels. Hours. Tons. Miles. Acres. Gallons. Dollars. Meters.

3. The denominations of United States money are mills (m.), cents (¢.), dimes (d.), dollars ($) and Eagles (E.).

UNITED STATES MONEY

One dollar is the standard

1 eagle = 10 dollars

1 dollar

1 dime = $\frac{1}{10}$ of a dollar

1 cent = $\frac{1}{100}$ of a dollar

1 mill = $\frac{1}{1000}$ of a dollar

4. Name each denomination of the following: $5.875. $10.125. $20.705.

5. Express 3 dollars as cents. As dimes. As eagles.

6. A 10-dollar gold piece is called an **Eagle.** A 20-dollar gold piece is called a **Double Eagle.** A 5-dollar gold piece is called a **Half Eagle.** What is the value of a **Quarter Eagle?**

7. Name the silver coins. What other coins are there?

8. How much money has a man who has 2 double eagles, an eagle, 3 half eagles, a quarter eagle, 2 dollars, 3 dimes, 2 nickels, and 3 cents?

9. Class Exercise. —— may name a certain number of different kinds of coins, and the class may find the amount of money which their sum equals.

10. Treasury or bank notes are also used as money. If a man has eight $100 bills, seven $20 bills, a $10 bill, a $5 bill, a $2 bill, and three $1 bills, how much less than $1000 has he?

11. How many mills in a dollar? In a half eagle? Why is there no coin to represent a mill?

12. The denominations of liquid measure are gills (gi.), pints (pt.), quarts (qt.) and gallons (gal.).

Liquid Measure

	4	2	4
gal.	qt.	pt.	gi.

Over the abbreviation of each denomination above you will find the number of units that equal a unit of the next higher denomination.

Give the table of liquid measure, beginning with the units of the lowest denomination.

13. Fill the blank in the following table of equivalent values:

1 gal. = 4 qt. = 8 pt. = — gi.

Suggestion to Teacher. Pupils should make actual measurements, so far as is practicable, in connection with the study of each table, and should learn to change rapidly units of one denomination into units of another.

14. Illustrate each of the following statements:

a As 4 gills equal a pint, any number of pints equals 4 times as many gills as pints.

b As 2 pints equal a quart, any number of quarts equals twice as many pints as quarts.

c As 4 quarts equal a gallon, any number of gallons equals 4 times as many quarts as gallons.

15. Express $\frac{3}{5}$ gal. as quarts. $\frac{5}{6}$ qt. as pints. $\frac{3}{4}$ pt. as gills.

16. Express $\frac{6}{7}$ gal. as quarts. $\frac{2}{3}$ qt. as pints. $\frac{8}{11}$ pt. as gills.

17. Express .75 gal. as quarts. .5 qt. as pints. .625 pt. as gills.

18. Express .375 gal. as quarts. As pints.

19. Express $5\frac{1}{2}$ gal. as quarts. As pints. As gills.

20. Express $7\frac{1}{4}$ gal. as pints.

21. Express $2\frac{3}{4}$ gal. as gills.

22. Express $3\frac{1}{2}$ gal. as quarts. 3 gal. and 2 qt. as quarts.

23. How many quarts in 5 gal. 2 qt.? 7 gal. 1 qt.?

24. How many pints in 7 qt. 3 pt.? In 1 gal. 3 pt.?

25. How many gills in 1 pt. 3 gi.? 1 qt. 3 gi.? 3 qt. 1 gi.?

26. 1 gi. equals what part of 1 qt. 1 pt.?

1 qt. 1 pt. = 3 pt., which equal 12 gi.
1 gi. $= \frac{1}{12}$ of 12 gi.

27. 1 gi. equals what part of 2 qt. 1 pt.? Of 3 qt. 1 pt.? Of 1 gal. 1 pt.?

28. Illustrate each of the following statements:

a As 4 gills make a pint, any number of gills equals $\frac{1}{4}$ as many pints as gills.

b As 2 pints equal 1 quart, any number of pints equals $\frac{1}{2}$ as many quarts as pints.

c As 4 quarts equal 1 gallon, any number of quarts equals $\frac{1}{4}$ as many gallons as quarts.

29. Express 32 gi. as pints. As quarts. As gallons.

30. Express 40 pt. as quarts. As gallons.

31. Express 25 gi. as pints. *Ans.* $6\frac{1}{4}$ pt.

32. Express 7 pt. as quarts. 9 qt. as gallons.

33. Express 11 pt. as quarts. As gallons.

34. Express 13 pt. as quarts and pints. *Ans.* 6 qt. 1 pt.

35. Express $15\frac{1}{2}$ pt. as quarts, pints, and gills.

36. Express $17\frac{1}{2}$ qt. as gallons, quarts, and pints.

37. Express $19\frac{1}{2}$ pt. as gallons, quarts, pints, and gills.

38. 1 gi. equals what part of a pint? Of a quart? Of a gallon?

39. 3 gi. equal what part of a quart? Of a gallon?

40. Express $1\frac{3}{4}$ pt. as quarts. As gallons.

41. Express 1 pt. 3 gi. as pints.

42. Express 2 qt. 1 pt. 2 gi. as pints. As quarts. As gallons.

43. Which is greater and how much, 2 gal. 1 qt. 3 pt. or 22 pt.?

44. Express 1 pt. as a decimal of a quart. 3 qt. as a decimal of a gallon.

45. At $6\frac{1}{4}$ cents a quart, how much will a gallon of cider cost? $3\frac{1}{2}$ gal.? $4\frac{3}{4}$ gal.? 1 gal. 3 qt.? 1 pt.? 3 qt. 1 pt.?

46. Name several articles that are measured by liquid measure.

47. Add:

gal.	qt.	pt.	gi.
2	3	1	2
7	1	0	3
4	2	1	2
14	3	1	3

We find the sum of the gills to be 7 gi. 7 gi. equal 1 pt. 3 gi. We place the 3 gi. under the column of gills and add the 1 pt. to the number of pints. The sum of the pints is 3 pt., equal to 1 qt. 1 pt. The 1 pt. is placed under the column of pints, and the 1 qt. is added to the number of quarts. The sum of the quarts is 7 qt., equal to 1 gal. 3 qt. The 3 qt. are placed under the column of quarts, and the 1 gal. is added to the number of gallons, making 14 gal. 3 qt. 1 pt. 3 gi.

Add:

	gal.	qt.	pt.	gi.
48.	1	3	1	3
	3	3	0	2

	gal.	qt.	pt.	gi.
49.	5	2	1	1
	7	3	1	3

	gal.	qt.	pt.	gi.
50.	8	3	0	3
	9	3	0	2

	gal.	qt.	pt.	gi.
51.	6	3	1	2
	7	2	1	3

	gal.	qt.	pt.	gi.
52.	11	3	1	1
	5	2	1	3

	gal.	qt.	pt.	gi.
53.	15	2	1	1
	11	3	1	3

54. A milkman leaves 25 gal. 3 qt. 1 pt. of milk at one hotel, and 33 gal. 2 qt. 1 pt. at another. How much does he leave at both hotels?

		gal.	qt.	pt.	gi.
55.	From	7	3	1	3
	take	5	1	1	2
		2	2	0	1

Subtract each number in the subtrahend from the corresponding number in the minuend.

56. Class Exercise. —— may write a compound denominate number consisting of gal., qt., pt., and gi. The class may use it as a minuend, making every number in the subtrahend less than its corresponding term in the minuend.

		gal.	qt.	pt.	gi.
57.	From	7	1	1	1
	take	2	3	0	3
		4	2	0	2

As 3 gi. cannot be taken from 1 gi., we reduce 1 pt. 1 gi. to gills, which gives 5 gi. 5 gi. minus 3 gi. equal 2 gi. As the 1 pt. has been taken from the column of pints and reduced to gills, there are no pints left in the minuend, from which 0 pt. are to be taken. As 3 qt. cannot be taken from 1 qt., we reduce 1 gal. to quarts, which, with the 1 qt., equal 5 qt. 3 qt. from 5 qt. leave 2 qt. As 1 gal. has been taken from the column of gallons and reduced to quarts, only 6 gal. remain. 6 gal. minus 2 gal. equal 4 gal. Hence the difference is 4 gal. 2 qt. 0 pt. 2 gi.

Subtract:

	gal.	qt.	pt.	gi.
58.	17	3	0	3
	11	1	1	1

	gal.	qt.	pt.	gi.
59.	16	3	1	1
	8	1	0	3

	gal.	qt.	pt.	gi.
60.	13	1	1	2
	6	3	1	1

	gal.	qt.	pt.	gi.
61.	15	2	1	0
	9	3	0	3

62. 17 gal. 1 qt. 1 pt. of oil were in a tank. 11 gal. 2 qt. 1 pt. were drawn out. How much remained?

63. Multiply 3 gal. 1 qt. 1 pt. 2 gi. by 9.

gal.	qt.	pt.	gi.
3	1	1	2
			9
30	3	1	2

9 times 2 gi. equal 18 gi., which equal 4 pt. 2 gi. The 2 gi. are written under the gills. 9 times 1 pt. plus the 4 pt. already found equal 13 pt., which equal 6 qt. 1 pt. 9 times 1 qt. plus the 6 qt. already found equal 15 qt. 15 qt. equal 3 gal. 3 qt. 9 times 3 gal. plus the 3 gal. already found equal 30 gal. Hence the product is 30 gal. 3 qt. 1 pt. 2 gi.

		gal.	qt.	pt.	gi.
64.	Multiply	5	3	1	2
	by				6

	gal.	qt.	pt.	gi.
65.	10	1	1	3
				9

66. Multiply 7 gal. 2 qt. 1 pt. 3 gi. by 3. By 5. By 7.

67. A milkman sold 99 gal. 3 qt. of milk on Monday. If he were to sell the same amount every day for a week, how much milk would he sell?

68. At 24¢ a gallon, how much would he receive for the milk? If the whole cost of the milk was $135, how much would he gain?

69. How many times is 3 gal. 3 qt. 1 pt. contained in 19 gal. 1 qt. 1 pt.?

Express both dividend and divisor in the same denomination before dividing.

70. How many bottles each containing 1 pt. 2 gi. can be filled from a flask containing 3 gal.?

71. How many bottles each containing 1 pt. 2 gi. can be filled from a 6-gallon tank?

72. Divide 21 gal. 3 qt. 1 pt. 3 gi. by 6.

In dividing denominate numbers, if there is a remainder after dividing, it is the custom to reduce that remainder to the next lower denomination instead of writing the quotient as a mixed number. In this way, fractions are avoided in all the denominations of the quotient except the lowest.

Dividing 21 gal. by 6, we have 3 gal. for the quotient and 3 gal. for the remainder. 3 gal. or 12 qt. plus 3 qt. equal 15 qt. 15 qt. divided by 6 give 2 qt. for a quotient and 3 qt. for a remainder. 3 qt. or 6 pt. plus 1 pt. equal 7 pt. 7 pt. divided by 6 give 1 pt. for a quotient and 1 pt. for a remainder. 1 pt. or 4 gi. plus 3 gi. equal 7 gi. 7 gi. divided by 6 equal $1\frac{1}{6}$ gi. Hence the quotient is 3 gal. 2 qt. 1 pt. $1\frac{1}{6}$ gi.

	gal.	qt.	pt.	gi.
6)	21	3	1	3
	3	2	1	$1\frac{1}{6}$

73. Divide

	gal.	qt.	pt.	gi.
2)	8	2	1	2

74.

	gal.	qt.	pt.	g
3)	6	3	1	2

75. Divide 9 gal. 3 qt. 1 pt. 1 gi. by 4. By 5. By 6. By 8.

76. How many half-pint bottles can be filled from a 10-gallon can of milk?

77. The denominations of dry measure are pints (pt.), quarts (qt.), pecks (pk.), and bushels (bu.).

DRY MEASURE

	4	8	2
bu.	pk.	qt.	pt.

Give the table of dry measure, beginning with the units of the lowest denomination.

78. Fill the blanks: 1 bu. = — pk. = — qt. = — pt.

79. Give the ratio of 1 pt. to a unit of each denomination of dry measure.

80. Express 3 qt. 1 pt. as pints. As quarts. As pecks.

81. Express 2 pk. 5 qt. 1 pt. as pt. As qt. As pk. As bu.

82. Name several articles that are measured by dry measure.

83. At 20¢ a peck, how much does a bushel of tomatoes cost? 3 qt. 1 pt.? 5 qt. 1 pt.?

84. At 25¢ a quart, how much does a pint of strawberries cost? 3 qt. 1 pt.? 5 qt. 1 pt.?

85. At $12\frac{1}{2}$¢ a quart, how much does a peck of potatoes cost? 1 pk. 2 qt.? 1 bu.?

86. Express $\frac{4}{5}$ bu. as pk. $\frac{1}{6}$ pk. as qt. $\frac{7}{8}$ qt. as pt. $\frac{3}{4}$ bu. as qt.

87. Express $\frac{8}{9}$ pk. as bu. $\frac{8}{11}$ qt. as pk. $\frac{3}{7}$ pt. as qt. $\frac{5}{6}$ qt. as bu.

88. Express .125 pk. as qt. .875 bu. as pk. 55 qt. as pt.

89. Express .375 bu. as pk. As qt. As pt.

90. Express 4 qt. as a decimal of a peck. Of a bushel.

91. How many pints in $\frac{7}{8}$ of $\frac{5}{21}$ of $\frac{3}{5}$ of a bushel?

92. By selling apples at $.40 a peck, Mr. Allen doubled his money. How much did they cost him per bushel?

93. Express in pecks 7% of a bushel. Express in quarts 51% of a peck.

Add:

94.

bu.	pk.	qt.	pt.
20	3	5	1
4	1	1	1
5	2	6	1

95.

bu.	pk.	qt.	pt.
21	3	1	1
33	2	0	1
48	3	7	1

96.

bu.	pk.	qt.	pt.
6	2	5	1
3	3	4	0
6	1	7	1

97.

bu.	pk.	qt.	pt.
10	3	6	1
8	1	5	1
12	2	4	0

Subtract:

	bu.	pk.	qt.	pt.
98.	18	3	2	1
	14	1	6	1

	bu.	pk.	qt.	pt.
99.	40	1	5	0
	17	3	2	1

	bu.	pk.	qt.	pt.
100.	8	3	6	0
	1	2	4	1

	bu.	pk.	qt.	pt.
101.	19	1	2	1
	14	3	6	1

Multiply:

	bu.	pk.	qt.	pt.
102.	4	2	6	1
				5

	bu.	pk.	qt.	pt.
103.	8	3	7	1
				7

	bu.	pk.	qt.	pt.
104.	7	1	5	1
				6

	bu.	pk.	qt.	pt.
105.	8	3	7	1
				8

106. Multiply 6 bu. 3 pk. 3 qt. 1 pt. by 4. By 6. By 8.

107. A grocer has 3 bins, each holding 4 bu. 3 pk. 2 qt. of potatoes. How much do they all hold?

108. How much wheat is there in 10 bins, if each bin contains 40 bu. 2 pk. 6 qt.?

109. At 5¢ a quart, how much will a bushel of walnuts cost?

110. Divide 5)6 bu. 1 pk. 7 qt. 1 pt.

111. 6)8 bu. 2 pk. 5 qt. 1 pt.

112. Divide 9 bu. 2 pk. 7 qt. 1 pt. by 2. By 3. By 4.

113. How many boxes containing 2 bu. 3 pk. of sawdust can be emptied into a bin which will hold 13 bu. 3 pk.?

114. Place a cubic centimeter upon each of the corners of the upper surface of a cube that holds a liter, and find how many square centimeters there are in the surface of the figure thus formed.

115. The standard unit of metric measure of capacity is the **Liter,** equal to about .9 of a quart dry measure and 1.05 quarts liquid measure.

METRIC MEASURE OF CAPACITY

1 kiloliter (Kl.) = 1000 liters
1 hectoliter (Hl.) = 100 liters
1 decaliter (Dl.) = 10 liters
1 liter (l.)
1 deciliter (dl.) = .1 of a liter
1 centiliter (cl.) = .01 of a liter
1 milliliter (ml.) = .001 of a liter

Write table of equivalents.

1 Kl. = 10 Hl. = —Dl. = —l. = —dl. = —cl. = —ml.

116. Learn to give quickly, forward and backward, the denominations of this table, and the meaning of each prefix to the word "liter."

To help you remember the value of the units observe that D, H, and K, the abbreviations for Deca, Hecto, and Kilo, follow one another in alphabetical order.

117. Give the meaning of the following prefixes: Kilo, milli, Hecto, centi, Deca, deci. Of d, H, D, c, K.

118. Read as a decimal 1235.576 l. Give the denomination of each figure.

119. How many centiliters in 5.37 l.? In 8.4 l.? 10.25 l.? 6.875 l.?

120. How are liters reduced to centiliters?

121. Express 7 kiloliters as liters. As hectoliters. As centiliters. As milliliters.

122. Express 2 Hl. 5 Dl. as l. As cl. As Kl. As dl.

123. Reduce 2345.248 l. to units of each of the other denominations.

124. CLASS EXERCISE. —— may name a number of liters, and the class may reduce them to dl. To cl. To ml.

125. Write in one number, 5 Kl. 2 Hl. 5 Dl. 2 l. 0 dl. 7 ml.

126. Write and add: 3 Kl. 0 Hl. 2 Dl. 5 l. 0 dl. 2 cl. 7 ml.
2 Hl. 4 Dl. 0 l. 2 dl. 7 cl. 9 Kl. 3 Hl. 2 Dl. 6 l. 5 dl. 2 cl. 8 ml.

127. From 8 Kl. 2 Hl. 7 Dl. 4 l. 6 dl. 2 cl. 9 ml.
take 6 Hl. 4 Dl. 2 l. 3 dl. 8 cl. 5 ml.

128. 825.346 l. − 27.59 l. = ?

129. Multiply 125.275 l. by 5. By 8. By 12.

130. Multiply 341.626 l. by 10. By 100. By 1000.

131. Divide 239.268 l. by 4. By 6. By 12.

132. Under which system of denominate numbers is it easier to add, subtract, multiply, and divide, the metric system or the English system? Why?

133. How many liters in 7% of 132.5 l.? Of 178.7 l.?

134. 17 l. + 3% of 17 l. = how many liters? How many dl.? How many cl.?

135. How much will 7.5 l. of wine cost at $1.25 a liter?

136. What is the cost of a Dl. of wheat, at $7.25 per Hl.?

137. What is the cost of a hogshead of wine containing 225 l. at $.15 per liter?

138. What is the cost of 25 l. of vinegar at $15 per Hl.?

139. How much wheat is contained in 125 sacks, each holding 1 Hl. 2 Dl.?

140. The denominations of avoirdupois weight are ounces (oz.), pounds (lb.), hundredweights (cwt.), and tons (T.).

AVOIRDUPOIS WEIGHT

	20	100	16
T.	cwt.	lb.	oz.

Fill out the following table of equivalents.

1 T. = — cwt. = — lb. = — oz.

141. 1 oz. is what part of a pound? Of a hundredweight? Of a ton?

142. Express 5 lb. 8 oz. as ounces. As pounds. As hundredweights. As tons.

143. Express 3 T. 10 cwt. 25 lb. 12 oz. in each denomination of avoirdupois weight.

144. Express $\frac{7}{11}$ T. as cwt. $\frac{5}{9}$ cwt. as lb.

145. Express $\frac{8}{9}$ oz. as lb. $\frac{5}{5}\frac{0}{1}$ lb. as cwt. $\frac{3}{4}\frac{0}{3}\frac{0}{1}$ lb. as cwt.

146. Express .7 T. as cwt. .17 cwt. as lb. .125 lb. as oz. .75 T. as lb.

147. Express .625 T. as cwt. As lb. As oz.

148. Express 15 lb. as a decimal of a hundredweight.

149. At 20¢ a pound, how much will 2 lb. and 8 oz. of butter cost? 3 lb. 4 oz.? 5 lb.? 12 oz.? 7 lb. 2 oz.?

150. How many pounds in $2\frac{1}{2}$ T.? In 3% of a ton?

151. A farmer brought a ton of hay to market and sold 25% of it. How many pounds had he left?

152. How many cwt. in 5% of a ton? In 45%?

153. 1 lb. $-\frac{5}{8}$ of a pound = how many ounces?

Add:

154.

T.	cwt.	lb.	oz.
7	15	75	8
4	12	55	12
6	17	80	15

155.

T.	cwt.	lb.	oz.
10	19	67	5
12	14	25	13
15	16	84	11

156.

T	lb.	oz.
8	425	7
9	375	8
7	425	5

157.

T.	lb.	oz.
16	875	10
14	985	11
6	435	13

		T.	cwt.	lb.	oz.
158.	From	9	3	30	4
	take	2	7	41	7

		T.	cwt.	lb.	oz.
159.		10	0	5	0
		6	7	2	8

		T.	lb.	oz.
160.	From	8	201	8
	take	4	175	12

		T.	lb.	oz.
161.		18	700	12
		4	900	8

162. From 30 T. 800 lb. of hay there were sold 7 T. and 900 lb. How much was left?

163. Mrs. Harris brought 20 lb. 8 oz. of butter to market, and sold 18 lb. and 12 oz. How much remained unsold?

164. Name articles that are weighed by avoirdupois weight.

165. John weighs 87½ lb., Thomas 92½ lb., William 97¾ lb. How much do they all weigh? Give answer in pounds and ounces.

SUGGESTION FOR CLASS EXERCISE. Find sums and differences of weights of pupils.

		T.	cwt.	lb.	oz.
166.	Multiply	5	10	40	10
	by				8

		T.	cwt.	lb.	oz.
167.		15	15	75	15
					12

168. Multiply 15 T. 7 cwt. 25 lb. 8 oz. by 3. By 5. By 8. By 9.

Divide:

		T.	cwt.	lb.	oz.
169.	4)9	15	20	8	

		T.	cwt.	lb.	oz.
170.	5)16	14	50	10	

171. Divide 20 T. 12 cwt. 48 lb. 12 oz. by 6. By 8. By 12.

172. One Christmas day 40 T. of coal were equally distributed among 11 poor families. How many tons, hundredweights, and pounds did each receive?

173. Formerly 2240 lb. were considered a ton, and that standard is sometimes used now. In that case the ton was called a "long ton." How many pounds in 4½ long tons? In 12½% of a long ton?

174. A coal dealer buys 150 T. of coal, 2240 lb. each, at $4.50 per ton. He sells it at $4.75 per ton, giving 2000 lb. per ton. How much does he gain?

175. The standard unit of metric measure of weight is a **Gram**, equal to about $\frac{1}{29}$ of an ounce.

METRIC MEASURE OF WEIGHT

1 kilogram (Kg.) = 1000 grams
1 hectogram (Hg.) = 100 grams
1 decagram (Dg.) = 10 grams
1 **gram** (g.)
1 decigram (dg.) = .1 of a gram
1 centigram (cg.) = .01 of a gram
1 milligram (mg.) = .001 of a gram

Fill blanks:

1 Kg. = — Hg. = — Dg. = — g. = — dg. = — cg. = — mg.

176. The weight of a cubic centimeter of water is a gram. How many grams does a liter of water weigh?

177. What part of a kilogram is a decagram? Decigram? Milligram? Hectogram? Centigram?

178. Name each denomination of the expression 1978.347 g.

179. Write in one number, making the gram the unit: 4 Kg. 7 Hg. 6 Dg. 5 g. 2 dg. 6 cg. 3 mg.

180. Express 75 Kg. as grams. As cg. As mg. As Dg. As dg. As Hg.

181. Express 186 cg. as grams. As mg. As Dg. As dg. As Hg. As Kg.

182. Add 325 g., 55 Kg., 75 Dg.

183. How many grams in 20% of 425 g.?

184. 126 g. — 4% of 126 g. equal how many Dg.? dg.? cg.? mg.?

185. A nickel weighs 5 g. $5.00 in nickels weigh how many kilograms?

186. A kilogram is equal to about $2\frac{1}{5}$ lb. Find the approximate cost of a kilogram of a drug that costs \$.60 a pound.

187. What is the cost of 2242 g. of coffee at \$.60 a kilogram?

188. If a kilogram of wool costs \$ 1.75, how much will 6.5 kilograms cost?

189. The denominations of time measure are seconds (sec.), minutes (min.), hours (hr.), days (da.), and years (yr.).

TIME MEASURE

	100	365	24	60	60
century	yr.	da.	hr.	min.	sec.

Fill out the following table of equivalents:

1 century = — yr. = — da. = — hr. = — min. = — sec.

190. Find the ratio of one hour to a unit of each denomination of time measure.

191. Express 2 hr. 10 min. as sec. As min. As hr.

192. Express $\frac{3}{5}$ yr. as da. $\frac{5}{16}$ da. as hr. $\frac{3}{4}$ hr. as min.

193. Express $\frac{1}{7}$ of a year as da. As hr. As min. As sec.

194. Express $\frac{1}{16}$ min. as hr. $2\frac{2}{3}$ hr. as da. 100 sec. as min.

195. Express .12 yr. as da. $.33\frac{1}{3}$ da. as hr. .75 hr. as min. $.66\frac{2}{3}$ min. as sec. .17 centuries as yr.

196. Express 108 sec. as a decimal of a minute. Of an hour.

197. How many minutes in 5% of an hour? In 65%? 95%?

198. 1 hr. + 15% of an hour = how many minutes?

199. At 6% how much interest will \$300 gain in 1 yr.? In $1\frac{1}{2}$ yr.? 3 yr. 6 mo.? 2 yr. 9 mo.?

200. Express in years, 1 yr. 1 mo. 15 da. Find the interest of \$ 400 for that time at 6%. At 8%. At 5%.

201. Usually every fourth year has 366 da., and is called a leap year. In leap year the month of February, which usually has 28 da., has 29 da. How many hours in February of a leap year?

202. Every year whose number is divisible by 4 is a leap year, excepting the years whose number ends in two ciphers, as 1500, 1700, 1800. Such years are leap years only if their number is divisible by 400, as 1200, 1600, 2000. Which of the following are leap years?

1848 1862 1892 1900 1904 2000 2108 2200 1000

203. At a dollar a day, how much could a man earn, working 6 da. in a week, in the month of February, 1896, which began on Saturday? How much in February, 1898?

204. At $1.50 per day, how much could a man earn in February of a leap year, in which the 1st of February fell on Sunday?

205. A man's heart beats at the rate of about 72 beats in a minute. At that rate how many times does it beat in an hour? In a day of 24 hr.? In a common year?

206. At that rate how many times would a man's heart beat in a lifetime of 80 yr., $\frac{1}{4}$ of which are leap years?

207. When a person born in 1883 is 20 yr. old, how many years of his life have been leap years?

208. The time in which the earth passes once around the sun is 365 da. 5 hr. 48 min. 46 sec. That is how much more than 365 da.? To how much will the extra time amount in 4 yr.? How much does it lack of being a whole day?

209. In adding the whole day to every fourth year or leap year, how many more minutes and seconds are added to the year than rightly belong to it?

210. Find how nearly the error caused in a century by this arrangement is corrected by omitting the extra day in February at the end of the century.

211. Find the amount of error at the end of the fourth century, and see how nearly it is corrected by restoring the extra day in February of that year.

212. Henry rose at 5.50 A.M. and went to bed at 8.20 P.M. How long was his day?

213. CLASS EXERCISE. —— may give a time for rising and a time for going to bed, and the class may find the length of the included day.

214. How many hours were there in the year 1800?

"Thirty days hath September,
April, June, and November."

215. Learn the above rhyme and remember the fact that all the rest of the months except February have 31 days.

216. Write the names of the months in order, beginning with January, using abbreviations. Write opposite the name of each month the number of days it contains.

217. Find the number of days from May 1st to June 7th.

There are 30 days after the 1st in May, which, with the 7 days in June, make 37 days.

Find the number of days between the following dates:

218.	May 7, 1896,	July 4, 1896.
219.	Jan. 1, 1900,	Mar. 1, 1900.
220.	Sept. 28, 1899,	Nov. 5, 1899.
221.	Dec. 15, 1899,	Jan. 31, 1900.

222. How many days from to-day until next Christmas? Next 4th of July?

223. Class Exercise. —— may give the date of his next birthday, and the class may find the number of days intervening.

224. Thirty days from the 4th of July, 1876, was what date?

225. What was the date 60 days after Oct. 14th, 1492? Christmas, 1897?

226. On the 17th day of June, Mr. Herbert borrowed from a bank $ 100 to be paid in 60 days. When was it due?

227. What will be the date 30 days after to-day? 60 da.?

228. What will be the date 33 days from next Monday? 63 da.? 93 da.? 105 da.?

229. What was the date 30 days before the first of May, 1891? 60 days before the 1st of March, 1892?

230. How many days since the 1st of January of this year?

231. Class Exercise. —— may give the date of his last birthday, and the class may find how many days have passed since then.

Add:

	da.	hr.	min.	sec.
232.	17	20	30	40
	20	16	40	10
	40	18	50	20

	da.	hr.	min.	sec.
233.	19	19	45	30
	25	20	15	30
	16	12	30	30

Add:

	da.	hr.	min.	sec.
234.	19	14	30	45
	70	20	45	15
	6	18	15	15
	4	2	30	30

	da.	hr.	min.	sec.
235.	15	2	30	30
	30	10	45	25
	40	6	15	35
	9	8	15	15

236. Mr. Cox earns $ 2.00 for each day of 10 hours that he works. On Monday he worked 8 hr. 30 min.; on Tuesday, 9 hr. 10 min.; Wednesday, 7 hr. 40 min.; Thursday, 8 hr. 30 min.; Friday, 7 hr. 50 min; Saturday, 4 hr. 30 min. How much did he earn in that week?

237. At $ 2.00 per day of 8 hours, how much would Mr. Cox have earned?

Find the differences:

	da.	hr.	min.	sec.
238.	47	18	2	10
	25	20	1	40

	da.	hr.	min.	sec.
239.	62	10	30	15
	48	20	19	45

240. Which is the 7th month? 12th? 3d? 5th?

241. Which month is February? September? June? November?

242. Read the following dates: 9/5/99. 10/5/98. 7/4/76.

243. Find the difference of time between March 5, 1898, and Oct. 21, 1902.

Write as below and subtract:

1902 10 21

1898 3 5

In finding the difference between two dates in years, months, and days, we assume that 30 days = a month.

American Authors

Ralph Waldo Emerson, born May 25, 1803 — died April 27, 1882.
John G. Whittier, born Dec. 17, 1807 — died Sept. 7, 1892.
Henry W. Longfellow, born Feb. 27, 1807 — died March 24, 1882.
James Russell Lowell, born Feb. 22, 1819 — died Aug. 12, 1891.

English Authors

Alfred Tennyson, born Aug. 6, 1809 — died Oct. 6, 1892.
Charles Dickens, born Feb. 7, 1812 — died June 9, 1870.

244. Find the number of years. months, and days since each of the authors mentioned above was born. Since each died.

245. Find the age of each author at his death.

246. Find the time between the birth of Emerson and that of each of the other authors.

247. Find the time between the death of Dickens and that of each of the other authors.

248. Multiply

	da.	hr.	min.	sec.
	7	18	20	10
by				7

249.

da.	hr.	min.	sec.
11	15	8	30
			9

250. Multiply 5 da. 10 hr. 20 min. 30 sec. by 5. By 6. By 8.

251. If you spend 4 hr. 30 min. in school every day, how many hours and minutes do you spend in a school week? In a school month of 4 wk.? In a school year of 10 mo.?

Divide:

252.

da.	hr.	min.	sec.
4)21	16	2	40

253.

da.	hr.	min.	sec.
6)13	13	13	30

254. Divide 15 da. 12 hr. 40 min. 30 sec. by 5. By 6. By 8.

255. The denominations of linear measure are inches (in.), feet (ft.), yards (yd.), rods (rd.), and miles (mi.).

LINEAR OR LONG MEASURE

mi.	rd.	yd.	ft.	in.
	320	$5\frac{1}{2}$	3	12

Fill out the table of equivalent values.

1 mi. = —rd. = —yd. = —ft. = —in.

256. How many inches in 2 yd. 1 ft. 7 in.? 5 yd. 3 ft. 7 in.?

257. Express 2 yd. 1 ft. 6 in. as in. As ft. As yd.

258. Express 2 mi. 20 rd. as mi. As rd. As yd. As ft.

259. 1 ft. is what part of a yard? Of a rod? Of a mile?

Reduce complex fractions to simple fractions.

260. Express 1 ft. 6 in. as yd. As rd. As mi.

261. Express 1 yd. 1 ft. 9 in. as yd. As rd. As mi.

262. Express $\frac{5}{16}$ mi. as rd. $\frac{8}{11}$ rd. as yd. $\frac{5}{9}$ yd. as ft. $\frac{3}{5}$ ft. as in. $\frac{4}{11}$ rd. as ft.

263. Express $\frac{8}{9}$ in. as ft. $\frac{9}{11}$ ft. as yd. $\frac{11}{12}$ yd. as rd. $11\frac{3}{7}$ rd. as mi. $1\frac{3}{8}$ ft. as rd. $1\frac{5}{7}$ in. as ft. $4\frac{1}{5}$ ft. as yd.

264. Express .875 mi. as rd. 3.6 rd. as yd. 5.5 yd. as ft. .9 ft. as in. $1.66\frac{2}{3}$ yd. as ft. .64 rd. as yd.

265. Express 7.2 in. as a decimal of a foot. As a decimal of a yard.

266. Express 115.5 ft. in yd. In rd. In mi.

267. How many rods in 5% of a mile? In 15%? 35%?

Add:

268.

yd.	ft.	in.
2	1	$11\frac{1}{2}$
5	1	$2\frac{1}{4}$

269.

yd.	ft.	in.
6	2	$7\frac{1}{3}$
3	2	$11\frac{5}{6}$

270.

mi.	rd.	yd.	ft.	in.
6	200	2	1	10
8	120	1	2	6

271.

mi.	rd.	yd.	ft.	in.
10	20	1	2	10
7	300	1	1	8

272.

mi.	rd.	yd.	ft.	in.
8	150	4	2	11
4	100	1	1	5

273.

mi.	rd	yd.	ft.	in.
10	180	4	2	3
7	40	2	1	11

274.

mi.	rd.	yd.	ft.	in.
16	200	3	2	4
14	150	1	1	10

275.

mi.	rd.	yd.	ft.	in.
18	2	4	2	10
5	319	2	1	7

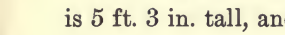
276.

mi.	rd.	yd.	ft.	in.
15	3	2	2	3
10	319	1	1	9

277. Mr. Smith's lot is 1 yd. 1 ft. shorter than Mr. Brown's lot, which is 30 yd. long. How long is Mr. Smith's lot?

278. An elm tree is 32 ft. 9 in. high. How high is a fir tree that is 6 ft. 10 in. less in height?

279. Find the difference between the height of John, who is 5 ft. 3 in. tall, and his sister, who is 3 ft. 11 in.



280. If it were $20\frac{3}{4}$ miles from A to C, and $5\frac{1}{2}$ miles from A to B, how far would it be from B to C?

281. If it were 20 mi. 20 rd. from A to C, and 5 mi 80 rd. from A to B, how far would it be from B to C?

Multiply:

282.

yd.	ft.	in.
3	1	2
		3

283.

mi.	rd.	yd.	ft.	in.
5	2	1	0	9
				4

284.

mi.	rd.	yd.	ft.	in.
7	80	0	2	6
				2

285.

mi.	rd.	yd.	ft.	in.
10	160	4	1	11
				7

286. Multiply 2 mi. 240 rd. 3 yd. 5 ft. 6 in. by 2. By 3.

287. How long is the perimeter of a regular pentagon, each of whose sides is 3 yd. 1 ft. 9 in. long?

288. A summer house is built in the shape of a regular hexagon, each side being 2 yd. 1 ft. 6 in. What is the entire distance around it?

289. How long is the edge of the border of a flower bed in the shape of an octagon, if each side is 1 yd. 1 ft. 8 in. long?

290. Divide 3 mi. 16 rd. 2 yd. 1 ft. 8 in. by 2. By 3. By 4.

291. Divide 19 mi. 10 rd. 5 yd. 2 ft. 6 in. by 5. By 6. By 7.

292. Divide 7 mi. 160 rd. 4 yd. 1 ft. 8 in. by 2. By 3. By 4.

293. Henry measured a cornstalk and found it to be 5 ft. 6 in. long. How many such stalks laid in a continuous line would it take to extend a mile?

294. Harry has two dogs, Don Quixote and Sancho Panza. Don measures 4 ft. 3 in. from the end of his nose to the tip of his tail, weighs 77 lb. 12 oz., and is 3 yr. 7 mo. 9 da. old. Sancho measures 3 ft. 8 in., weighs 53 lb. 14 oz., and is 2 yr. 11 mo. 28 da. old. Find the difference of the lengths of the dogs. Of their weights. Of their ages.

295. Harry's dog, Sancho Panza, chased a rabbit 40 yd. and then gave up the chase. The rabbit had 10 yd. the start of the dog and ran twice as fast as he ran. How far apart were the animals when Sancho Panza gave up the chase? Represent.

296. If from a string 2 yd. 2 ft. long, 2.5 ft. is broken off at one end, and 3.7 ft. at the other, how long a string is left?

297. The denominations of square measure are square inches (sq. in.), square feet (sq. ft.), square yards (sq. yd.), square rods (sq. rd.), acres (A.), and square miles (sq. mi.).

SURFACE OR SQUARE MEASURE

	640	160	$30\frac{1}{4}$	9	144
sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in

Write a table of equivalent values.

1 sq. mi. = — A. = — sq. rd. = — sq. yd. = — sq. ft. = — sq. in.

SUGGESTION TO TEACHER. Let a diagram of the square rod be drawn on the floor, a square yard in one corner of the square rod being subdivided into square feet, and one of the square feet into inches, so that literally each square unit may be a part of the units of higher denominations. Let pupils make many practical problems upon the figures.

298. 1 sq. ft. equals what part of a sq. yd.? Of a sq. rd.?

299. How many square feet in 2 sq. rd.? 3 sq. rd.? 5 sq. rd.? 7 sq. rd.? 10 sq. rd.? 12 sq. rd.?

300. How many square inches in 5 sq. ft. 60 sq. in.? In 4 sq. ft. 20 sq. in.? In 2 sq. ft. 80 sq. in.?

301. How many square feet in 4 sq. yd. 6 sq. ft.? In 20 sq. yd. 7 sq. ft.? In 18 sq. yd. 5 sq. ft.?

302. How many square yards in 8 sq. rd. 15 sq. yd.? In 10 sq. rd. $4\frac{3}{4}$ sq. yd.? In 24 sq. rd. 19 sq. yd.?

303. Express 1 sq. mi. 320 A. 80 sq. rd. in sq. rd. In A. In sq. mi.

304. Express 10 sq. yd. 7 sq. ft. 72 sq. in. in sq. in. In sq. ft. In sq. yd.

305. Express 2 sq. rd. 15 sq. yd. in sq. yd. In sq. rd. In sq. ft. In A.

306. Express $\frac{11}{32}$ sq. mi. as A. $\frac{5}{80}$ A. as sq. rd.

307. Express $\frac{8}{11}$ sq. rd. as sq. yd. $\frac{2}{7}$ sq. yd. as sq. ft. $\frac{15}{72}$ sq. ft. as sq. in. $\frac{12}{5}$ sq. in. as sq. ft.

308. Express .625 sq. mi. as A. .375 A. as sq. rd. .48 sq. rd. as sq. yd. .175 sq. yd. as sq. ft. .7 sq. ft. as sq. in. 4.32 sq. in. as sq. ft. .18 sq. ft. as sq. yd.

309. Express 345.6 sq. in. as a decimal of a square foot.

Add:

310.

sq. yd.	sq. ft.	sq. in.
21	2	100
7	8	6
16	5	40

311.

sq. yd.	sq. ft.	sq. in.
36	3	70
15	7	60
20	7	20

Find difference:

312.

sq. yd.	sq. ft.	sq. in.
17	8	85
4	3	75

313.

sq. yd.	sq. ft.	sq. in.
21	6	100
6	8	75

Multiply:

314.

sq. yd.	sq. ft.	sq. in.
24	3	140
		6

315.

sq. yd.	sq. ft.	sq. in.
16	4	96
		7

316. Multiply 2 A. 40 sq. rd. 10 sq. yd. 4 sq. ft. 20 sq. in. by 3. By 4. By 5.

317. How many acres in 5% of a square mile? 7%? 15%?

318. 1 sq. ft. $- 37\frac{1}{2}\%$ of a square foot = how many square inches?

319. Divide by 3, 48 sq. yd. 7 sq. ft. 4 sq. in.

320. Divide by 5, 25 sq. yd. 6 sq. ft. 2 sq. in.

321. Divide 20 A. 80 sq. rd. 20 sq. yd. 4 sq. ft. 72 sq. in. by 4. By 8. By 6.

322. How many square feet in a square $\frac{3}{4}$ of a foot in dimensions? How many square inches?

323. How long is the perimeter of a square $\frac{5}{6}$ of a foot in dimensions? What fraction of a square foot is its area? How many square inches in its area?

324. Mr. Gilbert owns 400 A. 120 sq. rd. of land in Gibson County, 225 A. and 10 sq. rd. in Warrick County, and 14 A. 40 sq. rd. in Vanderburgh County. How much does he own in those counties?

325. A farmer had 80 A. 50 sq. rd. of land. After selling 30 A. 10 sq. rd., how much had he left?

326. Mr. Carter owns three times as much land as his cousin, who owns 120 A. 80 sq. rd. How much land do both own?

327. A garden 180 ft. long. 150 ft. wide is surrounded by a tight board fence 6 ft. high. How much will it cost to paint the fence on both sides @ 12¢ per square yard?

Suggestion to Teacher. Pupils who cannot imagine the conditions of this problem may be required to inclose a surface on their desks by a strip of paper folded so that its divisions represent the parts of the fence.

328. From each corner of a square, a side of which is 2 ft. 5 in., a square measuring 5 in. on a side is cut out. Represent and find the area of the remaining figure.

329. Find the area of the walls of a room 12 ft. long, 10 ft. wide, 8 ft. high.

Find the areas of the walls and ceiling of rooms of the following dimensions, and the cost of plastering them at 20 cents a square yard, no allowance being made for openings.

	a			*b*		
	Length	Width	Height	Length	Width	Height
	ft.	ft.	ft.	ft.	ft.	ft.
330.	20	18	10	40	30	12
331.	30	25	9	21	20	9
332.	25	21	9	30	18	8
333.	18	15	8	24	20	9
334.	15	12	8	16	15	8

335. Which is greater, a rectangle 12 in. by 12 in. or a rectangle 16 in. by 9 in.? What is the difference in the length of their perimeters?

336. Give dimensions of several rectangles, each equal to a square foot. Compare the lengths of their perimeters.

337. If two rectangles have equal areas but different shapes, which will have the longer perimeter, the one which is more nearly square or the other? Illustrate.

338. Find the area of a square whose perimeter is 24 in. 28 in. 36 in. 40 in.

339. Find the perimeter of a square whose area is 9 sq. in.

340. The denominations of cubic measure are cubic inches (cu. in.), cubic feet (cu. ft.), and cubic yards (cu. yd.).

CUBIC MEASURE

	27	
cu. yd.	cu. ft.	1728 cu. in.

Write table of equivalent values.

1 cu. yd. = — cu. ft. = — cu. in.

341. How many cubic inches in 5 cu. ft. 192 cu. in.? In 1 cu. yd. 624 cu. in.?

342. How many cubic inches in .875 cu. ft.? In .625 cu. ft.?

343. John may draw a square yard on the floor in a corner of the room. How many blocks of ice 1 foot in dimensions would it take to cover that square yard?

344. If another layer of cubic feet of ice were laid upon the first, how many cubic feet of ice would there be? How high would the ice be piled?

345. If a third layer of cubic feet of ice were placed upon the other two, how many cubic feet of ice would there be?

346. What name is given to a solid which is 3 ft. long, 3 ft. wide, and 3 ft. high?

347. In the square yard which John drew William may set a yard stick upright at that corner which is not against a wall. Two other boys may place sticks in such a position that a cubic yard is outlined in the corner of the room.

SUGGESTIONS TO TEACHER. Devote a certain space in the room to the imaginary cubic yard. See that every member of the class images a cubic yard in that particular place. Let pupils show divisions of the cube by outlining them with their hands in the space devoted to it.

Let pupils model before the class with inch cubes the figures given in the following exercises.

348. Image a cubic yard with one cubic foot cut from the upper layer at a corner that is not against a wall. Model the figure. Outline in the cubic yard in the corner the part taken out. What is the ratio of the part taken out to the whole cubic yard? What is the ratio of the part taken out to the part left?

349. Take two more cubic feet from the upper layer, one on each side of the vacant space. Model. Outline in the cubic yard. Tell the ratio of the vacant space to the whole cubic yard. Of the vacant space to the filled space.

350. Take out the two cubic feet that were directly under the cubic foot first removed. Model. Outline. Find the ratio of the vacant space to the cubic yard. Of the filled space to the cubic yard. Of the vacant space to the filled space.

351. Restore the whole cubic yard. Take away the middle cubic foot on each side of the upper layer. Model. Outline. Tell the ratio of the vacant space to the cubic yard. Of the filled space to the cubic yard. Of the vacant space to the filled space.

352. Class Exercise. —— may give directions for taking away parts of the cubic yard. The class may tell the ratios of the spaces to one another. Some members may model the figures, and some one else may show the outline of the space imaged as vacant in the cubic yard.

353. How many feet in the sum of all the edges of a cube 1 yd. in dimensions?

354. How much will a cubic yard of building stone cost at $2.50 a cubic foot?

355. How many cubic feet in a right prism 6 ft. long, 2 ft. wide, and 1 ft. high? Model the prism.

356. How many cubic feet in a right prism,

a 4 ft. long, 2 ft. wide, 2 ft. high?

b 5 ft. long, 3 ft. wide, 1 ft. high?

c 8 ft. long, 2 ft. wide, 2 ft. high?

d 10 ft. long, 4 ft. wide, 2 ft. high?

357. How many cubic feet in a tank 14 ft. long, 10 ft. wide, 5 ft. high?

358. How many cubic yards in a wall 81 ft. long, 3 ft. thick, and 9 ft. high? In a wall 30 ft. long, 6 ft. high, and 3 ft. thick?

359. A monument is in the shape of a right prism, 7 ft. long, 4 ft. wide, and 3 ft. high. How much will it cost at $3.50 per cubic foot?

360. If the engraving on it costs $62.50, what will be the entire cost of the monument?

361. If a cake 3 in. long, 3 in. wide, and 3 in. high, has icing all over it except on the under side, how many square inches of icing has it?

362. To cut the cake into inch cubes, how many cuts would be necessary? How many inch cubes would there be?

363. How many of the cubes would have icing on three sides? On two sides? On one side? On no side?

364. How many inch cubes can be placed on a square foot? How many layers of those cubes would it take to make a cubic foot? How many cubic inches in a cubic foot? What is the ratio of 1 cu. in. to a cubic foot?

365. Imagine a cubic foot of marble with 1 cu. in. cut from each of the upper corners. What would be the ratio of the part cut out to the part left?

366. Which is greater, a right prism 12 in. by 12 in. by 12 in., or one 24 in. by 12 in. by 6 in.? Compare their surfaces.

367. Give dimensions of several right prisms, each of which equals a cubic foot. Compare their surfaces.

368. How many cubic feet in a stick of timber 12 in. wide, 9 in. thick, and 24 ft. long?

369. How many cubic feet in a cistern 5 ft. square and 6 ft. deep? How many cubic inches? How many gallons will the cistern hold? (231 cu. in. = 1 gal.)

Add:

	cu. yd.	cu. ft.	cu. in.		cu. yd.	cu. ft.	cu. in.
370.	5	10	1700	371.	3	5	1400
	4	8	129		10	24	300

Add:

	cu. yd.	cu. ft.	cu. in.
372.	9	11	1720
	6	18	10

	cu. yd.	cu. ft.	cu. in.
373.	4	15	1600
	8	20	200

Subtract:

	cu. yd.	cu. ft.	cu. in.
374.	120	13	1700
	65	15	1125

	cu. yd.	cu. ft.	cu. in.
375.	41	10	1634
	25	18	1507

Subtract:

	cu. yd.	cu. ft.	cu. in.
376.	81	3	208
	40	20	125

	cu. yd.	cu. ft.	cu. in.
377.	16	4	800
	4	21	525

Multiply:

	cu. yd.	cu. ft.	cu. in.
378.	20	5	1160
			3

	cu. yd.	cu. ft.	cu. in.
379.	15	10	989
			4

380. Multiply 2 cu. yd. 20 cu. ft. 1000 cu. in. by 2. By 4.

Divide:

	cu. yd.	cu. ft.	cu. in.
381.	5) 6	20	72

	cu. yd.	cu. ft.	cu. in.
382.	8) 10	4	36

383. Divide 10 cu. yd. 15 cu. ft. 180 cu. in. by 3. By 5.

384. Imagine a cubic rod of marble. Why do we have no such measurement as a cubic mile?

385. A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is a cord of wood. How many cubic feet in a cord of wood?

386. Represent a cord of wood by drawing or by placing blocks or toothpicks.

387. How many cords in a wood pile 16 ft. long, 8 ft. wide, and 8 ft. high?

388. At $5.00 per cord, what is the value of a pile of wood 20 ft. long, 4 ft. wide, and 4 ft. high? Of a pile 18 ft. long, 8 ft. wide, and 8 ft. high?

389. The standard unit of metric linear measure is a **Meter,** which is 39.37 in. This length was obtained by calculating one ten-millionth of the distance from the equator to a pole of the earth. A kilometer is about $\frac{5}{8}$ of a mile.

METRIC LINEAR MEASURE

1 kilometer (Km.) = 1000 meters
1 hectometer (Hm.) = 100 meters
1 decameter (Dm.) = 10 meters
1 **meter** (m.)
1 decimeter (dm.) = .1 of a meter
1 centimeter (cm.) = .01 of a meter
1 millimeter (mm.) = .001 of a meter

Write table of equivalents:

1 Km. = — Hm. = — Dm. = — m. = — dm. = — cm. = — mm.

390. Express 42 m. as centimeters. As decimeters. As decameters. As hectometers.

391. Express 375 m. as kilometers. As decameters. As decimeters. As millimeters.

392. Express 4287 m. as kilometers. As decimeters. As hectometers.

393. How many centimeters in 11% of 12 m.? Of 25 dm.?

394. 7% of 192 m. = how many meters? Decameters?

SUGGESTION TO TEACHER. Let pupils find in meters and decimals the length and width of room. Length of blackboards. Length of diagonal of room or blackboard. Heights of pupils.

395. About how many inches in a decameter? In a decimeter?

396. Find approximately the number of inches in a kilometer. In 4 Hm. In 12 Dm. In 7 dm.

397. What is the cost of 12 m. of cloth at \$.75 per meter? Is the cloth cheaper or dearer than at \$.75 per yard? Explain.

398. At the rate of 36 Km. per hour, how far will a train run in 3 hr. 30 min.?

399. What is the value of a decameter of silk at \$1.65 per meter?

400. How many centimeters long is the perimeter of a regular octagon, one side of which is 8 mm.?

401. Find the length in decimeters of one side of a regular pentagon whose perimeter is 75 cm.

402. How long is the base of an isosceles triangle whose perimeter is 4 dm. and whose equal sides are each 12 cm.? Represent.

403. How long is each of the equal sides of an isosceles triangle whose perimeter is 3 dm. and base 8 cm. Construct.

404. A kilometer is about what fraction of a mile?

405. Find approximately the number of miles in 40 Km. In 72 Km. In 3.2 Km. In 6.72 Km.

406. Find the approximate number of miles in 9288 m.

SUGGESTION. Express 9288 m. as kilometers before finding its equivalent in miles.

407. Find approximately the number of miles in 45864 m. In 63824 dm. In 59888 Dm. In 71848 Hm.

408. Find approximately the number of kilometers in 75 mi. In 235 mi. In 84.5 mi.

409. How many square millimeters in a rectangle 1 cm. long and 1 cm. wide?

410. In a square decimeter, how many square centimeters? Square millimeters?

411. In a square meter how many square decimeters? Square centimeters? Square millimeters?

412. A square decameter equals how many square meters? Square decimeters? Square centimeters? Square millimeters?

413. A square hectometer equals how many square decameters? Square meters? Square decimeters?

414. A square kilometer equals how many square hectometers? Square decameters? Square meters? Square decimeters? Square centimeters? Square millimeters?

415. In long measure, under the metric system, what is the ratio of a unit of each denomination to a unit of the next higher denomination?

416. In square measure, metric system, what is the ratio of a unit of each denomination to a unit of the next higher denomination?

417. Write a table of square measure, metric system. Write a table of equivalents of units of square measure, metric system.

418. Express 3 sq. Km. 2 sq. Dm. 50 sq. m. in square meters. In square decameters. In square kilometers.

419. Express 7 sq. m. 20 sq. dm. 30 sq. cm. in square millimeters. In square centimeters. In square decimeters.

420. Express 1 sq. m. 2 sq. dm. 3 sq. cm. in sq. mm. In sq. cm. In sq. dm. In sq. m. In sq. Dm. In sq. Hm. In sq. Km.

421. What is the area of a square whose perimeter is 24 cm.? 20 cm.? 40 mm.?

422. How long is the perimeter of a square whose area is 81 sq. cm.? 49 sq. cm.? 64 sq. dm.?

423. What is the area and the perimeter of a rectangle which is 35 cm. long and $\frac{1}{5}$ as wide as long?

424. A land measurement, 10 meters square, or its equivalent, is called an **Are** (a.). How many square meters in an are? How long is the perimeter of an are in the form of a square?

425. What is the cost of $\frac{3}{5}$ a. of land at $12.50 per are?

426. The standard unit of metric land measure is an **Are**, which is equal to a square decameter or approximately to $\frac{1}{40}$ of an acre.

METRIC LAND MEASURE

1 hectare (Ha.) = 100 ares
1 **are** (a.)
1 centare(ca.) = .01 are

Notice that the final vowel of "hecto," and "cento" is dropped before the word "are."

427. How many meters of fence would be required to inclose a hectare in the form of a square?

428. Draw on the floor a square containing a centare. How long is its perimeter?

429. How many square decimeters in a centare? In an are? In a hectare?

430. What is the cost of 24.7 Ha. at $425 a Ha.? Of 63.25 Ha. at $1032 a Ha.?

431. Approximately, how many acres in 280 a.? In 160 a.? In 240 a.? In 120 a.?

432. Find the approximate value in ares of 30 A. 75 A. 17½ A. 8 A. 120 sq. rd. 12 A. 80 sq. rd. 6 A. 350 A. 500 A. 40 sq. rd.

433. Image a cubic centimeter and a cubic decimeter. How many cubic centimeters are equal to the cubic decimeter?

434. Draw a square meter in one corner of the room. Imagine it covered with a layer of cubic decimeters or liters. How many are there?

435. With a meter stick outline a cubic meter. How many layers of cubic decimeters are there in it? How many cubic decimeters?

436. In cubic measure, metric system, how many units of each denomination make one unit of the next higher denomination?

437. Write the table of cubic measure, metric system.

438. A cubic meter, or its equivalent, is called a **Stere** (s.). Image a stere of ice, of cubical form. How many square decimeters in all its surfaces?

439. Image a stere of marble, 2 m. long and 1 m. wide. How high is it? How many decimeters in all its edges? Represent with blocks.

440. A box which holds a stere is full of packages of Breakfast Food, each of which holds a liter. How many packages are there?

441. At 15¢ a liter, what is the value of the contents of the box?

442. At 2¢ a liter, what is the cost of 3 s. of wheat? 4726 s.? 8347 s.?

443. The standard unit of metric wood measure is a **Stere**, which is a little over $\frac{1}{4}$ of a cord.

METRIC WOOD MEASURE

1 decastere (Ds.) = 10 steres
1 stere (s.)
1 decistere (ds.) = .1 of a stere

444. A pile of wood 7 m. long, 6 m. wide, and 5 m. high contains how many steres? How much is it worth at $1.50 a stere?

445. At 70¢ a stere, what is the value of a pile of wood 4.5 Dm. long, 3.5 m. wide, and 300 cm. high?

446. About how many cords are there in 20 s.? In 32 s.? In 42 s.? In 12.8 s.? In 6.36 s.?

447. How many cubic feet in a cord? About how many cubic feet in a stere?

448. Approximately how many steres in 5 cd.? In 7 cd.? In $9\frac{1}{2}$ cd.? In 3 cd. 64 cu. ft.? In 6 cd. 32 cu. ft.? In 12 cd. 16 cu. ft.? In 4 cd. 8 cu. ft.? In 11 cd. 4 cu. ft.? In 24 cd. 8 cu. ft.?

449. Which is greater and how much, a stere or a kiloliter? Explain.

450. About how many liquid quarts equal a liter?

(A liter equals approximately 1.1 qt.)

451. About how many quarts are there in a decaliter? In a hectoliter? In a kiloliter?

452. About how many liters in 7.7 qt.? In 132 qt.? In 39 qt.? In 17 qt.?

453. A cask of oil containing 187 l. was bought at 20¢ a liter and sold at 25¢ a quart, a liter being counted as 1.1 qt. How much was gained?

454. How much is gained by buying 209 l. of wine at 30¢ a liter and selling them at 40¢ a quart, counting a liter as 1.1 qt.?

455. Image a milliliter. What else is it called?

(See page 81.)

456. The weight of a cubic centimeter of pure water at its greatest density is called a **Gram** (g.).

457. Image a glass vessel of cubical shape containing a liter of pure water. How many grams would it contain?

458. Approximately 1000 g. equal $2\frac{1}{5}$ lb. What is the approximate equivalent of a gram?

MISCELLANEOUS EXERCISES

1. Divide 62.5 by .0025.
2. Find the g. c. d. of 567 and 637.

3. Reduce to lowest terms: $\frac{6}{7}\frac{5}{8}\frac{7}{8}$. $\frac{8}{5}\frac{6}{7}\frac{8}{6}$.

4. Find the l. c. m. of 24 and 57.

5. Add $\frac{3}{8}\frac{3}{8}$ and $\frac{4}{5}\frac{5}{7}$.

6. From $\frac{7}{7}\frac{7}{5}$ take $\frac{2}{2}\frac{2}{5}$.

7. Resolve into prime factors 26,460 and 60,060.

8. Write the improper fraction that expresses the ratio of the first prime number after 40 to the first prime number after 20, and reduce it to a mixed number.

9. How many 6ths in $\frac{1}{2}$ of 10? In $\frac{1}{2}$ of 7?

10. One eighth of 88 is how many times 3? $4\frac{1}{2}$? $5\frac{1}{2}$?

11. If 13 is a divisor and 39 a dividend, what is the quotient? If both divisor and dividend are multiplied by 4, what is the quotient?

12. From $49\frac{7}{8}$ subtract a number which is $\frac{1}{7}$ as large.

13. A minuend is 15, and a subtrahend 11. What is the difference? If 3 is added to both minuend and subtrahend, what is the difference?

14. If $3\frac{1}{5}$ is added to both minuend and subtrahend in the preceding question, what is the difference?

15. Find difference between $\frac{5}{7}$ of $\frac{1}{2}\frac{4}{5}$ of 40 and $\frac{3}{8}$ of $\frac{1}{2}\frac{6}{7}$ of 3.

16. Square $\frac{5}{7}$. $\frac{3}{5}$. $\frac{2}{9}$. $\frac{1}{2}\frac{1}{5}$.

17. What fraction multiplied by itself will give $\frac{4}{9}$?

18. What is the square root of $\frac{2}{8}\frac{5}{1}$? $\frac{8}{1}\frac{1}{0}\frac{}{0}$? $\frac{6}{8}\frac{4}{1}$? $\frac{2}{6}\frac{5}{4}$? $\frac{9}{1}\frac{}{0}\frac{}{0}$?

19. Class Exercise. —— may give a fraction that is a perfect square, and the class may give its square root.

20. How many cubic yards of earth must be removed to make a reservoir 120 ft. long, 44 ft. wide, and 9 ft. deep?

21. How much will it cost to dig a cellar 36 ft. long, 18 ft. wide, and 6 ft. deep, at $ 2.50 a cubic yard?

22. How many cords of wood in a pile 36 ft. long, 4 ft. wide, and 8 ft. high? At $3.50 a cord, how much would it cost?

Find the cost of plastering ceilings of the following rooms at 20¢ a square yard:

23. 18 ft. × 20 ft.

24. 16 ft. × 17 ft.

25. $14\frac{1}{2}$ ft. × 20 ft.

26. 9 ft. × $16\frac{1}{2}$ ft.

27. 10 ft. × $13\frac{1}{2}$ ft.

28. $15\frac{1}{3}$ ft. × $18\frac{1}{4}$ ft.

29. 27 ft. × 36 ft.

30. 3 yd. × 16 ft.

31. Estimate the cost of plastering the ceiling of your schoolroom at 25¢ a square yard.

Find the cost of plastering the walls and ceiling of rooms of the following dimensions:

	length	width	height
32.	6 m.	5 m.	2.8 m.
33.	8 m.	7 m.	3 m.
34.	6.5 m.	5 m.	3 m.
35.	7 m.	6.5 m.	3 m.

The price is 25¢ per square meter, and no allowance is made for openings.

36. An arc which is $\frac{1}{8}$ of a circumference is 1 yd. 1 ft. 3 in. long. How long is the circumference? The diameter? The radius?

37. How many acres in a field 56 rd. long and 40 rd. wide?

38. How much will it cost to pave a walk, 60 ft. long and 15 ft. wide, at $1.25 a square yard?

39. How many trees can be planted on 3 A. of ground if only 1 tree is planted on each square rod?

40. How many cubic feet in a pile of wood 24 ft. long, 3 ft. wide, and 8 ft. high? How many cords?

41. Express in grams the weight of the following measurements of pure water at its greatest density:

1 cu. dm. 3 l. 15 cu. cm. 1 ms. 1 ml. 1 s. 4 cu. m.

42. Express the measurements above in kilograms.

43. Taking $2\frac{1}{5}$ lb. as the equivalent of a kilogram, what is your weight in kilograms?

44. Express the following in avoirdupois on the basis of $2\frac{1}{5}$ lb. to the kilogram:

75 Kg. 88 Hg. 15 Dg. 175 g. 395 cg.

45. Express the following in avoirdupois weight:

33 Kg. 275 g. 924 mg. 99 Hg. 16 Dg.

46. How much is gained by buying a barrel of flour (196 lb.) for \$ 6.00 and selling it at 7¢ a kilogram?

47. How much is gained by buying 99 lb. of sugar at 5¢ a pound and selling it at 13¢ a kilogram?

48. How much is gained or lost by buying 440 lb. of dried fruit at 10¢ a lb. and selling it at 22¢ a Kg.?

49. How much is gained by buying 100 Kg. of coffee at 50¢ a Kg. and selling it at 30¢ a lb.?

50. How much is gained by buying 500 Kg. of raisins at 12¢ a Kg. and selling them at 8¢ a lb.?

51. Imagine a cubic decimeter cut from each corner of the upper layer of a cubic meter, and find the surface of the figure thus formed.

52. A sector whose arc is a quadrant was cut from a circle. If the area of the whole circle was $4\frac{4}{5}$ sq. in., what was the area of the part that was left?

53. A California woman took 300 lb. of honey from her hives in a month. What was its value at \$ 5.00 per hundredweight?

54. Earning $.75 per day, how long will it take a boy to earn enough to buy a $ 12.00 watch?

55. Mr. Taylor bought 3 prize pigs whose respective weights were 3 cwt. 73 lb. 12 oz., 4 cwt. 99 lb. 15 oz., 5 cwt. 12 oz. How much did they all weigh?

56. How many baskets, each holding $2\frac{1}{4}$ pk., can be filled with 18 bu. of apples?

57. A garden containing 1089 sq. yd. is $49\frac{1}{2}$ yd. long. How wide is it?

58. A fisherman had a line 24 yd. 2 ft. long. A fish broke off 3 yd. 1 ft. 6 in. of it. How much was left?

59. A dealer bought 2 T. 3 cwt. of carpet tacks in 8-oz. papers. How many papers of tacks were there?

60. How long is one side of an equilateral triangle whose perimeter is 5 yd. 1 ft. 3 in.? Of a regular pentagon having an equal perimeter? Of a regular octagon of equal perimeter?

61. A string, 4 yd. 2 ft. 6 in. long, was used to outline a regular hexagonal flower bed. How long was each side?

62. A farmer sold 5 loads of hay, each containing 17 cwt. 85 lb. How much did he sell?

63. How far will a man walk who begins walking at 9 A.M. and walks until 3.30 P.M., at the rate of 5 mi. an hour?

64. A family started to go in a wagon to St. Louis from a town 132 miles away. They rode 24 miles a day for 5 days. On the morning of the sixth day, they started at 9 o'clock to ride the remaining distance at the rate of 6 miles an hour. At what time did they reach St. Louis?

65. Mr. A has a lot 40 rd. square, and Mr. B has a lot containing 40 sq. rd. How many more square rods in Mr. A's lot than in Mr. B's?

66. A flight of stairs in Mrs. Long's house consists of 18 steps, each 1 ft. wide and 8 in. high. How much will the stair carpet cost at $.75 per yard, if 3 in. is allowed at each step for the turning in of the carpet?

67. How much can be earned in two weeks by a person who earns $ 2.34 every working day?

68. General McClellan was born Dec. 3, MDCCCXXVI, and died Oct. 29, MDCCCLXXXV. How old was he when he died?

69. April 4th, 1898, was Monday. At the close of that day, Ruth Mayo found that there were 8 weeks and 3 days left of the school term. On what day did the term close?

70. She entered college Sept. 14, 1898. The first term closed Dec. 21. How long was it?

71. Her expenses for the term were $ 95.75. What was the average per week?

72. The Thanksgiving vacation began Nov. 24 and ended Nov. 28, and there was no other vacation in the term. She attended a Saturday class. How many working days had she in that term?

73. Her second term began Jan. 3, 1899, and ended March 25, 1899. How long was it?

74. Her expenses for that term averaged $ 7.50 per week, and she earned $ 25 during the term by outside work. Her expenses were how much more than her earnings?

75. The floor of Mrs. Reed's dining room, which is 15 ft. long and 14 ft. wide, is laid with parquetry flooring. How much did it cost at $.62½ per square yard?

76. The wainscoting is 3 ft. high. There are 4 doors, each 3 ft. wide. Two windows, each 3 ft. wide, extend down into the wainscoting 1½ ft. There is a fireplace 4 ft. wide. How many square yards in the wainscoting?

77. Her dining table is 6 ft. long and 4 ft. wide. How many square yards in the top of it?

78. A rug under the table is 12 ft. long and covers 12 sq. yd. of the floor. How wide is it?

79. Make a problem about the dimensions of a room.

80. Fourteen cords of wood are piled evenly on an open car 28 ft. long and 8 ft. wide. How high is the wood piled?

81. If a leaf of a book is 12 cm. long and 9 cm. wide, how many square centimeters in the surfaces of both sides of the leaf?

82. If your schoolroom were 36 ft. long and 30 ft. wide, how many square yards could be drawn on the floor, provided no two overlapped?

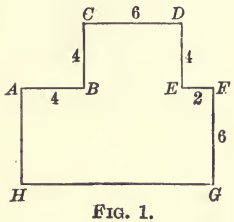


FIG. 1.

83. In Fig. 1 the angles are all right angles. How long is the line represented by *GH*? *HA*?

84. Copy Fig. 1, making the dimensions inches or centimeters. Draw a construction line *BE*. How long is it? Find the area of Fig. 1 by finding the sum of the areas of the two rectangles that are thus formed.

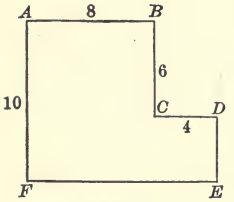


FIG. 2.

85. Copy Fig. 2. Find its area by drawing a construction line from *C* perpendicular to *FE* and finding the area of the two rectangles that are thus formed.

86. Find the area of Fig. 2 by drawing a construction line from *C* perpendicular to *AF* and finding the area of two rectangles thus made.

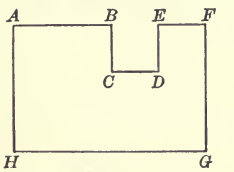


FIG. 3.

87. Copy Fig. 3, making *AB* 6 in., *BC* 3 in., *CD* 3 in., *DE* 3 in., *EF* 3 in., *FG* 8 in. Find length of *GH* and *HA*.

88. Find area of Fig. 3 by drawing construction lines that will divide it into three rectangles and finding the area of the rectangles. Show different ways of dividing it.

89. Find area of Fig. 3 by finding area of a rectangle *AFGH* and subtracting the square *BEDC*.

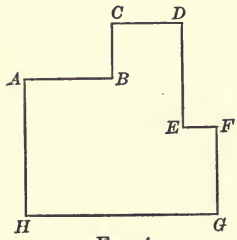


Fig. 4.

90. Copy Fig. 4, making *AB* 5 in., *BC* 3 in., *CD* 4 in., *DE* 6 in., *EF* 2 in., *FG* 4 in. *GH* = ? *HA* = ?

91. Show four different ways of dividing Fig. 4 into three rectangles. Find its area.

92. Beginning at a point marked A, draw to the right 4 in., down 3, to the right 3, down 4, to the left 3, down 3, to the left 4, up 3, left 3, up 4, right 3, up to A. Find perimeter of the figure. Find its area.

93. Beginning at A, draw down 7, to the right 3, up 3, right 3, down 3, right 4, up 4, left 2, up 2, left 3, up 1, left 5. Show several different ways of dividing the figure into rectangles. Find its area.

94. Class Exercise. —— may give directions to the class for drawing a figure which has only straight lines and right angles. The class may divide the figure by different construction lines and find their length and the area of the figure.

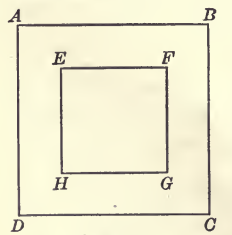


Fig. 5.

95. *ABCD* is a square 9 in. in dimensions, and *EFGH* is a square 5 in. in dimensions. How many square inches in the surface lying between the perimeter of the squares?

96. The frame of a mirror is 28 in. long and 20 in. wide on the outside edge. The glass in the center of the frame is 20 in. long and 12 in. wide. Represent. What is the width

of each side of the frame? How many square inches in the surface of the frame?

97. A rug 12 ft. long and 9 ft. wide was laid on the floor, leaving a margin 3 ft. in width all around the rug. What was the area of the floor? Of the rug? Of the uncovered part?

98. A picture 18 in. long and 15 in. wide has a frame each side of which is 6 in. wide. How many square inches in the surface of the frame?

99. A door 7 ft. high and 3 ft. wide has a 6-inch casing around it. How many square inches in the surface of the casing?

100. Measure a door and the width of its casing and find the number of square feet in the surface of the casing.

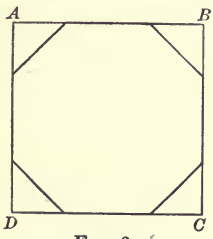


FIG. 6.

101. *ABCD* represents a square 15 in. in dimensions. The altitude and the base of each triangle is 4 in. Find the area of the octagonal figure left when the triangles are cut away.

102. Two lines are respectively 6 in. and 10 in. What is their average length?

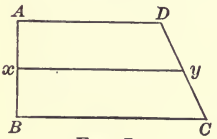


FIG. 7.

103. Reproduce the trapezoid *ABCD*, making *AB* 4 in., *BC* 8 in., and *AD* 6 in. Let x be the middle point of *AB*, and y the middle point of *DC*. xy represents the average length of the parallel sides of the trapezoid. How long is xy?

104. Through the point y draw the line *EF* parallel to *AB*. *EF* is the altitude of the trapezoid. Find the area of the rectangle *AEFB*. Cut off the triangle *yFC* and apply it to the triangle *yED*. How does the area of the rectangle *AEFB* compare with the area of the trapezoid *ADCB*?

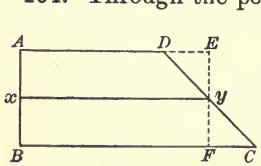


FIG. 8.

105. Draw another figure and show the reasons for the following rule.

To find the area of a trapezoid —

Multiply the average length of the parallel sides by the altitude.

106. Draw a trapezoid whose parallel sides are respectively 9 in. and 5 in., and whose altitude is 6 in. Find its area.

107. A board is 16 ft. long, 2 ft. wide at one end and 1 ft. wide at the other, tapering gradually. How many square feet in the surface of the board?

108. A farmer has a field in the shape of a trapezoid. One of the parallel sides is 40 rd. long, the other is 24 rd. long. The distance between them is 25 rd. Find area of the field.

Make diagrams to illustrate the following problems:

109. Mrs. Hall's parlor has a bay window, the floor of which is in the shape of a trapezoid. The longer of the parallel sides is 12 ft., the shorter 9 ft., and the distance between them is 4 ft. How much will it cost to cover it with parquetry flooring at $1.25 per square yard?

110. Her sitting room is 18 ft. long and 15 ft. wide. How many strips of carpet 1 yd. wide, running lengthwise of the room, will be required to cover the floor? How many yards in each strip? How much will the carpet cost at $.87½ a yard?

111. How many strips of carpet 1 yd. wide, running lengthwise, will be required for a floor 24 ft. wide? If the floor is 30 ft. long, how many yards will be required?

112. How much will it cost to carpet a room 30 ft. long 24 ft. wide with ingrain carpet at $.75 per yard, if a margin of ½ yard is left uncovered?

113. Mrs. Ross covered the floor of her parlor, 20 ft. long and 18 ft. wide, with velvet carpet 27 in. wide. How many strips were used? What was the cost of the carpet at $1.65 per yard?

114. Her sitting room, 16 ft. by 12 ft., is carpeted with ingrain carpet at $.67 per yard. In order to match the figures, the carpet layer was obliged to cut off or waste a piece of carpet $\frac{1}{3}$ of a yard long from each strip except the first. How much did the carpet cost?

115. Allowing $\frac{1}{4}$ of a yard to be turned in or cut off from each strip except the first in order to match the figures, how much will it cost to carpet a room 21 ft. by 18 ft. with carpet 27 in. wide, worth $1.35 per yard?

SUGGESTION FOR CLASS EXERCISE. Let pupils give dimensions of floors and estimate cost of covering them with carpet of different widths.

116. What is the volume of an 8-inch cube?

117. What number cubed equals 27? 216?

118. Find value of x: $x^3 = 64$. $x^3 = 125$. $x^3 = 343$.

119. Image a box in the shape of a cube whose volume is 8 cu. in. Suppose the box to be entirely covered with blue velvet. How many square inches of the velvet are there?

120. When a 3-inch cube is built of inch cubes, how many inch cubes are there that cannot be seen from the outside in whatever position the cube may be placed?

121. How many of the cubic centimeters that make a cubic decimeter have any of their surfaces on the outside of the figure? How many have one surface? Two surfaces? Three surfaces? No surface?

122. If a pipe discharges 245 gal. 2 qt. 1 pt. in 1 hr., how much will it discharge in the time from Tuesday, 6 P.M., to Wednesday, 11 A.M.?

123. A school that uses 12 crayons in a week will use how many gross of crayons in 40 weeks?

124. A stationer bought 8 gross of lead pencils and sold 50 dozen of them. How many lead pencils had he left?

125. If he bought them at $3 per gross and sold them at 5¢ apiece, how much did he gain on each pencil? On 50 doz. pencils?

126. At 10¢ a square yard, how much will it cost to sod a lawn 40 ft. long and 36 ft. wide?

127. If you had $23.70 in the bank at 4% interest, how much interest would it yield you each year?

128. Mr. Thomas Wheeler bought of Mr. Frank W. Barton a horse, the price of which was $100. Mr. Wheeler gave in payment a check for that amount on the California State Bank of Sacramento, of which the following is a copy:

Sacramento, March 1 1904 No. 22

California State Bank

Pay to the order of

Frank W. Barton $100 00/100

One Hundred 00/100 Dollars

Thomas Wheeler

SUGGESTIONS TO TEACHER. Explain the method of using checks to make payments. Get a check book from a bank. Select some pupil to act as banker and let pupils make imaginary purchases from one another, giving checks for the necessary amounts.

129. What is the advantage of keeping money in a bank and drawing it out as it is needed?

130. Mr. Dow borrowed $800 at 5% from Mr. Howe, kept it 3 yr., and then gave Mr. Howe his check for the amount due. Make out the check.

131. Mr. Ford had $427 in bank. He drew out $135.87, deposited $77.50, then drew out $35.25. How much remained to his credit in the bank?

132. Mr. Arnold had $1200 in the bank. On Monday he drew out $60. On Tuesday he drew out $30 more than on Monday. On Wednesday he drew out $90 more than on Tuesday. How much had he left in the bank?

133. Mr. Monroe lent $100 at 5% interest. At the end of 7 yr. the principal (that is the sum lent) and the interest were both paid. To how much did they both amount?

134. What amount will Mr. Day receive from $228 which he lent 2 yr. ago at 6%, if both principal and interest are paid? How much if the rate of interest is $6\frac{1}{2}\%$? $7\frac{1}{2}\%$? $5\frac{1}{2}\%$?

135. Class Exercise. —— may name a sum of money supposed to be lent at $3\frac{1}{2}\%$ for 2 yr., and the class may find the interest and the amount of principal and interest.

136. Mr. Shaw borrowed $750, kept it until it had gained $78.75 interest, and then paid $300. How much did he still owe?

137. Mr. Shaw borrowed $600 and gave his note for it, due in 2 yr. with 6% interest. How much was due at the end of the two years? At that time he made a partial payment of the note, paying only $200. How much did he still owe?

138. Mr. Shaw borrowed $700, giving his note at 5% interest. At the end of 2 years how large a partial payment must he make that only $500 may be due?

139. Mr. Shaw borrowed $900 at 6%. At the end of the first year he paid $154. How much was still due? That sum went on gaining interest until the end of the second year; then he paid $148. At the end of the third year he paid all that was due. How much did he pay?

140. Class Exercise. —— may name a sum of money and a number of years for which it was borrowed. Other members of the class may suggest partial payments to be made at different times, and the class may find the amount due after each payment.

141. Make out a bill for the following goods and receipt it:

C. H. Wilson bought from J. G. Cooper & Co., at Columbus, Ohio, on the tenth day of June, 1875: 13 lb. coffee @ 30¢; 4 lb. butter @ 35¢; 10 lb. flour @ 6¢; 12 lb. dried beef @ 24¢; 25 lb. sugar @ 18¢; 3 lb. starch @ 20¢.

142. From a field containing 400 sq. rd., the owner sold a piece of land 15 rd. square and another piece containing 15 sq. rd. How many square rods had he left?

143. Mr. Ray sold his house and a farm of 75 A., receiving $7500 for both. If the house was worth $2000, how much did he receive per acre for the land?

144. If Mr. Rudd earns $15 a week and spends $7, in how many weeks will he save $104?

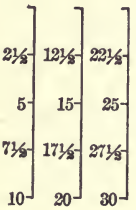
145. Mrs. Hall's sitting room has a picture rail extending all around it $1\frac{1}{2}$ ft. from the ceiling. The room is 18 ft. long and 15 ft. wide. How much did the picture rail cost at $7\frac{1}{2}$¢ per foot?

CHAPTER VI

ALIQUOT PARTS

1. Numbers, either integral or fractional, by which a given number is divisible are called **Aliquot Parts** of that number. For example, 5 and $2\frac{1}{2}$ are aliquot parts of 10.

Give three numbers which are aliquot parts of 100.



2. Draw three vertical lines each 10 in. long and divide them into lengths each $2\frac{1}{2}$ in. long. Number the lengths consecutively as in the diagram. How many $2\frac{1}{2}$ in. lengths in 10 in.? In 20 in.? In 30 in.?

3. Beginning with $2\frac{1}{2}$, count quickly by intervals of $2\frac{1}{2}$ to 30. Count back from 30 to $2\frac{1}{2}$ by intervals of $2\frac{1}{2}$.

NOTE TO TEACHER. The following exercises are for rapid drill, which should be given frequently until pupils learn the ratios of the smaller aggregations of $2\frac{1}{2}$ to one another. This kind of work leads to expertness in business calculations.

4. How many times $2\frac{1}{2}$ is $7\frac{1}{2}$? $17\frac{1}{2}$? $27\frac{1}{2}$? $12\frac{1}{2}$? 25? 15? $22\frac{1}{2}$? 10? 20? 30?

5. Give quickly the 4th multiple of $2\frac{1}{2}$. The 7th. 10th. 5th. 3d. 6th. 9th. 11th. 8th. 12th. 2d.

6. Which multiple of $2\frac{1}{2}$ is $7\frac{1}{2}$? 15? 25? 10? $17\frac{1}{2}$? $22\frac{1}{2}$? 30? $12\frac{1}{2}$? $27\frac{1}{2}$? 20? 5?

7. How many times $2\frac{1}{2}$ must be added to $7\frac{1}{2}$ to make 20? 30? $12\frac{1}{2}$? $17\frac{1}{2}$? $27\frac{1}{2}$? $22\frac{1}{2}$? 15? 25? 10?

8. How many times $2\frac{1}{2}$ must be taken from $27\frac{1}{2}$ to leave 15? $7\frac{1}{2}$? $22\frac{1}{2}$? $12\frac{1}{2}$? $2\frac{1}{2}$? 10? 5? $17\frac{1}{2}$? 25? 20?

9. With 15 as a starting point find how many times $2\frac{1}{2}$ must be added to it or subtracted from it to equal $27\frac{1}{2}$. $12\frac{1}{2}$. $2\frac{1}{2}$. 30. 20. $7\frac{1}{2}$. $22\frac{1}{2}$. 10. $17\frac{1}{2}$. 5. 25.

10. Learn to give quickly the ratio of 5 to each of the multiples of $2\frac{1}{2}$ that are less than $32\frac{1}{2}$.

11. Take Ex. 10, substituting for 5 each multiple of $2\frac{1}{2}$ that is greater than 5 and less than $32\frac{1}{2}$.

12. CLASS EXERCISE. —— may name two multiples of $2\frac{1}{2}$, and the class may give the ratio of the greater to the less, then the ratio of the less to the greater.

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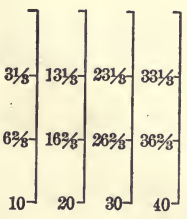
13. $\frac{10 \times 2\frac{1}{2} \times 6}{7\frac{1}{2} \times 15} = ?$

14. $\frac{12\frac{1}{2} \times 7\frac{1}{2} \times 4}{25 \times 15 \times 3} = ?$

15. $\frac{17\frac{1}{2} \times 25}{7 \times 2\frac{1}{2} \times 12\frac{1}{2}} = ?$

16. $\frac{12\frac{1}{2} \times 22\frac{1}{2} \times 6}{25 \times 9 \times 2\frac{1}{2} \times 7} = ?$

17. At $2\frac{1}{2}$ ¢ per yard what is the cost of 8 yd. of lace? 12 yd.? 7 yd.? 9 yd.? 6 yd.? 11 yd.?



18. At $2\frac{1}{2}$ ¢ per yard, how many yards of lace can be bought for 10 ¢? 20 ¢? 30 ¢? 40 ¢? 50 ¢? $12\frac{1}{2}$ ¢? 25 ¢?

19. Draw four vertical lines each 10 in. long and divide them into lengths of $3\frac{1}{8}$ in. Number the lengths consecutively. 40 in. equals how many times $3\frac{1}{8}$ in.?

20. Learn to count quickly from $3\frac{1}{8}$ to 40 by intervals of $3\frac{1}{8}$. From 40 to 0 by intervals of $3\frac{1}{8}$.

NOTE TO TEACHER. Give exercises upon $3\frac{1}{8}$ similar to those in Exs. 4–12 upon $2\frac{1}{2}$.

21. At $3\frac{1}{8}$ ¢ per yard, what is the cost of 7 yd. of lace? 10 yd.? 8 yd.?

22. At $3\frac{1}{8}¢$ per yard, how many yards of calico can be bought for $20¢$? $40¢$? $10¢$? $30¢$?

23. How many more yards of ribbon can be bought for $40¢$ when the price is $2\frac{1}{2}¢$, than when it is $3\frac{1}{3}¢$?

Cancel:

24. $\dfrac{3\frac{1}{8} \times 7\frac{1}{2} \times 5}{6\frac{2}{3} \times 3 \times 2\frac{1}{2} \times 7} = ?$

25. $\dfrac{2\frac{1}{2} \times 6\frac{2}{3} \times 12\frac{1}{2}}{10 \times 3\frac{1}{3} \times 9} = ?$

26. $\dfrac{12\frac{1}{2} \times 6 \times 10}{25 \times 7\frac{1}{2} \times 11} = ?$

27. $\dfrac{13\frac{1}{3} \times 2\frac{1}{2} \times 2}{6\frac{2}{3} \times 7\frac{1}{2} \times 5} = ?$

28. A merchant bought goods at $8¢$ a yard, and sold them for $10¢$ a yard. How much did he gain on each yard? What is the ratio of the gain to the cost?

29. Find ratio of gain to cost of goods:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Bought at	$6¢$	$8¢$	$9¢$	$5¢$	$7\frac{1}{2}¢$	$10¢$	$10¢$	$12\frac{1}{2}¢$
Sold at	$9¢$	$12¢$	$12¢$	$7\frac{1}{2}¢$	$10¢$	$12\frac{1}{2}¢$	$15¢$	$15¢$

30. When goods are bought for $9¢$ a yard and sold for $6¢$ a yard, how much is lost? What is the ratio of the loss to the cost?

31. Find ratio of the loss to the cost of goods:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Bought at	$8¢$	$10¢$	$15¢$	$20¢$	$18¢$	$7\frac{1}{2}¢$	$10¢$	$12\frac{1}{2}¢$
Sold at	$6¢$	$8¢$	$10¢$	$15¢$	$15¢$	$5¢$	$7\frac{1}{2}¢$	$10¢$

32. Find ratio of gain or loss to cost of goods:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Bought at	$45¢$	$20¢$	$25¢$	$36¢$	$22¢$	$33¢$	$36¢$	$60¢$
Sold at	$50¢$	$15¢$	$30¢$	$40¢$	$33¢$	$22¢$	$32¢$	$72¢$

33. Find ratio of gain to cost of goods:

	a	*b*	*c*	*d*	*e*	*f*	*g*
Cost,	$10¢$	$20¢$	$30¢$	$6\frac{2}{3}¢$	$16\frac{2}{3}¢$	$26\frac{2}{3}¢$	$13\frac{1}{3}¢$
Selling price,	$13\frac{1}{3}¢$	$23\frac{1}{3}¢$	$33\frac{1}{3}¢$	$10¢$	$20¢$	$30¢$	$16\frac{2}{3}¢$

34. How much is gained on each apple bought at the rate of 2 for 5¢, and sold at 3¢ apiece?

35. On the 4th of July, Andrew bought lemons at the rate of 3 for a dime. He used one lemon and $\frac{2}{3}$ of a cent's worth of sugar to make each glass of lemonade. How much did he gain on each glass of lemonade sold at 10¢ a glass? How much on $1\frac{1}{2}$ doz. glasses?

36. For $1.00 William bought ice cream enough to fill 36 ice cream plates. He sold it at 10¢ a plateful. How much did he gain on each plateful of ice cream? He sold $1\frac{1}{2}$ doz. platefuls. How much did he gain?

37. The rest of his ice cream was unsold and was wasted. Did he gain or lose, and how much, on his whole transaction?

38. Beginning at $16\frac{2}{3}$, count quickly to 100 and back to 0 by intervals of $16\frac{2}{3}$.

39. How many times is $16\frac{2}{3}$ contained in $66\frac{2}{3}$? In $33\frac{1}{3}$? In 100? In $83\frac{1}{3}$?

40. How many times $16\frac{2}{3}$ must be added to 50 or subtracted from it to make $83\frac{1}{3}$? 100? $66\frac{2}{3}$? $16\frac{2}{3}$?

41. Give the ratio of $16\frac{2}{3}$ to each of its multiples that is less than 101.

42. Learn to give quickly the ratio of each multiple of $16\frac{2}{3}$ that does not exceed 100, to every other multiple of $16\frac{2}{3}$ that does not exceed 100.

43. Class Exercise. —— may name two multiples of $16\frac{2}{3}$, and the class may give the ratio of the less to the greater, and of the greater to the less.

44. At $16\frac{2}{3}$¢ per yard, what is the cost of 3 yd. of lawn? 5 yd.? 7 yd.? 2 yd.? 6 yd.? 4 yd.?

45. At $16\frac{2}{3}$¢ per yard, how many yards of lawn can be bought for $1? $2? $7? $10? $33\frac{1}{3}$¢? $83\frac{1}{3}$¢? $66\frac{2}{3}$¢?

46. Beginning with $8\frac{1}{3}$, count quickly to 100 and back to 0 by $8\frac{1}{3}$'s.

SUGGESTION TO TEACHER. Exercises should be given upon $8\frac{1}{3}$ similar to those in Exs. 39–43 upon $16\frac{2}{3}$.

47. CLASS EXERCISE. —— may name two multiples of $8\frac{1}{3}$, and the class may give their reciprocal ratios.

48. $8\frac{1}{3}¢ = 25\%$ of what? 50%? 10%?

49. At $8\frac{1}{3}¢$ per yard, what is the cost of 6 yd. of muslin? 3 yd.? 12 yd.? 4 yd.? 8 yd.? 10 yd.? 5 yd.? 9 yd.?

50. At $8\frac{1}{3}¢$ per yard, how many yards of lace can be bought for \$1.00? \$.83$\frac{1}{3}$? \$1.08$\frac{1}{3}$? \$.33$\frac{1}{3}$? \$.66$\frac{2}{3}$? \$.50?

51. Find gain and ratio of gain to cost.

	a	*b*	*c*	*d*	*e*	*f*	*g*
Cost,	25¢	50¢	$16\frac{2}{3}¢$	$33\frac{1}{3}¢$	$66\frac{2}{3}¢$	75¢	$91\frac{2}{3}¢$
Selling price,	$33\frac{1}{3}¢$	$66\frac{2}{3}¢$	25¢	50¢	75¢	$83\frac{1}{3}¢$	\$1.00

52. How much would Mr. Lee gain by buying 12 yd. of cassimere, at \$.66$\frac{2}{3}$ a yard, and selling it at \$1.00 a yard?

53. Would he gain or lose, and how much, by buying 12 yd. of silk at \$.33$\frac{1}{3}$ per yard, selling half of it at \$.50 per yard, and the rest at \$.25?

54. $\frac{1}{12}$ of $100\% = ?$

55. Complete the following table and learn it:

$8\frac{1}{3}\% = \frac{1}{12}$	$33\frac{1}{3}\% =$ —	$58\frac{2}{3}\% =$ —	$83\frac{1}{3}\% =$ —
$16\frac{2}{3}\% =$ —	$41\frac{2}{3}\% =$ —	$66\frac{2}{3}\% =$ —	$91\frac{2}{3}\% =$ —
$25\% =$ —	$50\% =$ —	$75\% =$ —	$100\% =$ —

56. Copy the six-pointed star given on page 92 and divide it into 6 equal rhombuses.

57. If the perimeter of the star were 100 in., how long would the perimeter of one of the rhombuses be?

58. What is the ratio of each rhombus to the star? Express the ratio in per cent. What per cent of the star would remain if one rhombus were erased?

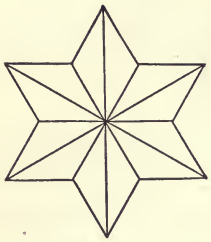


FIG. 1.

59. Draw a line from each point of the star to the center of the figure as in Fig. 1. Into what kind of figures is each rhombus divided?

60. What is the ratio of each isosceles triangle to the rhombus of which it is a part? To the star?

61. If one triangle were shaded, what per cent of the star would be unshaded?

62. Place letters at the center and at the vertices of the angles of the star and name a figure that is 50% of the star. 75%. $8\frac{1}{3}\%$. $16\frac{2}{3}\%$. $33\frac{1}{3}\%$. $66\frac{2}{3}\%$. $83\frac{1}{3}\%$.

63. Kate drew a square foot on the board and marked it off into inches. She erased $8\frac{1}{3}\%$ of it. How many square inches were left?

64. How many cubic inches in $8\frac{1}{3}\%$ of a cubic foot?

65. To gain $8\frac{1}{3}\%$, at what price must goods be sold that cost 12¢? 48¢? 60¢? 30¢? 45¢? 72¢? 75¢? $1.08?

66. Mr. Barnett drew out of the bank $56, which was $16\frac{2}{3}\%$ of the money he had in the bank. How much had he in the bank before he drew any out? How much afterward?

67. A line a yard long was lengthened $8\frac{1}{3}\%$. How long was it then?

68. How many times is $8\frac{1}{3}$ contained in the second multiple of $16\frac{2}{3}$? In the 5th? 3d? 6th?

69. How many times is $3\frac{1}{3}$ contained in the second multiple of $16\frac{2}{3}$? In the 3d? 6th? 4th?

70. Find ratio of gain or loss to cost of goods at the following prices. Express ratios in per cent.

	Cost	Selling Price		Cost	Selling Price
a	\$.50	\$.66⅔	*f*	\$.83⅓	\$.66⅔
b	.33⅓	.50	*g*	.66⅔	.33⅓
c	.83⅓	1.00	*h*	.33⅓	.16⅔
d	1.00	.66⅔	*i*	.66⅔	1.00
e	.66⅔	.50	*j*	.33⅓	.66⅔

71. How much is $\frac{1}{8}$ of 100? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$?

72. Beginning at $12\frac{1}{2}$, name all the multiples of $12\frac{1}{2}$ that are less than 101. Write them in a vertical column.

Suggestion to Teacher. Exercises should be given upon $12\frac{1}{2}$ similar to those in Exs. 39–43 upon $16\frac{2}{3}$.

73. Name all the multiples of $12\frac{1}{2}$, less than 101, that are also multiples of $8\frac{1}{3}$, and tell how many times they contain $8\frac{1}{3}$; also how many times they contain $12\frac{1}{2}$.

74. If 6 yd. of calico are required for Mary's dress, how much more would it cost at $12\frac{1}{2}¢$ per yard than at $8\frac{1}{3}¢$?

75. Jennie has \$1.25 to spend in ribbon. How many more yards can she buy at $8\frac{1}{3}¢$ per yard than at $12\frac{1}{2}¢$?

76. Find the amount of gain on each article, and the ratio of the gain to the cost of the goods at the following prices:

	a	*b*	*c*	*d*	*e*	*f*	*g*
Cost,	25¢	50¢	62½¢	75¢	12½¢	50¢	62½¢
Selling price,	37½¢	62½¢	87½¢	87½¢	25¢	87½¢	75¢

77. How much did Mr. Luce gain on 14 yd. of silk bought at \$.87½ per yd. and sold for \$ 1.25 per yd.?

78. Count from 0 to 100 and back from 100 to 0 by intervals of $6\frac{1}{4}$.

Suggestion to Teacher. Exercises should be given upon $6\frac{1}{4}$ similar to those in Exs. 39–43 upon $16\frac{2}{3}$.

79. Class Exercise. The multiples of $6\frac{1}{4}$ being written upon the board, —— points to one of them, and the class give quickly its ratio to $6\frac{1}{4}$.

80. At $6\frac{1}{4}$ cents a yard, what is the cost of 8 yd. of calico? 12 yd.? 3 yd.? 5 yd.? 7 yd.? 9 yd.? 11 yd.? 6 yd.?

81. At $6\frac{1}{4}$ cents a yard, how many yards of calico can be bought for $25¢$? $50¢$? $\$1$? $\$.12\frac{1}{2}$? $\$.37\frac{1}{2}$? $\$.62\frac{1}{2}$?

82. Find gain and ratio of gain to cost:

	a	*b*	*c*	*d*	*e*	*f*
Cost,	$12\frac{1}{2}¢$	$25¢$	$25¢$	$37\frac{1}{2}¢$	$50¢$	$18\frac{3}{4}¢$
Selling price,	$18\frac{3}{4}¢$	$31\frac{1}{4}¢$	$37\frac{1}{2}¢$	$50¢$	$56\frac{1}{4}¢$	$25¢$

83. Find loss and ratio of loss to cost:

	a	*b*	*c*	*d*	*e*		*g*
Cost,	$18\frac{3}{4}¢$	$25¢$	$37\frac{1}{2}¢$	$50¢$	$75¢$	$68\frac{3}{4}¢$	$62\frac{1}{2}¢$
Selling price,	$12\frac{1}{2}¢$	$18\frac{3}{4}¢$	$31\frac{1}{4}¢$	$43\frac{3}{4}¢$	$68\frac{3}{4}¢$	$62\frac{1}{2}¢$	$56\frac{1}{4}¢$

84. How many times is $6\frac{1}{4}$ contained in the 4th multiple of $12\frac{1}{2}$? In the 7th multiple of $12\frac{1}{2}$? In the 3d multiple of $12\frac{1}{2}$?

85. How many more yards of goods can be bought for $\$1.00$ when the price is $6\frac{1}{4}¢$ than when it is $12\frac{1}{2}¢$?

86. How much did Mr. Colton gain on 8 yd. of denim, bought at $\$.12\frac{1}{2}$ a yard and sold at $\$.18\frac{3}{4}$?

87. How much did he lose on 20 yd. of calico, bought at $\$.06\frac{1}{4}$ a yard and sold at $\$.05$ a yard?

88. Find loss or gain and its ratio to cost:

	a	*b*	*c*	*d*	*e*	*f*	*g*
Cost	$56\frac{1}{4}¢$	$37\frac{1}{2}¢$	$62\frac{1}{2}¢$	$75¢$	$25¢$	$43\frac{3}{4}¢$	$68\frac{3}{4}¢$
Selling price,	$62\frac{1}{2}¢$	$43\frac{3}{4}¢$	$68\frac{3}{4}¢$	$50¢$	$12\frac{1}{2}¢$	$50¢$	$75¢$

89. How much did Mr. Hale gain on 8 yd. of ribbon, bought at $\$.37\frac{1}{2}$ a yard and sold at $\$.50$ a yard?

90. Write all of the first 12 multiples of $8\frac{1}{3}$ that are also multiples of $6\frac{1}{4}$, and tell how many times each of them contains $8\frac{1}{3}$, and how many times it contains $6\frac{1}{4}$.

91. How many more yards of ribbon can be bought at $6\frac{1}{4}\,¢$ than at $8\frac{1}{3}\,¢$, if the sum spent is \$1.00? \$.50? \$.75? \$.25?

92. $\dfrac{8\frac{1}{3} \times 6\frac{1}{4}}{25 \times 25} = ?$

93. $\dfrac{8\frac{1}{3} \times 2\frac{1}{2} \times 6\frac{1}{4}}{16\frac{2}{3} \times 10 \times 25} = ?$

94. $\dfrac{2\frac{1}{2} \times 3\frac{1}{3} \times 6\frac{1}{4} \times 8\frac{1}{3}}{5 \times 10 \times 25 \times 16\frac{2}{3}} = ?$

95. $\dfrac{8\frac{1}{3} \times 12\frac{1}{2} \times 2\frac{1}{2}}{33\frac{1}{3} \times 6\frac{1}{4} \times 7\frac{1}{2}} = ?$

96. $\dfrac{12\frac{1}{2} \times 8\frac{1}{3} \times 2\frac{1}{2}}{7\frac{1}{2} \times 25 \times 6\frac{1}{4}} = ?$

97. $\dfrac{8\frac{1}{3} \times 25 \times 12\frac{1}{2}}{25 \times 75 \times 62\frac{1}{2}} = ?$

98. $\dfrac{75 \times 25 \times 2\frac{1}{2}}{62\frac{1}{2} \times 6 \times 5} = ?$

99. $\dfrac{87\frac{1}{2} \times 5 \times 3}{14 \times 12\frac{1}{2} \times 2\frac{1}{2}} = ?$

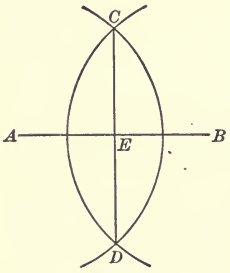


FIG. 2.

100. Draw the line AB of any convenient length and bisect it.

To bisect the line AB. With A as a center and any convenient radius greater than one half of AB, describe an arc. With B as a center and the same radius describe an arc cutting the first arc at C and D. Join C and D. The point E where CD cuts AB is the middle point of the line AB. CD is perpendicular to AB.

101. Bisect AE and mark its middle point F. Bisect EB and mark its middle point G.

102. Mark on each new division what per cent it equals of the line AB.

103. What per cent of the line AB is AF? AG? GB? EB? FG? FB?

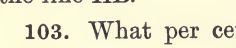


FIG. 3.

104. Bisect each division of the line AB and mark it as in Fig. 3. Write on each division its per cent of the whole line AB.

105. What per cent of the whole line is Ay? Am? Az? Fz? xy? xm? Fm? xB? yB? ym? xz?

106. Draw a regular octagon and divide it into 8 equal isosceles triangles. Each triangle is what per cent of the octagon?

107. Divide each triangle into 2 equal right triangles. Each triangle is what per cent of the octagon?

108. Complete the following table and learn it:

$6\frac{1}{4}\% = \frac{1}{16}$	$31\frac{1}{4}\% =$ —	$56\frac{1}{4}\% =$ —	$81\frac{1}{4}\% =$ —
$12\frac{1}{2}\% =$ —	$37\frac{1}{2}\% =$ —	$62\frac{1}{2}\% =$ —	$87\frac{1}{2}\% =$ —
$18\frac{3}{4}\% =$ —	$43\frac{3}{4}\% =$ —	$68\frac{3}{4}\% =$ —	$93\frac{3}{4}\% =$ —
$25\% =$ —	$50\% =$ —	$75\% =$ —	$100\% =$ —

109. How many square inches in $12\frac{1}{2}\%$ of a rectangle 1 ft. long and $\frac{1}{2}$ ft. wide?

110. How much is $37\frac{1}{2}\%$ of $72¢$? Of $\$1.44$? Of $\$4.80$?

111. To gain $12\frac{1}{2}\%$, how must goods be sold that cost $8¢$? $40¢$? $60¢$? $50¢$? $\$1.20$? $\$1.60$? $\$3.20$? $\$6$?

112. $\$5$ is $12\frac{1}{2}\%$ of Charles's weekly salary. How much does he receive each week?

113. Mr. Owen sold 15 acres of land, which was $12\frac{1}{2}\%$ of his farm. How large was his farm before the sale? After the sale?

114. In Mrs. Abbot's parlor there is a rug 6 ft. long and 5 ft. wide, which covers $12\frac{1}{2}\%$ of the floor. How many square feet are there in the floor?

115. What is the ratio of the uncovered part of the floor to the whole floor? Express that ratio in per cent.

116. Find loss or gain and ratio to cost:

	Cost	Selling Price		Cost	Selling Price
a	$\$.25$	$\$.37\frac{1}{2}$	e	$\$1.00$	$\$.75\frac{1}{2}$
b	.50	$.62\frac{1}{2}$	f	1.00	$.62\frac{1}{2}$
c	.75	$.87\frac{1}{2}$	g	.75	$.62\frac{1}{2}$
d	$.87\frac{1}{2}$	1.00	h	$.37\frac{1}{2}$	$.12\frac{1}{2}$

By observing some aliquot parts of numbers we may find short methods of multiplying.

117. Annex two ciphers to 48 and divide the result by 4. Compare the result with the product of 48×25.

Observe that by annexing two ciphers to any integer we multiply it by 100, which gives a result 4 times as great as when the integer is multiplied by 25.

118. Take Ex. 117, substituting other numbers for 48. What general truth connected with that work can you state?

119. Illustrate the following rules and give reasons for them:

To multiply by $3\frac{1}{3}$. — *Annex a cipher to the multiplicand and divide the result by* 3.

To multiply by $33\frac{1}{3}$. — *Annex two ciphers to the multiplicand and divide the result by* 3.

120. Tell how you would multiply a number by $333\frac{1}{3}$ by the method of aliquot parts.

121. Multiply 72 by 25. By $33\frac{1}{3}$. By $333\frac{1}{3}$.

122. Annex a cipher to 36 and divide the result by 4. Why is the quotient thus obtained equal to the product of 36 and $2\frac{1}{2}$?

123. If you annex a cipher to 24 and divide the result by 3, the quotient thus obtained equals how many times 24? Why?

124. By $2\frac{1}{2}$ multiply 84. 56. 128. 31.

125. By $3\frac{1}{3}$ multiply 27. 81. 43. 28. 15.

126. Give a rule for finding the product of $2\frac{1}{2}$ and any integer without using either $2\frac{1}{2}$ or the integer as a multiplier. Give reasons for the rule.

127. Give a similar rule for finding the product of $3\frac{1}{3}$ and any integer. Give reasons.

128. How many ciphers must be annexed to an integer, and by what must the result be divided, in order to multiply the integer by $16\frac{2}{3}$? By $12\frac{1}{2}$? Illustrate.

MISCELLANEOUS EXERCISES

1. What number is 6 more than $1\frac{1}{2}$ times $16\frac{2}{3}$?

2. Square 2.1. 3.2. 1.25. 1.02. 2.003.

3. Cube .02. .8. 1.2. 1.03. 1.05.

4. Change $\frac{1}{2}$ and $\frac{5}{8}$ to decimals and find their product.

5. Change to decimals and find their continued product:

a			*b*			*c*		
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{8}$

6. Find sum of all the prime numbers between 20 and 50.

7. How many times 4.37 equal 17.48?

8. What number besides 137 will exactly divide 11,371?

9. $\frac{2}{3}$ of $8\frac{1}{2} \times \frac{6}{6} \times \frac{28}{34} \times \frac{16}{25}$ of $\frac{15}{75}$ of $\frac{25}{100} = ?$

10. Reduce:

$$\frac{15}{7\frac{1}{2}} \quad \frac{10}{3\frac{1}{3}} \quad \frac{16\frac{2}{3}}{3\frac{1}{3}} \quad \frac{33\frac{1}{3}}{3\frac{1}{3}} \quad \frac{40}{3\frac{1}{3}} \quad \frac{13\frac{1}{3}}{3\frac{1}{3}}$$

11. A man worked 5 da. at the rate of \$ $9\frac{3}{4}$ a week (6 da.). How much did he receive for the 5 da. work?

12. $6\frac{3}{8} \times \frac{16}{17} \times \frac{5}{33} \times \frac{11}{25} = ?$ $\quad 5\frac{5}{8} \div \frac{9}{16} \div 1\frac{2}{3} \div 2\frac{7}{10} = ?$

13. Give a fraction whose reciprocal is less than the fraction itself. Is the original fraction proper or improper? What kind of a fraction is the reciprocal?

14. Find $33\frac{1}{3}\%$ of the largest odd number that can be written with two figures.

15. Find 11% of the only prime number between 89 and 101.

16. Find $16\frac{2}{3}\%$ of 24^2.

17. Find 25% of $\sqrt{64}$. Of $\sqrt{144}$.

18. Find $66\frac{2}{3}\%$ of 6^3. Of 8^3.

19. Find 50 % of the cube root of the following numbers:

27 64 343 125 216 729 1728

20. Divide twenty-four thousandths by sixteen millionths.

21. A lady buys a dollar's worth of soap at $6\frac{1}{4}¢$ a bar. If she uses 8 bars in 1 mo., how long will it last?

22. How long is the circumference of a circle whose diameter is $1\frac{5}{16}$ cm.? The arc that is $\frac{4}{11}$ of that circumference?

23. At $12\frac{1}{2}¢$ a qt., how much will 2 gal. of molasses cost? 8 gal.? 3 gal. 3 qt.? 1 pt.? 2 qt. 1 pt.? 1 gi.? 3 gi.?

24. It costs $6\frac{1}{4}¢$ to make a gill of a certain kind of medicine. If it is sold for a dollar a quart, how much is gained on a quart?

25. Express in per cent the ratio of a peck to a bushel.

26. Express in per cent the ratio of 7 m. to a dm. 6 m. to a Hm. A square meter to a hectare. 9 sq. dm. to a sq. m. 700 cu. dm. to a stere.

27. Reduce to a common fraction in its lowest terms .075. .875. .5625.

28. Write five fractions whose numerators are each $6\frac{1}{4}$ and whose denominators are multiples of $6\frac{1}{4}$, and reduce them to lowest terms.

29. Write six fractions whose numerators are each $8\frac{1}{3}$ and whose denominators are multiples of $8\frac{1}{3}$, and reduce them to lowest terms.

30. Divide 50% of 216.48 by 33.

31. $$\frac{177.76 - 25\% \text{ of } 177.76}{1.2} = ?$$

32. Change to hundredths expressed as a decimal:

$\frac{7}{15}$ $\frac{5}{6}$ $\frac{19}{24}$ $\frac{17}{18}$ $\frac{1}{16}$

33. Express in per cent $\frac{1}{9}$. $\frac{2}{9}$. $\frac{3}{9}$. $\frac{4}{9}$. $\frac{5}{9}$. $\frac{6}{9}$. $\frac{7}{9}$. $\frac{8}{9}$.

34. A regiment in marching takes 128 steps in a minute, each step $2\frac{3}{4}$ ft. long. How many feet does it advance in an hour?

35. How many rods of fencing will it take to inclose a lot 45 rd. 3 yd. long and 30 rd. 1 yd. wide?

36. A room is $17\frac{2}{3}$ ft. long and $12\frac{5}{8}$ ft. wide. What will be the cost of a molding around it at $3\frac{1}{2}¢$ per foot?

37. Divide the sum of $\frac{2}{3}$ and $\frac{3}{4}$ by $\frac{7}{8}$.

38. A farmer's wife sold to a grocer 30 doz. eggs at $18\frac{3}{4}¢$ per dozen, receiving in payment a barrel of flour at $5.50 per barrel, and the balance in cash. How much cash did she receive?

39. A grocer bought apples at $1.50 per bushel, and sold them at 50¢ a peck. How much did he gain on each bushel?

40. What is the cost of excavating 437.24 cu. yd. of earth at $1.65 a cubic yard?

41. A man sold $\frac{2}{5}$ of his farm of 216 A. to one neighbor, and $\frac{3}{8}$ of it to another. How many acres were left?

42. Four men, A, B, C, and D, together bought a ship for $16,256. A paid $4756, B paid $763 more than A, and C paid $256 less than B. How much did D pay?

43. How many quarts of water can be poured into a tin box that is 11 in. long, 6 in. wide, and 7 in. deep? (231 cu. in. = 1 gal.)

44. Seven boys pick 4 bu. 3 pk. 7 qt. of berries and share them equally. What is each boy's share?

45. How many more pounds of candy can be bought for $1, at $6\frac{1}{4}¢$ per pound, than at $33\frac{1}{3}¢$?

46. How many axes, each weighing 3 lb. 8 oz., can be made from a ton of iron?

47. If a man walks 65 ft. in 1 min., how many miles will he walk in $10\frac{1}{2}$ hr.?

48. How many steps, each 2 ft. 6 in. long, will a boy take in going around a lot 5 ft. square?

49. A man has 285 bu. 3 pk. 6 qt. of grain, which he wishes to take to market in 15 equal loads. How much must he put into each load?

50. A worker in a cotton mill weaves 6 cuts of cloth in a day, receiving $16\frac{2}{3}¢$ a cut. How much does she earn in a week?

51. The circumference of a bicycle wheel is 3 ft. How many times does it turn in running 18 ft.? 15 yd.? 4 rd.?

52. If a horse eats 2 bales of hay, costing $60¢$ a bale, and 2 bushels of oats, costing $30¢$ a bushel, in 1 week, how much does it cost to feed him 1 year of $52\frac{1}{7}$ weeks?

53. An automobile ran $67\frac{3}{5}$ mi. in one day, and $1\frac{1}{4}$ times as far the next day. How far did it run in both days?

54. If Lucy washes dishes 3 times a day, how many times will she wash dishes in the winter months, beginning Dec. 1, 1903?

55. One of the parallel sides of a trapezoid is $9\frac{5}{12}$ ft. long. The other is $3\frac{7}{12}$ ft. longer. Each of the non-parallel sides is $5\frac{5}{12}$ ft. long. How long is the perimeter of the trapezoid?

56. Multiply 26 by 11 by a short method.

This can be done by writing the sum of the digits between them. Eight is the sum of the digits 2 and 6. This written between them gives the number 286, which is the product of 26 and 11.

57. By 11 multiply:

33 72 54 81 45 71 60 22 70

58. Think of a number the sum of whose digits is less than 10. Multiply it by 11 by the ordinary written method, and try to discover why the short method gives the same result.

59. Try to discover how the product should be written when the sum of the digits is more than 9.

60. By 11 multiply:

36 48 79 85 91 87 58 69 75

61. A man had two plots of land fronting a street. The first piece was 600 ft. wide, the second 950 ft. wide. He divided them into house lots of the greatest possible equal width. How wide was each lot? How many lots were there?

62. A man hoed a piece of land in $9\frac{1}{3}$ da., hoeing $\frac{3}{4}$ of an acre each day. How many acres were there in the piece?

63. If 18 suits of clothing can be made from 101 yd. of cloth, how many yards will be needed for 30 suits?

64. Mr. Wade, dying, left $ 9600 to be divided equally among his three sons, Arthur, Henry, and Joseph. How much did each receive?

65. Mr. Arthur Wade bought a lot for $ 1000, built a house costing $ 2200, and rented it for $ 24 a month. The taxes on the property for the first year were $ 75. The insurance was $ 8. The house was vacant two months. At the end of the year he sold the property for $ 2800. Did he gain or lose, and how much?

66. Mr. Henry Wade bought $ 3200 worth of mining stock, on which his taxes were $ 13.75. He visited the mine at an expense of $ 22.50. His yearly profits were $ 173.75. At the end of the year he sold his mining stock for $ 3000. Did he gain or lose, and how much?

67. Mr. Joseph Wade bought a farm of 50 A. at $ 40 an acre, with a house worth $ 1200. The taxes on the farm were $ 41.50. He rented the farm, receiving as rent $\frac{1}{3}$ of the value of the crops. The crops sold for $ 935.75. At the end of the year the farm was worth as much as at the beginning. Did he gain or lose, and how much?

68. Scott & Co. of St. Louis bought a bill of goods in New York amounting to $ 836.75. They bought them with the agreement that they need not pay for them for three months,

but that if they chose to pay for them at once, 5% of the bill would be taken off. They chose to pay at once. How much did they pay?

69. The same firm bought another bill of goods amounting to $1573.84, and were allowed a discount of 5% for cash payment. How much did they pay?

70. Kinkle, Barbour & Co. of Springfield, Ill., sold a bill of goods to Luther Johnson of Jackson, Tenn., as follows:

1 doz. pairs Men's Shoes @ $1.50 per pair.

1 case (12 pairs) Children's Oxfords @ $.75 per pair.

4 gross of Leather Laces at $.50 per gross.

2 cases Children's and Misses' Oil Grain @ $1.00 per pair.

1 doz. Men's Kangaroo Calf @ $1.60 per pair.

½ doz. Misses' Sandals @ $.90 per dozen.

Make out the bill and receipt it, allowing 5% off for cash.

71. Mr. Lang borrowed $700 at 7%. At the end of $3\frac{3}{4}$ years how much did he owe?

72. Mr. Lang borrowed $800 at 5% and at the end of 2 years made a payment of $200. How much did he still owe?

73. Mr. Davis invested $625 in a mine, gained 7% on his investment, and paid $5.25 in taxes on it. How much of his profit remained?

74. Find loss and ratio of loss to cost of goods:

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
Bought at	6¢	12¢	8¢	8¢	16¢	20¢	30¢	25¢	40¢
Sold at	5¢	9¢	7¢	5¢	14¢	15¢	25¢	20¢	30¢

Express in per cent.

CHAPTER VII

PERCENTAGE

1. The Latin phrase "per centum," meaning "by the hundred," is shortened to "per cent," and is represented by the sign "%."

Express 15% as a common fraction. As a decimal.

2. Arrange the following in the order of their size, placing the smallest first: $\frac{35}{100}$. .33. 34%.

3. Express 21% in three different ways.

4. Find 7% of \$20 by the following rule:

To find any per cent of a number —

Multiply the number by the per cent expressed as a decimal.

5. Give reasons for the rule.

6. By this rule find 11% of \$48. Of 96.

7. By the rule you have been using find 50% of 8. Show an easier way of finding 50% of 8.

8. Whenever a given per cent can be expressed as a common fraction, a convenient method of finding that per cent of a given number is to multiply the number by that fraction expressed in its lowest terms.

In this way find $16\frac{2}{3}$% of 1200. Of 2400. Of 72.6. Of $\frac{6}{7}$.

9. Give the value of each of the following fractions in per cent:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{6}$	$\frac{2}{7}$	$\frac{5}{8}$	$\frac{2}{9}$	$\frac{7}{9}$	$\frac{3}{10}$	$\frac{1}{12}$
$\frac{1}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{4}{9}$	$\frac{8}{9}$	$\frac{7}{10}$	$\frac{1}{16}$
$\frac{2}{3}$	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{1}{7}$	$\frac{3}{8}$	$\frac{1}{9}$	$\frac{5}{9}$	$\frac{1}{10}$	$\frac{9}{10}$	$\frac{1}{20}$

10. How much is 100% of a dollar? Of a day? If you had only a dollar, could you spend 150% of it? Explain.

11. What is 100% of 5 books? 100% of 2 watermelons?

12. The number or quantity of which a per cent is taken is call the **Base** of percentage.

With 72 as a base find $11\frac{1}{9}\%$. $44\frac{4}{9}\%$. $88\frac{8}{9}\%$. $16\frac{2}{3}\%$.

13. Of a school of 40 children 20% were absent. How many pupils were absent? How many were present? What per cent of them were present?

14. When you have written 40% of a spelling lesson of 20 words, how many words have you written? How many are yet to be written?

15. Think of \$1000; decrease it 10%, then increase the remainder $33\frac{1}{3}\%$, decrease that result 25%, decrease \$100, increase 50%, decrease \$200, increase 50%. What is the last result?

16. How many different bases of percentage were given in the preceding problem? Name each.

17. Mr. Smith had \$500, gained 20%, then lost 50% of what he had, gained $33\frac{1}{3}\%$, lost 25%, gained \$300, lost $16\frac{2}{3}\%$. How much had he then?

18. Mr. A. had \$600, lost $16\frac{2}{3}\%$, gained 25%, lost \$25, gained $33\frac{1}{3}\%$, gained $12\frac{1}{2}\%$, lost $33\frac{1}{3}\%$, gained 10%, lost \$160, gained 25%. How much more had he than at first?

19. Make a problem similar to the above.

20. A laborer was earning \$1.60 a day when his wages were cut 25%. What were his daily wages then? After a time his wages were raised 25%. How much less wages did he receive then than he received before the reduction?

21. If the rate of decrease and the rate of increase were each 25%, why do they not balance?

22. Tell which of the following per cents are most easily found by reducing to a common fraction:

a	*b*	*c*	*d*	*e*	*f*	*g*
31 %	$16\frac{2}{3}\%$	$16\frac{1}{2}\%$	$37\frac{1}{2}\%$	$11\frac{1}{9}\%$	35%	25%
h	*i*	*j*	*k*	*l*	*m*	*n*
$87\frac{1}{2}\%$	26 %	27 %	$55\frac{5}{9}\%$	$83\frac{1}{3}\%$	85%	$88\frac{8}{9}\%$

23. Use 144 as a base with each of the rates given in Ex. 22.

24. Find 101% of 135. Of 146. Of 168. Of 179.

25. We may find 10% of 75 by placing a decimal point between the 7 and the 5. Explain.

26. Find in this way 10% of 25. Of 88. Of \$17.50.

27. A cavalry regiment took 960 horses into a battle. If $8\frac{1}{3}\%$ were killed, $12\frac{1}{2}\%$ were wounded, and $16\frac{2}{3}\%$ were caught by the enemy, how many were left for use?

28. Mr. Ellis willed 25% of his property to his son, $62\frac{1}{2}\%$ to his daughter, and the remainder to a library. After the estate was settled there remained \$153,600. How much did each legatee receive?

29. Express each of the following ratios, first as a common fraction, then as hundredths in decimal form, and then as per cent:

3:6 7:10 8:40 1:11 9:72 6:24 24:36 36:48

30. What per cent of 35 is 18?

SOLUTION. 18 equals $\frac{18}{35}$ of 35. 35 = 100% of 35. $\frac{18}{35}$ of 100% =

$$\frac{18}{\cancel{35}_{7}} \times \frac{\overset{20}{\cancel{100}}}{1} = \frac{360}{7}, \text{ or } 51\tfrac{3}{7}. \quad \textit{Ans.}\ 51\tfrac{3}{7}\%.$$

31. Illustrate the following rule:

To find what per cent one number is of another —

Express the ratio of the one to the other as a common fraction, and multiply that fraction by 100.

32. What per cent of 72 is 12? 9? 24? 36? 48? 72?

33. Mary is 7 yr. old, and her brother is 21 yr. old. Mary's age equals what per cent of her brother's age?

34. Find what per cent each number in the first column is of each number in the second column.

35. Find what per cent each number in the second column is of each number in the first column.

5	10
6	30
7	20
8	25
9	40
10	18
4	16

36. What per cent of 25 is each number greater than 10 and less than 20?

37. Find what per cent 9 is of each number between 20 and 30. Between 2 and 12.

38. John bought and sold peanuts, gaining 4¢ on every 6¢ that he invested. What per cent did he gain?

39. What per cent is gained on goods bought at 12¢ and sold at 14¢? Sold at 15¢? 16¢? 18¢? 20¢? 21¢? 22¢?

40. What per cent is gained on goods:

	a	*b*	*c*	*d*	*e*	*f*	*g*
Bought at	10¢?	11¢?	8¢?	15¢?	5¢?	16¢?	16¢?
Sold at	15¢?	14¢?	15¢?	18¢?	$7\frac{1}{2}$¢?	25¢?	32¢?

41. What per cent is lost on goods:

	a	*b*	*c*	*d*	*e*	*f*	*g*
Bought at	20¢?	18¢?	25¢?	15¢?	7¢?	10¢?	10¢?
Sold at	15¢?	16¢?	20¢?	10¢?	5¢?	7¢?	6¢?

42. Some pupils were finding out facts about evaporation. They placed a quart of water in a shallow pan and left it in the sun until 3 gi. of it had evaporated. What per cent was left?

43. A pan with straight sides was filled with water to the depth of 6 in. The water was left to evaporate until it was only 5 in. deep. What per cent of the water evaporated?

44. One evening Mrs. Eaton prepared batter for buckwheat cakes. She set it to rise in a jar 10 in. high, which it half filled. The next morning the batter filled $\frac{9}{10}$ of the space in the jar. What was the per cent of increase?

45. Russell caught 12 fish, and Walter caught 8. What per cent of all the fish caught did each boy catch?

46. Two boys formed a partnership to mend bicycle tires. The material cost $5.00, and they received $7.50. What per cent was gained?

47. At 8 o'clock one morning Mr. Field started on a trip of 60 mi. on a train running 25 mi. an hour. What per cent of the distance had he traveled by 9 o'clock?

48. Vincent and Fred bought a rabbit house. Vincent paid $1.50, and Fred $.50. What per cent of it does each boy own?

49. In playing ball, John caught the ball 23 times out of 25. What per cent of the throws did he miss?

50. Two boys, wishing to know how much water dry bricks would absorb, put 30 lb. of brick into water. When the bricks were taken out they weighed 32 lb. What per cent of that weight was water?

51. Milk was poured into a straight glass jar to the height of 10 in. The next morning the cream in the jar was $\frac{1}{2}$ in. thick. What per cent of the fluid was cream?

52. Mr. Bates owed Mr. Weber $7.50 and gave him a $10 bill in payment. What per cent of the ten dollars was returned in change?

53. What per cent of a cubic foot are 512 cu. in.? 576 cu. in.? 824 cu. in.? 1024 cu. in.? 504 cu. in.? 1584 cu. in.?

54. What per cent of $3.75 is $1.50?

If the decimal point in each term of the fraction $\frac{1.50}{3.75}$ is moved two places to the right, the value of the fraction is unchanged. $\frac{150}{375}$ can then be reduced to its lowest terms, $\frac{2}{5}$.

$$\frac{2}{5} \times \overset{20}{\cancel{100}} = 40. \qquad \textit{Ans.}\ 40\%.$$

It is often more convenient not to reduce the fraction to its lowest terms, as

$$375)\,15000(40$$
$$\underline{1500}$$

Ans. 40%.

55. What per cent of $62.50 is $12.50? $37.50? $18.75? $31.25?

56. Mr. Hull bought some goods for $87.50. How much would he gain by selling them for $112.50? $93.75? 118.75? $131.25? $121.875?

57. Mr. Gordon bought groceries costing $218.75. What per cent would he gain or lose by selling them for $243.75? $193.75? $206.25? $212.50? $256.25? $225?

58. What per cent of $133\frac{1}{3}$ is $16\frac{2}{3}$?

59. Write 5 numbers that are multiples of $8\frac{1}{3}$, and find what per cent each one is of $116\frac{2}{3}$.

60. Write 5 numbers that are multiples of $3\frac{1}{3}$, and find what per cent each one is of 150.

61. What per cent is gained by buying goods at $.16\frac{2}{3}$ a yard, and selling them at $.25 per yard? $.30? $.33\frac{1}{3}$?

62. What per cent is lost on goods by buying them at $.75 and selling them at $.70? $.62\frac{1}{2}$? $.66\frac{2}{3}$? $.50? $.56\frac{1}{4}$? $.45?

63. After a line 5 in. long was increased 20%, what per cent of 1 ft. was its length?

64. Edwin lost 7¢, which was $33\frac{1}{3}\%$ of his money. How much had he at first?

Consider $33\frac{1}{3}\%$ as $\frac{1}{3}$.

65. Five cents is 25% of what sum of money? $12\frac{1}{2}\%$? $16\frac{2}{3}\%$?

66. Find the number of which 15 is $66\frac{2}{3}\%$.

Consider $66\frac{2}{3}\%$ as $\frac{2}{3}$.

67. $66\frac{2}{3}\%$ of a railroad is in Missouri. If that part is 252 miles long, what is the length of the railroad?

68. 35 is $87\frac{1}{2}\%$ of what number? $83\frac{1}{3}\%$ of what number?

69. 6 is 3% of what number?

SOLUTION

If 3% = 6
then 1% = 2
and 100% = 200 *Ans.*

70. By the same reasoning find the number of which 8 is 4%. The number of which 21 is 3%.

71. Tell of what number I am thinking if 35 is 7% of it. If 4 is 3% of it.

72. CLASS EXERCISE. —— may think of a number and tell the class how much a certain per cent of it is. The class may find the number.

73. Can you see the reason for the following rule?

To find the number of which a given number is a certain per cent —

Multiply the given number by 100 and divide the result by the number of per cent.

74. Sixteen equals 8% of what number? 40% of what number?

75. Find the number of which $24 is 12%. 50%. 15%.

76. Find the number of which $45.75 is 5%. 15%. 25%.

77. How much money has a man if 7% of it is $3500? $6216? $3110.80? $4498.20?

78. In a certain city, 75% of the telegraph wires are under ground. If 300 mi. are under ground, how many miles are above ground?

79. Mr. Allen sold Mr. Cummings 1240 bu. of corn, which was 62% of his entire crop. How much was his entire crop?

80. During a storm at sea 600 bu. of grain were thrown overboard. How many bushels were there on board before the storm, if the number of bushels thrown overboard equaled 24% of the whole number?

81. Nine hundred acres of a Florida plantation are marsh. How many acres are there in the whole plantation if the marsh is 75% of it?

82. Mr. Leeds lost in a business transaction $ 1850.50, which was 15% of his entire property. What was the value of his property before his loss? After his loss?

83. Sixty years is 150% of Mr. Harvey's age. How old is he?

84. Eighteen months is 75% of Edgar's age. How old is he?

85. Forty per cent of a load of hay was clover. If 1 T. was clover, what was the weight of the load?

86. A rectangle 9 in. long and 8 in. wide equals 3% of another rectangle. What is the area of the larger rectangle?

87. A rectangle 8 in. by 6 in. equals 4% of a rectangle which is 6 ft. long. How wide is it?

88. Selling a bicycle for $30 more than I gave for it, gives me a gain of 25%. How much did I give for it?

89. I gained 20% by selling a watch for $ 5.75 more than I paid for it. How much did I pay for it?

90. I sold some goods at a profit of $ 418, which was 38% of their cost. How much did they cost me? For how much did I sell them?

91. By selling a house for $ 100 less than it cost I lost 5%. How much did the house cost me?

92. By selling a stove for $ 5.80 less than cost, I lost 29%. What was the cost?

93. I gained $ 340 by selling wheat at an advance of 20%. How much did I pay for the wheat?

94. A merchant sold silk at $ 2.20 per yard, which was 110% of what it cost him. How much did it cost him? What per cent did he gain on it?

95. What is the cost of goods sold at 18¢ a yard, which is 20% more than the cost?

SOLUTION. As the selling price is 20% more than the cost, it is 120% of the cost. If 18¢ is $\frac{120}{100}$ of the cost, the cost is $\frac{100}{120}$ or $\frac{5}{6}$ of 18¢, which is 15¢.

96. Find the cost of tea sold at 85¢ per pound, which was 25% above cost.

97. Find the cost of goods sold at the following prices, which are 25% above cost:

a Calico,	@ $.05 per yard.	*d* Velvet,	@ $ 1.85 per yard.
b Ribbon,	@ .65 per yard.	*e* Lace,	@ 2.75 per yard.
c Flannel,	@ 1.35 per yard.	*f* Silk,	@ 4.75 per yard.

98. Thirty per cent less than the cost of anything equals what per cent of its cost? Illustrate.

99. A wheel was sold for $ 28, which was 30% less than cost. Find the cost.

100. A stove sold for $ 6 brought 40% less than cost. Find the cost.

101. Arthur had $ 4.20, which was 40% more than his brother had. How much had his brother?

102. Mr. X raised 350 bushels of wheat this year, which is $16\frac{2}{3}$% more than the number of bushels that he raised last year. How much wheat did he raise last year?

103. A grocer has 180 lb. of dry sugar. In drying, it lost 10% of its weight. What was its weight before drying?

104. There are 240 A. in Mr. King's farm, which is 20% less than the number of acres in his uncle's farm. How many acres has the latter?

105. A merchant had 380 yd. of cloth after it had shrunk. What was its length before shrinking, if the shrinkage was 5%?

106. Twenty per cent of a pole was broken off. The part remaining was 16 ft. long. What was the length of the pole before it was broken?

107. After using 10% of a load of coal I find there are 900 lb. left. How many pounds were in the load?

108. In Mr. Crosby's orchard there are 336 fruit trees; $16\frac{2}{3}\%$ of them are plums, $12\frac{1}{2}\%$ of them are pear trees, and the remainder are peach trees. How many are there of each kind?

109. A man whose wages are \$14 per week, spent 8% of his earnings last year for music lessons. How much did he spend for the lessons if he worked 50 weeks?

110. A man buys a horse for \$200 and sells it for \$46 more than it cost him. What per cent does he gain?

111. A bicycle is bought for \$75 and sold at a gain of \$13.50. What per cent is gained?

112. A watch costing \$25 is sold at an advance of \$4.75. What per cent is gained?

113. A pile of wood 50 ft. long, 6 ft. wide, and 5 ft. high was 25% oak, $33\frac{1}{3}\%$ maple, 21% beech, and the rest walnut. How many cords were there of each?

114. The average lung capacity of men is about 320 cu. in. If your lungs hold 75% as much, how much will they hold? If your clothing were so tight as to reduce your lung capacity 40 cu. in., what per cent would the reduction be?

115. The amount of food required by a man is about $5\frac{1}{2}$ lb. a day. If you ate 80% as much, in how many days would you eat one ton and two pounds of food?

116. A man's heart weighs about 11 oz. If your heart weighs 70% as much as that, how much does it weigh?

117. Of a certain fence, 200 rd. were wire fencing, and the rest rail fencing. If 60% was rail, what was the length of the fence?

118. Forty-five miles of country road is macadamized. If this is 20% of the whole road, how much remains to be macadamized?

119. How much will it cost to plaster a room 20 ft. 4 in. long, 18 ft. wide, 8 ft. high, at $.20 a square yard, if 12½% of the surface is covered with wainscoting?

120. A certain school enrolls 23 boys and 22 girls. The girls equal what per cent of the school?

121. The same school has an average attendance of 42. What per cent of the enrollment is the attendance?

122. What per cent of the pupils in your school are boys?

123. If 3 pupils belonging to your school were absent every day, the attendance would be what per cent of the enrollment?

124. A boy having broken his bicycle sold it for $20, which was 33⅓% of the cost. What was the cost?

125. I buy goods at $1.20 and sell them at a gain of 10%. What is the selling price?

126. I sell goods at $1.32, gaining 10%. Find the cost. The gain.

127. I sell goods at a gain of $12, which is 10% of the cost. Find the cost.

128. I buy goods for $120 and sell them for $12 more than they cost me. What per cent do I gain?

129. I buy goods for $120 and sell them for $132. What per cent do I gain?

130. Your age in years is what per cent of the age of a person who is 4 yr. older than you? 6 yr. older? 10 yr. older?

131. A man sold two horses for $150 each, gaining 20% on one and losing 20% on the other. Did he gain or lose by the two transactions, and how much?

132. An inkstand that cost 50¢ was sold at a gain of 100%. What was the selling price?

133. If berries that cost 5¢ a quart are sold for 10¢ a quart, what is the per cent of gain?

134. How long will it take a dollar to gain 100% if it is at interest at 5%? At 6%? 8%? 9%? 12%?

135. Walter and Thomas sold lemonade at a fair. Walter furnished $.25 worth of lemons, and Thomas $.50 worth of sugar. To what per cent of the profits was each boy entitled? If they took in $2.64, how much was each boy's share of the gain?

136. Three men raised a fund for charity. One man gave $15, another $20, another $25. What per cent of the whole did each man give?

137. Mr. Low had his money invested in three houses as follows: in the first $1620, in the second $8100, in the third $3240. What per cent of his money was invested in each house?

138. Mr. Eves insured Mr. Croft's building for $1600, which is 75% of its value. What is its full value? If Mr. Croft paid 1% upon the amount insured, how much did he pay?

139. Mr. Jones's yearly income from a mine is $4000, which is 15% of the sum he invested in it. How much did he invest?

140. A store is rented for $60 a month. The yearly rent is $8\frac{1}{3}$% of the value of the property. What is its value?

141. A lawyer collected some money for his client, receiving for his services $80, which was 5% of the sum collected. How much did he collect and how much did he pay over to his client?

142. Mr. Roy gained 8% by selling his cow for $20 more than it cost him. For how much did he sell it?

143. Mr. Litch received $500 a year rent for one of his houses, which was 7% of its value. What was its value?

144. After Mr. Lane had paid $37\frac{1}{2}\%$ of his debts, he found that $3568 would pay the remainder. What was his total indebtedness?

145. Mr. Bingham sold two houses for $7000 each; for the first he received $12\frac{1}{2}\%$ less than its value and for the second $16\frac{2}{3}\%$ more than its value. What was the value of each?

146. Mr. Allen bought for $3200 a store which had depreciated 40% in value. What was the original value of the store?

147. When the Twentieth Century Novelty Company failed in business they had $3350, which paid 67% of their debts. What was the amount of their debts?

148. A grocer sold his stock for $2000, which was at a loss of 10%. How much did the stock cost?

149. Mr. Drew bought 5 doz. oranges. Of these 30% were not good. How many oranges were good?

150. A florist used in January 4000 bu. of coal. This quantity was 36% of the number of bushels used during the winter. How many bushels were used in all?

151. Mr. M. receives $150 each year as the interest on a sum of money which pays 6%. What is the sum of money?

152. How much money must one have at interest that he may receive from it $750 a year when the rate of interest is 5%? 6%? 8%? 4%? 3%?

MERCHANDISING

153. Buying and selling goods for profit is called **Merchandising.** Those who carry on merchandising are called merchants. Mention several lines of merchandising.

154. A merchant bought $6000 worth of dry goods, and in the first year gained 20% of his capital. How much did he gain that year?

155. He used $900 that year for living expenses, and put the rest into his business. What was his capital at the beginning of the second year?

156. In the second year he cleared 33⅓% on his capital. What was his gain?

157. The second year he used $1200 for his living expenses, and put the balance into his business. What was his capital at the beginning of the third year?

158. In the third year he gained 37½% on his capital. His expenses outside of his business that year were $1800. The rest of his gain went into the business. What was his capital at the end of the third year?

159. In the fourth year he gained 44⅘% on his capital, and used $1700 for outside expenses. What was his capital at the end of the fourth year?

160. January 1, 1899, Mr. A. went into business with a capital of $10,000. At the end of the year he found that the amount of his sales for the year had been $18,448; that his business expenses, such as rent, clerk hire, advertising, etc., had been $6000; that he had spent $10,625 of his receipts in replenishing his stock of goods, and that he had $12,000 worth of stock on hand. How much had he gained that year? His living expenses were $1000. What was his capital Jan. 1, 1900?

161. In 1900 he netted 10% on his capital, and his living expenses were $1500. What was his capital Jan. 1, 1901?

162. In 1901 his profits were 7% on his capital, and his living expenses were $1300. What was his capital Jan. 1, 1902?

Suggestion to Teacher. Lead pupils to realize some of the conditions of merchandising, and let them make similar problems, tracing the course of simple business ventures.

163. A merchant buys broadcloth at $2.40 per yard. How shall he mark it that he may sell it at a gain of 40%? 33⅓%? 35%? 50%?

164. How much would he gain by selling 20 yd. at each of those rates of advance?

165. Mr. B. bought 40 yd. of novelty goods at $1.60 per yard. He sold $\frac{1}{4}$ of it at an advance of 75%, $\frac{1}{4}$ of it at an advance of 50%, and the rest at 25% advance. How much did he gain on the whole piece?

166. Mr. C. bought 60 yd. of goods at $1.20 per yard. He sold $\frac{1}{5}$ of the piece at a gain of 60%, $\frac{1}{2}$ of the rest at a gain of 50%, 14 yd. at $1.30 per yard, and the rest at $.75 per yard. How much did he gain on that piece?

167. Mr. Evans bought goods at $.75 a yard and marked them to sell at an advance of $33\frac{1}{3}\%$. Later in the season he sold them at a reduction of 10% from the marked price. What was the actual selling price?

Find actual selling price, and amount of gain or loss on goods:

	Bought	Marked	Sold
168.	$.60	50% above cost	10% below marking
169.	.80	25% above cost	30% below marking
170.	.90	$33\frac{1}{3}\%$ above cost	$16\frac{2}{3}\%$ below marking
171.	1.75	$2.00 above cost	$12\frac{1}{2}\%$ below marking
172.	1.60	25% above cost	$12\frac{1}{2}\%$ below marking

173. Class Exercise. —— may mention prices at which goods might be bought, marked, and sold, and the class may find the amount of gain or loss.

COMMISSION

174. Instead of buying and selling for themselves, some merchants buy or sell goods for others at a given per cent of their value. The money which they receive for buying or selling is called their **Commission**, and they are called commission merchants, agents, or brokers.

What is the commission on $900 at 10%?

175. Mr. Ward sold a piano for $575, receiving a commission of 10% on the selling price. How much was his commission, and how much money should he send to the owner of the piano?

176. Mr. Clark is an agent for the Elliott Piano Co. which pays him a commission of 25% on all his sales. During the month of January, 1899, he sold one piano at $325, one at $400, two at $250, and three at $200. How much were his commissions, and how much should he return to the Elliott Piano Co.?

177. The expenses of his store for rent, heating, lighting advertising, clerk hire, etc., for January, were $178.50. How much did Mr. Clark make that month above expenses?

178. His living expenses for that month were $109.75. How much did he save? Would it be safe to reckon his yearly income on this basis? Why?

179. A cotton broker receives a shipment of cotton from Alabama consisting of 400 bales. He sells it at $20 a bale, receiving $1\frac{1}{2}\%$ commission. How much is his commission on that sale, and how much should he return to the owners of the cotton?

180. John sold his sister's sled for her, receiving $1 for it. When he gave his sister the dollar, she handed him a dime for selling the sled. What per cent was his commission?

181. Mr. Adams sold a wagon for the Melvin Wagon Co. at $200, and after taking out his commission, sent the Wagon Co. $180. How much was his commission? What per cent of the value of the wagon?

182. Mr. Gordon buys chickens for a Baltimore firm, receiving a commission of 15% on all the money spent in buying them. If he buys $12,000 worth of chickens in a year, what is his income for that year?

183. Mr. Wilson travels in the South, selling stoves. He receives a commission of 10% on sales. When he has sold $ 18,000 worth of stoves, how much has he earned?

184. Mr. Wood buys hogs for a pork-packing establishment, receiving a commission of 3% on the amount spent for the hogs. If he buys $ 7500 worth in a month, what are his earnings for that month?

185. A commission merchant bought $ 5000 worth of peanuts in Tennessee, and shipped them to a Chicago firm. The cost of hauling them to the depot in drays was $ 2.75, the freight charges were $ 47.50. The buyer's commission was 4% on the amount paid for the peanuts. What was the amount of his commission? How much did the peanuts cost the Chicago firm?

186. A commission merchant in Kentucky bought for a New York firm 3000 lb. of pecans at $.06½ per pound. How much was his commission at 2%?

187. Class Exercise. —— may fill out the following, and the class may find the amount received as commission:

A commission merchant { sold / bought } —— worth of ——, receiving —— % commission.

188. People are sometimes hired to collect money due to other people. These collectors receive a percentage on the amount collected.

How much does Mr. Cox earn by collecting a bill of $ 600 at 2%?

189. Mr. Howell gave Mr. Scott a list of bills to be collected, 5% of the amount of which Mr. Scott was to receive as commission. How much did he earn by collecting the following bills, and how much did he turn over to Mr. Howell?

John Andrews, Groceries, $ 28.75.

Charles Stockton, 21 lb. Sugar, @ $.05.

George Baldwin, 12½ lb. Turkey, @ $.15.

Peter Garrison, 10 gal. Molasses, @ $.45.

190. Class Exercise. —— may fill out the following, and the class may find the collector's receipts:

A collector collects ——, receiving for his work —— %.

191. A collector undertook to collect $2125 worth of bills due a physician. He collected 67% of them, receiving 10% as his fee. How much did he collect? How much did he keep? How much did he send to the physician?

192. Mr. Blair sold a carriage for the Hale Carriage Co. at $450. His commission was 15%. The purchaser paid $300 cash, and Mr. Blair was obliged to pay a collector 10% on the balance for collecting it. How much did Mr. Blair gain on the sale?

TRADE DISCOUNT

193. A reduction in the selling price of goods is called a **Discount.**

Mr. Reed found that he could buy the bicycle that he wanted for $70, on 3 months' time, or for $66.50 cash. How much was the discount?

194. How much is paid for a bill of goods invoiced at $18.75, with a discount of 30%?

195. The employees of a certain large dry goods store are allowed to buy goods from it at a discount of 10%. What amount was paid for a bill of $17.25 bought by an employee?

196. A man received a bill for $75. Printed on the bill head were the words, "5% discount if paid within 30 days." The bill was paid at the end of 4 weeks. What amount was paid?

197. Hale & Co., Springfield, Ill., bought of D. W. Lamont & Co., St. Louis, Mo., 3 doz. plain gold rings, @ $20 per doz.; 4 gold rings, diamond settings, @ $50; 6 gold watches, @ $15; 4 sets teaspoons, @ $6. Make out the bill, allowing a discount of 25%.

198. Mr. K. sold Mr. D. furniture for his home to the amount of $75, on 60 days' time; but Mr. D. accepted the offer of $3\frac{1}{3}\%$ off for cash. How much did Mr. K. receive for the furniture?

Many manufacturers and wholesale dealers have a fixed price for their goods, called the list price. They sell to the retail dealers at a discount from the list price. For instance, a manufacturer of carriages sends out a catalogue containing a list of the different kinds of carriages he makes, and their prices, with the discounts he gives to retail dealers.

199. Howard, Cowperthwait, & Co., who sell carriages, select from a manufacturer's list a carriage whose list price is $750. The discount on this carriage is 55%. How much do Howard, Cowperthwait, & Co. pay for the carriage? If Mrs. Douglas buys the carriage for $700, how much do they make on it?

200. At the same rate of discount, how much will be gained by Howard, Cowperthwait, & Co. if they buy four carriages whose list price is $600, and sell them at the list price?

201. Frequently more than one discount is given upon the same purchase. If Howard, Cowperthwait, & Co. buy a $1000 carriage at 55% off, its cost is $450. If they get an additional discount of 10%, it is reckoned on the $450 and deducted from it. In that case, how much will the carriage cost them?

Observe that when there is more than one discount, each successive discount is made upon a smaller sum of money than the preceding.

202. How much is left of $800 when it is discounted 25%, then $33\frac{1}{3}\%$, then 25%, then $33\frac{1}{3}\%$?

Find cost of the following:

	List price	Discount		List price	Discount
203.	$175	45 and 10 off	**206.**	$500	20, 10, and 5 off
204.	$370	35 and 5 off	**207.**	$600	40, 30, and 2 off
205.	$350	15 and 10 off	**208.**	$800	50, 10, and 5 off

209. If you were buying goods should you prefer discounts of 15% and 10%, or a straight discount of 25%? Find the discount on different sums at these rates, and state which terms are more advantageous to the buyer, and why.

210. A dealer buys wagons whose list price is $60, at a discount of 30%, 20%, and 5% off for cash. How much do these wagons cost him? He sells them at the list price with 5% off for cash. How much does he make on a cash sale? On a time sale?

If a time sale is made in which the time exceeds a certain limit, the buyer gives his note for the amount and pays the interest upon it. This time limit varies with different firms, being usually not less than 60 da., nor more than 6 mo.

211. How much is paid for a bill of goods invoiced at $37.25, discounts 10, 15, and 5 off. How much is gained if the goods are sold 10% above the list price?

212. Ball & Co., piano dealers, select from the manufacturer's catalogue, 3 pianos listed at $400, $450, and $700, and order 6 pianos of each kind. The trade discount is 60, 10, and 5. What is the cost of that shipment of pianos?

213. Mrs. Fox pays cash for one of the $400 pianos sold at list price, and gets a discount of 5%. How much do Ball & Co. make on that sale?

214. Mr. Shaw buys a $700 piano at list price, pays $75 cash, and gives his note for the balance, payable in 6 mo. At the end of that time, finding himself unable to pay for it, he returns the piano to Ball & Co., who spend $2 in polishing and tuning it, and then sell it for $650. How much do Ball & Co. make on that piano?

215. Mr. King buys one of the pianos whose list price is $450, on the installment plan, paying $10 a month. Ball & Co. charge him $500 for the piano. How much do they make on that sale?

216. Mrs. Lee buys from Ball & Co. a piano listed at $700. She agrees to pay for it $750, in monthly payments of $15 each. After making 7 payments she returns the piano. Ball & Co. spend $5 for repairs and sell it for $625. What is their gain on it?

SUGGESTION TO TEACHER. Require the pupils to bring to class similar problems, describing business occurrences. Let them find out facts which will enable them to keep the conditions of their problems within the range of probabilities.

Find gain on the following goods bought at discounts of 10, 20, and 5 off, and sold at an advance of 20% on list price:

	Items	List price
217.	1300 yd. Carpet	@ $.75
218.	800 yd. Drapery Silk	@ $3.50
219.	100 prs. Lace Curtains	@ $6.75

220. On a bill of $675, what is the difference between discounts of 40% and 20%, and a straight discount of 60%?

221. Mr. Dow, a merchant in Kentucky, goes to New York twice a year to buy goods. He had been getting discounts of 30, 20, and 5% off for cash, but on his last trip he found that he could get discounts of 30, 40, and 5%. If he bought goods to the amount of $9000, how much was he benefited by the change of discounts?

222. The list price of an article with three different houses is $400. One house offers discounts of 20, 10, and 5%; the second 5, 10, and 20%; the third 10, 5, and 20%. Which is the best offer, and why?

INTEREST

223. Money paid for the use of money is called **Interest.** The sum on which interest is paid is called the **Principal.**

If $250 is loaned at 6% for 1 yr., how much is the interest? What sum is the principal?

224. When you know the interest of a sum of money for 1 yr., how may you find the interest of the same sum at the same rate for 2 yr.? 7 yr.? $3\frac{1}{2}$ yr.? $\frac{1}{12}$ of a year? 2 yr. 1 mo.?

Find interest:

	Prin.	Rate	Time		Prin.	Rate	Time
225.	$ 200	6%	4 yr.	232.	$ 1800	5%	1 yr. 8 mo.
226.	$ 700	8%	$2\frac{1}{2}$ yr.	233.	$ 400	7%	2 yr. 9 mo.
227.	$ 300	7%	$2\frac{1}{3}$ yr.	234.	$ 600	4%	1 mo.
228.	$ 800	8%	2 yr. 6 mo.	235.	$ 800	6%	7 mo.
229.	$ 900	5%	3 yr. 4 mo.	236.	$ 1200	3%	11 mo.
230.	$ 600	4%	2 yr. 1 mo.	237.	$ 1800	9%	5 mo.
231.	$ 5000	6%	3 yr. 2 mo.	238.	$ 1500	8%	10 mo.

There are many ways of calculating interest, all depending upon this fact. *Principal* × *Rate* × *Time* = *Interest.* A 6% method and a cancellation method are given in this book.

Six Per Cent Method

239. From the first equation reason out the equations which follow it. Learn them.

At 6% the interest of $ 1 for 12 mo. = $.06
the interest of $ 1 for 2 mo. = $.01
the interest of $ 1 for 1 mo. = $.005
the interest of $ 1 for 6 da. = $.001
the interest of $ 1 for 1 da. = $.000$\frac{1}{6}$

At 6% what is the interest of $ 1.00 for:

	yr.	mo.	da.		yr.	mo.	da.
240.	1	2	6	246.	8	10	18
241.	2	1	6	247.	9	3	1
242.	3	1	12	248.	7	5	5
243.	4	3	6	249.	8	9	7
244.	5	3	12	250.	6	11	8
245.	7	8	12	251.	7	7	9

252. CLASS EXERCISE. —— may give a number of years, months, and days, and the class may find the interest on $1 for that time at 6%.

253. Find the interest of $1 at 6% for 2 yr. 6 mo. 24 da. and then find the interest of $2 for that time. $3. $7. $25.

254. Find the interest of $25.75 for 5 yr. 8 mo. 12 da. at 6%.

25.75
.342
5150
10300
7725
$8.80650

The interest of $1 for 5 yr. 8 mo. 12 da., at 6%, is $.342. The interest of $25.75 is 25.75 times $.342. In practice it is more convenient to multiply 25.75 by .342. Of course the result is the same.

Find the interest of $125.37 at 6% for:

	yr.	mo.	da.		yr.	mo.	da.
255.	3	4	6	260.	2	9	7
256.	7	9	12	261.	3	10	8
257.	8	2	18	262.	6	5	9
258.	7	5	24	263.	5	5	10
259.	6	11	7	264.	11	2	11

Find interest at 6%:

265. $175.25 for 1 yr. 6 mo. 7 da.

266. $210.60 for 3 yr. 2 mo. 9 da.

267. $625.48 for 2 yr. 9 mo. 9 da.

268. $330.27 for 5 yr. 7 mo. 9 da.

269. $ 45.60 for 8 yr. 3 mo. 10 da.

270. $910.75 for 1 yr. 6 mo. 13 da.

271. $712.25 for 2 yr. 8 mo. 14 da.

272. $861.60 for 3 yr. 8 mo. 14 da.

273. $520.40 for 7 yr. 4 mo. 14 da.

274. What is the interest of $100 for 1 yr. at 6%? At 3%?

275. What is the ratio of the interest of a sum of money for a given time at 3%, to the ratio of the same sum of money for the same time at 6%?

276. Find the interest of the following amounts for 1 yr. 8 mo. 24 da., first at 6% and then at 3%:

$276 $24.76 $13.25 $417 $625

277. Find by the 6% method the interest of $318 at 5% for 2 yr. 7 mo. 12 da.

SOLUTION. The interest of $318 for 2 yr. 7 mo. 12 da. at 6% is $49.926. At 1% the interest is $\frac{1}{6}$ as much or $8.321. At 5% the interest is 5 times as much as at 1% or $41.605.

Required interest:

	Prin.	Rate	Time
278.	$ 28.35	6%	1 yr. 6 mo. 16 da.
279.	$ 49.36	7%	2 yr. 9 mo. 12 da.
280.	$ 30.75	8%	6 yr. 15 da.
281.	$252.00	6%	4 yr. 7 mo. 27 da.
282.	$160.00	7%	8 mo. 26 da.
283.	$ 72.00	5%	6 yr. 8 mo. 13 da.
284.	$ 75.00	4%	8 yr. 3 mo.
285.	$112.00	5%	6 yr. 7 mo. 22 da.
286.	$ 46.75	3%	2 yr. 11 mo. 20 da.

287. If you borrowed $100 and paid it back at the end of one year, with the interest on it at 6%, how much would you pay?

288. The sum of the principal and interest is called the **Amount.**

What amount must be paid back when $200 is borrowed at 6% interest and kept 2 yr. 10 mo. 18 da.?

Find the amounts of the following:

	Prin.	Rate	Time
289.	$900	6%	7 yr. 4 mo. 6 da.
290.	$800	3%	3 yr. 5 mo. 7 da.
291.	$600	4%	11 yr. 2 mo.
292.	$144	5%	8 mo. 12 da.
293.	$672	7%	3 yr. 8 mo. 15 da.
294.	$145.36	7%	2 yr. 2 mo. 1 da.
295.	$816.35	5%	3 yr. 3 mo. 3 da.
296.	$696	8%	5 yr. 7 mo. 13 da.
297.	$216.25	4%	4 yr. 18 da.
298.	$625	7%	9 yr. 9 mo. 9 da.

299. Class Exercise. —— may mention a sum of money, and the class may find the amount of it for any length of time and at any rate which he may decide.

300. Find the time from Jan. 1, 1898, to July 7, 1899.

301. Mr. Monroe borrowed $300 Jan. 1, 1897, at 6%. What was the interest March 1, 1898? Sept. 1, 1899?

302. On Nov. 7, 1896, what amount was due on $600 borrowed May 1, 1892, with interest at 6%?

303. Find the amount of $376.25 borrowed July 1, 1883, and paid Nov. 13, 1887, with interest at 6%.

At 6%, what is the amount of $700?

304.	Borrowed Sept. 7, 1898	Paid April 19, 1899
305.	Borrowed June 15, 1895	Paid Oct. 3, 1898
306.	Borrowed Dec. 12, 1891	Paid May 15, 1894
307.	Borrowed Aug. 6, 1880	Paid May 12, 1885
308.	Borrowed Feb. 29, 1896	Paid June 30, 1897

Cancellation Method

309. If the interest of a sum of money for a certain time is \$ 72, what will be the interest of that sum for $\frac{3}{4}$ of that time? $\frac{5}{6}$ of that time?

310. The interest of a certain sum of money for a certain time is \$ 96. Find by cancellation the interest of that sum for $\frac{17}{24}$ of that time. $\frac{31}{32}$ of it. $\frac{47}{48}$ of it.

311. Find the interest of \$ 400 for 2 mo. 20 da. at 3%.

SOLUTION. The interest of \$ 400 for 1 yr. at 3% is $400 \times \frac{3}{100}$. The interest of the same sum for 2 mo. 20 da., or 80 da., is $\frac{80}{360}$ as much. Hence the interest $= \frac{80}{360}$ of \$ 12.

Canceling we have $\overset{4}{\cancel{400}} \times \frac{\cancel{3}}{\cancel{100}} \times \frac{\overset{2}{\cancel{80}}}{\cancel{360}\ \cancel{120}\ 3} = \frac{8}{3} = \$ 2.66\frac{2}{3}$

Find the interest of the following by the cancellation method:

	Prin.	Rate	Time
312.	\$ 276	12%	3 mo. 9 da.
313.	\$ 184.50	4%	5 mo. 27 da.
314.	\$ 1200	6%	1 mo. 21 da.
315.	\$ 1400	5%	3 mo. 15 da.
316.	\$ 1800	7%	3 mo. 20 da.
317.	\$ 625	8%	90 da.
318.	\$ 800	7%	63 da.
319.	\$ 900	8%	100 da.
320.	\$ 1100	6%	33 da.
321.	\$ 2175	3%	93 da.
322.	\$ 4150	6%	63 da.

323. Find the interest of \$ 840 at 5% for 1 yr. 3 mo.

$840 \times \frac{5}{100} \times \frac{15}{12}$. When no days are given, find the number of months and express them as twelfths of a year.

Find interest:

	Prin.	Rate	Time
324.	$560	3%	1 yr. 8 mo.
325.	$218.64	6%	2 yr. 1 mo.
326.	$175.25	8%	1 yr. 6 mo.
327.	$165.36	4%	4 yr. 2 mo.
328.	$500	5%	7 mo. 6 da.

When days are given, reduce the whole time to days and express as 360ths of a year.

Find interest of:

	Prin.	Rate	Time
329.	$700	2%	1 yr. 1 mo. 6 da.
330.	$750	6%	3 yr. 1 mo. 15 da.
331.	$420	7%	2 yr. 1 mo. 10 da.
332.	$800	9%	2 yr. 3 mo. 6 da.
333.	$875	10%	1 yr. 8 mo. 20 da.
334.	$630.25	7%	1 yr. 4 mo. 20 da.

335. There are some special rules for calculating interest which are derived from the principles of the cancellation method as,

To compute interest at 8% —

Multiply the principal by the number of days, move the decimal point of the product two places to the left, and divide the result by 45.

Find interest of $200 for 53 da. at 8%.

By cancellation method

$$\overset{2}{\cancel{200}} \times \frac{8}{\cancel{100}} \times \frac{53}{\underset{45}{\cancel{360}}} = \frac{106}{45} = 2.35+$$

It will be seen that dividing by 45 gives the same result as multiplying by 8 and dividing by 360.

By special rule

$$\begin{array}{r} 53 \\ 200 \\ \hline 45)106.00(2.35+ \\ 90 \\ \hline 160 \\ 135 \\ \hline 250 \\ 225 \end{array}$$

336. Give the reason for the special rule for computing interest at 8%.

337. Show how the following rules are derived from the cancellation method:

(*a*) **To compute interest at 5% —**

Multiply the principal by the number of days, move the decimal point of the product two places to the left, and divide the result by 72.

(*b*) **To compute interest at 6%**

Multiply the principal by the number of days, move the decimal point of the product three places to the left, and divide the result by 6.

338. Give a similar rule for computing interest at 9%. At 4%. At 12%. At 10%. At 3%.

Exact Interest

339. Usually 360 da. are considered one year, but sometimes calculations of interest are made, in which a year is considered as 365 da. This is called **Exact Interest.** To find the exact interest of a sum of money, use the cancellation method, expressing the exact number of days as 365ths of a year.

Find the exact interest of $900 from Dec. 1, 1898, to Feb. 12, 1899, at 8%.

SOLUTION. The exact number of days from Dec. 1, 1898, to Feb. 12, 1899, is 73 da., or $\frac{73}{365}$ of a year.

$$\overset{9}{\cancel{900}} \times \frac{8}{\cancel{100}} \times \frac{\overset{1}{\cancel{73}}}{\underset{5}{\cancel{365}}} = \frac{72}{5} = \$14.40 \quad \textit{Ans.}$$

Find exact interest of:

340. $300, 1 yr. 1 mo. 1 da. at 9%.

341. $240, 11 mo. 25 da. at 4%.

342. $336, 1 yr. 2 mo. 10 da. at 6%.

343. $430 from Oct. 15, 1897, to Jan. 11, 1898, at 8%.

PROMISSORY NOTES

344. July 1, 1900, Mr. James Allen bought a horse of Mr. William Brown for $125, paying $25 cash and giving a promissory note, like the following, for the balance. The note was paid in full when due. What amount was paid?

$100.00/100 New York, July 1, 1900
Three months after date I promise to pay
to the order of William Brown
One hundred 00/100 Dollars
with interest at six per cent per annum
at The National Park Bank of New York
value received
No. 1357 Due Oct. 1 James Allen

345. A **Promissory Note** is a written promise to pay money. Who is the maker of the above note?

346. Who should keep the note until it is paid? What should be done with it after the money is paid?

347. Write a note promising to pay Robert Ruskin $300 with interest at 6% in one year from date.

348. The person to whom the note is to be paid is called the **Payee.** Who is the payee of the note you have just written?

349. The sum mentioned in the note is called the **Face** of the note. What is the face of your note?

350. The date at which the note becomes due is called the date of **Maturity.** What is the date of maturity of your note?

351. Is it an interest-bearing note?

SUGGESTION TO TEACHER. Procure blank forms of promissory notes of different kinds, and let the differences between them be discussed in class.

352. What is the amount of a note for $425 that matures in 4 mo., interest being 5%?

353. In some states the law allows 3 days more than the specified time for the payment of a note, but interest is exacted for these 3 days, called **Days of Grace.** If a note is made payable Aug. 1, on what day is it really due, when grace is allowed?

In the problems of this book, days of grace are not to be considered unless mentioned.

354. A note for $400 is dated March 1, 1896, and made payable May 1, 1896, with grace. It is said to mature May 1/4, and interest is computed to May 4. What is the interest at 6%?

355. A note for $500, dated July 1, 1900, is made payable in 3 mo. with grace. When does it mature, and what is the interest at 6%? At 5%? At 7%?

356. How much must be paid for the use of $625 from June 1, 1897, to July 1, 1898, with grace, at 6%? At 3%?

357. How much must be paid for the use of $500 from Dec. 1, 1899, to March 1, 1900, with grace, at 6%? At 4%?

358. What is the interest on a note for $300 at 6%, dated Aug. 31, 1895, and made payable in 30 da., with grace?

359. June 17, 1897, Mr. Kent gave a note for $500 at 6%, payable in 60 da., with grace. When was the note due? What was its face? Its amount at maturity?

360. Notes, being promises, may be varied to suit the intentions of the parties concerned. Some notes draw interest from date, some after maturity, and some not at all. Some are made payable at a specified time, and are called **Time Notes.** Some are made payable upon the demand of the holder for payment, and are called **Demand Notes.** Some are made in such a way that they can be sold or transferred to other persons, and are called **Negotiable Notes.** Some are made payable only to a certain person, and are called **Non-negotiable** notes.

What is the special advantage of a negotiable note? Of a non-negotiable note?

Demand Note

$300.75. BOSTON, MASS., Sept. 20, 1900.

On demand, I promise to pay William D. Owen three hundred and $\frac{75}{100}$ dollars, with interest at 6 %. Value received.

EDWARD M. ARLINGTON.

361. What is due Jan. 19, 1901?

362. If the note above were not paid until May 20, 1902, how much would be due?

363. How much would be due on the above note if it were paid Sept. 25, 1902? January 11, 1903? March 1, 1903?

364. How does a demand note differ from a time note?

365. Write a time note for $500, due in 3 mo., at 4%.

Negotiable Note

$175.50. LOWELL, MASS., Sept. 15, 1899.

One year after date, I promise to pay to Henry Scott, or bearer, one hundred and seventy-five and $\frac{50}{100}$ dollars, with interest at 6%. Value received.

MARY GREEN.

366. What two words in the above note make it a negotiable note?

367. If the negotiable note given above was paid when it was 6 mo. past due, how much was paid?

368. Write a negotiable time note for $600, interest 6%, and find the amount of it when due. When 3 mo. past due. 11 mo. past due.

$1000. SAN FRANCISCO, CAL., Nov. 12, 1901.

One year after date, I promise to pay to the order of Ellen Eames, One Thousand Dollars, with interest.

JAMES PORTER.

369. When a note includes the words "with interest," but gives no specified rate, interest is computed at the rate legal in

the state in which it is dated. Copy the above note, dating it at the place where you live, and find the amount of it at maturity under the laws of your state.

370. Write a note for $700 due in 3 mo., with interest after maturity. Find the amount due on it 8 mo. after its date under the laws of your state.

PARTIAL PAYMENTS

371. May 7, 1896, Mr. James Smith gave Mr. John Brown a note for $700 payable on demand, with interest at 6%. How much was due May 7, 1897? At that time Mr. Smith made a partial payment of the amount due by giving Mr. Brown $442. On how much money ought Mr. Smith to continue to pay interest? How much was due May 7, 1898? At that time Mr. Smith made another partial payment, giving $218. How much was due on that note Nov. 7, 1898?

SOLUTION

May 7, '96, Mr. Smith owed Mr. Brown	$700	Prin.
Int. on prin. from May 7, '96, to May 7, '97	42	Int.
May 7, '97, Mr. Smith owed Mr. Brown	$742	Am't.
May 7, '97, Mr. Smith paid Mr. Brown	442	Pay't.
Mr. Smith still owed Mr. Brown	$300	New prin.
Int. on new prin. from May 7, '97, to May 7, '98 . .	18	Int.
May 7, '98, Mr. Smith owed Mr. Brown	$318	Am't.
May 7, '98, Mr. Smith paid Mr. Brown	218	Pay't.
Mr. Smith still owed Mr. Brown	$100	New prin.
Int. on last prin. to Nov. 7, '98	3	Int.
Nov. 7, '98, Mr. Smith owed Mr. Brown	$103	Am't due.

372. When a partial payment is made, the holder of the note writes upon the back of it the amount of money paid and the date of payment. The writing is called an **Indorsement**, and serves as a receipt for the amount paid. What were the indorsements that Mr. Brown wrote?

373. Aug. 1, '93, Mr. John Dow gave to Mr. Frank Rand his note for $800 at 6%. Aug. 1, '94, he paid $218. Feb. 1, '95, he paid $223.90. How much did he owe Aug. 1, '95?

SUGGESTION TO TEACHER. Let the pupils in one section of the class enact the part of Mr. Dow in writing the note, and those of another section take the part of Mr. Rand, making the indorsements upon the notes written by the others. Let class discuss justice of the settlements.

$800. NEW ORLEANS, March 1, 1898.

For value received, 60 da. after date, I promise to pay to the order of Amos Butler, Eight Hundred Dollars, with interest at 6%. HOWARD CURTIS.

374. On the back of this note these indorsements were written by Mr. Butler: Dec. 1, 1898, $300. June 1, 1899, $222.08. How much was due March 1, 1900?

375. Make a problem in which you suppose that you give a note for $900 due in 3 yr., with interest at 8%.

What amount would you owe at the end of 3 yr. if you made no payment before that time? On what principal would the yearly interest be reckoned?

But suppose that instead of waiting until the end of the three years you made a payment of $12 at the end of the first year. The interest then due would be $72.

If, now, in this case, as in the previous problems, the payment $12 were deducted from the amount $972, and if the difference, $960, were regarded as a new principal, observe that simply because you had made a payment on the note, you would be charged interest on a greater principal. Would that be just?

376. To prevent injustice in such cases, the Supreme Court of the United States has adopted the following rule:

UNITED STATES RULE. *When the payment is less than the interest due at the time of payment, no change of principal shall be made at that time, but the interest shall be computed upon the same principal until the sum of the payments shall equal or exceed the interest due.*

Make a problem in which the first payment on a note is less than the interest due when the payment is made.

$900. CINCINNATI, OHIO, Sept. 30, 1896.

One year from date, I promise to pay Henry Moore, or order, Nine Hundred Dollars, with interest at 8%. Value received.

MARTIN CAMPBELL.

Indorsements: March 30, 1897, $16; Sept. 30, 1897, $56.

377. How much was due June 1, 1898?

SOLUTION. The first payment, \$ 16, is less than the interest, \$ 36, that has accrued at the time this payment is made (int. of \$ 900 for 6 mo. at 8 % = \$ 36). Therefore, we compute interest to the time of the second payment. The interest of \$ 900 for 1 year at 8 % is \$ 72, and the amount is \$ 972. Subtracting from this amount the sum of the payments (\$ 16 + \$56 = \$ 72), we find that the new principal on Sept. 30, 1897, is \$ 900. The interest on \$ 900 from Sept. 30, 1897, to June 1, 1898 (9 months), is \$ 54. Therefore, the amount due June 1, 1898, is \$ 954.

378. A note of \$ 280 was dated June 25, '94, interest 6%, indorsed \$ 20, Jan. 25, '95. How much was due June 25, '95?

379. Face of note, \$ 700. Date, July 2, '95. Rate, 8%. Indorsed, Jan. 2, '96, \$ 225. Find the amount due Oct. 2, '96.

380. Face of note, \$ 200. Date, Sept. 7, 1896. Rate, 7%. Indorsed, March 7, 1897, \$ 30. June 7, 1897, \$ 40. Sept. 7, 1897, \$ 60. Find the amount due Dec. 7, 1897.

381. When settlement is made within a year, the following rule is generally used:

MERCANTILE RULE. *Find the amount of the principal from date to time of settlement. Find the amount of each payment from its date to the time of settlement. Subtract the amounts of the payments from the amount of the principal.*

Find by this rule the amount due at the end of a year on a note of \$ 500 with interest at 6%, if a payment of \$ 200 is made 4 months before settlement.

382. A note for \$ 1000 was given Feb. 7, 1898. Rate, 6%. Settlement was made 63 da. later. A payment of \$200 was made 30 da. before settlement, and 15 da. before settlement \$ 300 was paid. How much was paid at settlement?

SOLUTION. The amount of the principal, \$ 1000, from date of note to time of settlement (63 da.), is \$ 1010.50. The amount of first payment, \$ 200 (30 da.), is \$ 201; the amount of second payment, \$ 300 (15 da.), is \$ 300.75. Subtracting the amounts of the payments, \$ 501.75, from the amount of the principal, \$ 1010.50, there remains to be paid \$ 508.75.

Find amount paid at settlement applying Mercantile Rule.

383. Face of note, \$ 60. Date, June 20, '85. Rate, 8%. Indorsed, \$ 20, July 6, '85. Settled, Aug. 23, '85.

384. Face of note, $80. Date, Nov. 5, '91. Rate, 7%. Indorsed, $30, Dec. 5, '91. Jan. 5, '92, $25. Settled, Feb. 5, '92.

385. Face of note, $120. Date, Aug. 9, '93. Rate, 6%. Indorsed, Sept. 15, '93, $48. Oct. 1, '93, $45. Settled, Oct. 9, '93.

BANK DISCOUNT

386. Mr. James Gage sold a carriage to Mr. John Lyman, price $600, terms $100 cash, and the balance by a note due in 6 mo. without interest. As Mr. Gage wished to use the money in his business, he took the $500 note immediately to Mr. Peter Reed, who discounted it at 8%; that is, in exchange for the note, he gave Mr. Gage what remained after the interest of the $500 at 8% for 6 mo. had been deducted from the $500. How much did Mr. Gage receive for the note? At the end of the 6 mo. to whom should Mr. Lyman pay the $500? How much did Mr. Reed make by the transaction?

387. When a note is discounted, the payee indorses it, making it payable to the one who discounts it. The payee is then responsible with the maker of the note for its payment. Mr. Gage wrote on the back of the note when he transferred it to Mr. Reed,

Pay to the order of Peter Reed.

JAMES GAGE.

If when the note became due Mr. Reed should be unable to collect the amount of it from Mr. Lyman, to whom could he look for payment?

SUGGESTION TO TEACHER. Let three pupils enact the parts of Mr. Gage, Mr. Lyman, and Mr. Reed, one making, signing, and giving the note; another receiving, indorsing, and transferring it; the third discounting it. Let the class discuss the purpose and the justice of each step in the transaction.

388. To discount a note is to take from its face the simple interest on it for the time between the date of discounting and the date of maturity.

At 5%, what is the discount on a non-interest-bearing note for $700 due in 60 da.?

389. Discount that is found by computing interest for a certain time is called **Bank Discount.**

How does it differ from trade discount?

390. If you had a note which promised to pay you $300 at the end of a year's time without interest, would it be worth $300 now? If it were discounted at 8%, how much would the discount be, and how much would you receive for the note now?

391. The difference between the bank discount and the face of the note discounted is called the **Proceeds** of the note.

What are the proceeds of a non-interest-bearing note for $400 due in 6 mo. discounted at 10%?

392. Hale & Co. sold a carriage for $400; terms $50 cash, balance by note, payable in 60 da., without interest. As they wished to use the money at once, they sent the note to a bank where it was discounted at 8%. What were the proceeds of the note? How much did Hale & Co. really receive for the carriage?

Find bank discount and proceeds of non-interest-bearing notes for the following amounts:

393. $250, due in 90 da., discounted at 6%.

394. $450, due in 30 da., discounted at 9%.

395. $900, due in 60 da., discounted at 8%.

396. $750, due in 60 da., discounted at 9%.

397. $900, due in 30 da., discounted at 6%.

398. $650, due in 100 da., discounted at 7%.

399. A note for $800 due in 4 mo. was discounted at 6% 3 mo. before it was due. What were the proceeds?

As the note had only 3 mo. more to run, it was discounted for 3 mo.

Find proceeds of non-interest-bearing notes discounted at 8%:

	Am't	Date of note	Date of discount	Date of maturity
400.	$600	Feb. 1, '97	Mar. 1, '97	Apr. 1, '97
401.	$500	Feb. 15, '96	Mar. 31, '96	May 1, '96
402.	$870	Sept. 1, '95	Oct. 10, '95	Dec. 1, '95
403.	$660	June 4, '97	July 1, '97	Aug. 4, '97
404.	$745	Apr. 6, '84	June 3, '84	July 9, '84

405. Find proceeds of a note given April 15, 1894, due in 60 da., discounted May 15, 1894, at 7%.

Find proceeds of the following non-interest-bearing notes discounted at 8%:

	Face	Date of note	Time	Date of discount
406.	$ 38	June 7, '95	60 da.	July 1, '95
407.	$ 900	May 10, '84	90 da.	June 10, '84
408.	$ 850	Sept. 7, '90	60 da.	Sept. 21, '90
409.	$ 750	Mar. 15, '87	4 mo.	May 1, '87
410.	$1500	Oct. 2, '93	60 da.	Nov. 1, '93

411. If you had a note promising to pay you $400 in one year with interest at 6%, how much would it be worth at the end of the year? If that amount were discounted at 8%, what would the proceeds be?

As national banks do not usually discount long-time notes, if you wished to obtain the money in advance on this note you might apply to a savings bank or to one of those persons who deal in money and are called capitalists, money lenders, brokers, or loan agents. Their rates of discount and their ways of computing it vary. In the case of interest-bearing notes, the discount is sometimes reckoned upon the face of the note, sometimes upon the amount due at maturity, and sometimes the face of the note is discounted at a rate per cent equal to the difference between the rate of interest and the rate of discount.

412. A note for $1500 due in one year with interest at 7% was taken to a money lender, who deducted 8% from the

face. What were the proceeds? At the end of the year, how much did the money lender receive? How much did he gain?

413. A note for $1500 due in one year with interest at 7% was taken to another money lender, who calculated the amount due at maturity and discounted that amount at 8%. What were the proceeds?

414. A note for $1500 due in one year with interest at 7% was taken to another money lender, who agreed to discount it at 8%. He found the difference between the rate of interest and the rate of discount, 1%, and took 1% of the face. What were the proceeds?

Find the proceeds of the following notes, discounted at 8% by the method explained in Ex. 414:

In these problems, the notes are supposed to be discounted on the days on which they are dated.

	Face of note	Time	Rate
415.	$ 750	2 mo.	5%
416.	650	3 mo.	6%
417.	1700	1 mo.	6%
418.	1200	4 mo.	7%
419.	1000	60 da.	6%

420. Mr. Ashby bought an automobile for $1000 and sold it for $1250, receiving $1000 cash and the balance in a note due in 6 mo., with interest at 6%. On the day of the sale the note was discounted at 8% by the plan given in Ex. 414. How much did Mr. Ashby gain by the sale?

421. Mr. Day sold three bicycles, each of which cost him $75, on the following terms: For the first he received $50 cash and a note for $50 due in 4 mo., interest 6%. For the second he received $75 cash and a note for $25 due in 6 mo., interest 5%. For the third he received $99 cash. The notes were discounted at date at 8% by the method used in Ex. 414. Compare the profits on the three sales.

INSURANCE

422. Mr. Adams has a house worth $ 7000. He has made an agreement with the agent of an insurance company by which, if the house is destroyed by fire, he will receive from the company $ 5000, or, if it is injured but not destroyed, he will receive a sum in proportion to the damage done. For this insurance against loss by fire, he pays the company every year 1% of the sum for which the house is insured. How much does he pay for insurance?

423. The money paid for insurance is called a **Premium.**

Mr. Green, wishing to provide for his wife in case of his death, has taken, for her benefit, an insurance of $ 5000 upon his life. The company has agreed to pay her $ 5000 upon proof of his death. For this he pays a premium of $ 27.35 a year, for each thousand dollars. If he pays premiums for 25 yr., what amount will he pay to the company?

424. Mr. Strong, wishing to provide for his future, has taken out what is called an endowment policy. This agreement provides that if he is alive at the end of ten years, he shall receive $ 3682.25, and in case of his death at any time during the ten years, his heirs shall receive $ 2500. For this he pays an annual premium of $ 347.47. To how much will his premiums amount in the ten years?

425. There are two kinds of insurance. **Property Insurance** and **Personal Insurance.**

To which kind of insurance does each of the three preceding problems refer?

There are many kinds of property insurance, as insurance against loss by fire, tornadoes, shipwreck, theft, unpaid debts, etc.

426. Mr. Campbell has a house worth $ 1800. If it were insured for $\frac{2}{3}$ of its value at 1 % each year, what would be the annual premium? If the house were destroyed by fire, how much insurance would Mr. Campbell receive?

427. What would be Mr. Campbell's annual premium if his house were insured for $\frac{3}{4}$ of the value, and the rate of insurance were $1\frac{1}{4}\%$? How much would the premium be if the rate were $1\frac{1}{3}\%$, and the insurance covered $\frac{5}{6}$ of the value of the house?

428. A stock of goods invoiced at $10,500 was insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$. How much premium was paid?

429. A ship worth $75,000 was insured for $\frac{3}{5}$ of its value at $1\frac{1}{8}\%$. The cargo, valued at $7500, was insured for $\frac{4}{5}$ of its value at $2\frac{1}{2}\%$. Find amount of premiums.

430. Insurance companies generally insure property for a period of years, as 1, 3, or 5 yr., charging a certain number of cents on each hundred dollars insured. They also charge a certain amount, usually $1, for the written agreement to pay the insurance in case of loss. This written agreement is called a **Policy.**

An insurance company insured Mr. Allen's house, worth $1600, for $\frac{3}{4}$ of its value, for a period of 3 yr., charging $1.30 for each $100, and $1 for the policy. How much did the insurance cost him?

SUGGESTION TO TEACHER. Let an insurance policy be brought into the schoolroom to be discussed and examined by the pupils. After solving the following problems let pupils compose similar ones.

431. Mr. Stevens takes out an insurance policy of $7000 for a period of 3 years. The 3 yr. rate is twice the annual rate, which is $.65 for $100. Policy, $1. Find cost of insuring.

Find cost of insuring the following property at the above rates; the policy costing $1 in each case.

432. Barn, $600. Hay, $300. $\frac{3}{4}$ value insured, 1 yr.

433. House, $2700. Stable, $600. $\frac{2}{3}$ value insured, for 3 yr.

434. Stock of goods, value $6000, $\frac{3}{4}$ insured, 1 yr.

435. House, $5000, $4000 worth of insurance taken for 3 yr.

436. House, $9500, $7000 worth of insurance taken for 3 yr.

437. Mr. Rice takes $3000 worth of insurance from the Helena Fire Insurance Co. for a period of 3 yr. at the rates given above. Policy, $1. Six months later his house is damaged by fire to the extent of $1000, for which amount he receives a check from the company. How much better off is Mr. Rice than he would have been had he taken no insurance?

438. Mr. Wood insured his house five times successively for 3 yr. periods at $1.25 a hundred, each policy costing $1. During the first period the house was insured for $8000, and during the next period for $7500. He continued to reduce the amount of insurance $500 each time he renewed it. The house was never injured by fire. How much did he pay out for insurance on the house during those 15 yr.? What did he receive in return for his payments?

439. Mr. Charles Olney insures his furniture for $500 for 3 yr. The annual rate is $.55 per hundred. The rate for 3 yr. is twice the annual rate. What is the premium?

440. John Gibson took out two policies: $3000 on dwelling, and $1250 on furniture. Term, 5 yr. Annual rate, 40¢. 5 yr. rate three times the annual rates. What was the amount of both premiums?

441. The trustees of Perry Township hold a policy on a school building for $3250.00. Term, 5 yr. What premiums have they paid, the rate being as in the previous problem?

442. West & Co. insure their stock against wind storms, for 3 yr., for $20,000. Rate, 40¢ a hundred for 3 yr. What is the amount of premiums?

443. George Brown takes out a policy for 3 yr.: $3000 on his dwelling and $500 on his furniture. Rate, $.90 for 3 yr. Policy fee, $1. What is the cost of his insurance?

444. A house valued at $3000 and insured for $\frac{2}{3}$ its value was struck by lightning. The adjuster for the insurance com-

pany estimated that it was damaged 37½% of its value, and paid that per cent of the amount insured. How much did the owner of the house receive from the insurance company?

There are several kinds of personal insurance, as Life Insurance, Accident Insurance, Endowments, Annuities, etc.

445. Mr. Blake took out a life insurance policy of $5000 for the benefit of his wife, upon which he paid $33.30 per thousand, yearly premium. He lived 20 yr. How much more was paid to his widow than he had paid to the insurance company? What would have been the difference between the amount paid to the company by Mr. Blake and the amount received from it by Mrs. Blake, if he had taken an insurance of $10,000?

Successful insurance companies take the small savings of those who are not able to invest them to advantage, and, massing them, invest them profitably. The insured loses the interest of the money which he pays to the company, but receives an assurance that those for whose benefit he is insured will receive the full amount in case of his death at any time.

How much more or less is received from the insurance company than is paid to it in the following cases?

	Amount insured	Yearly premium per $1000	Years
446.	$7,000	$30.70	20
447.	6,000	44.84	40
448.	10,000	28.28	21
449.	30,000	27.38	50
450.	50,000	35.65	5
451.	200,000	39.00	15

452. Mr. Bland takes out a life insurance policy for $4000, paying $24 per thousand. What is the annual premium? If he pays it for 50 years, how much more does he pay than his heirs receive?

453. Mr. Corlen insured his life for $13,000 paying his first premium of $57.50 per thousand on Jan. 1, 1901. He died Mar. 1, 1901. If the company paid the agent a commission of

15% on the first premium for insuring Mr. Corlen's life, how much did it lose by the insurance?

454. Mr. Hill took out a life insurance policy for $25,000 at the rate of $32.75 per thousand. He died three months after paying the first premium. How much more did his heirs receive than he paid?

455. Many insurance companies divide a part of their earnings among those who are paying premiums, giving each one a certain per cent on the premium he pays. This amount is called a **Dividend.**

Find the value of a dividend of 5% on a premium of $250.

456. For 25 yr. Mr. Field paid an insurance premium on a life policy for $7000 at the rate of $27.50 per thousand. Ten per cent of the amount of the premiums was returned to him in dividends. How much did his insurance cost him?

457. Mr. A., who is a traveling salesman, carries an accident policy. When he had paid $205.75 in premiums, he was accidentally injured and received an allowance of $25 per week for 7 weeks. How much more or less did he receive from the company than he had paid to it?

458. At 25 yr. of age Mr. B. took out an endowment policy by which he will receive $5000 when he is 45 yr. old. Annual premium, $240.38. How much more will he receive from the company than he pays to it? How can the company afford to do business in that way?

SUGGESTION TO TEACHER. Let pupils make problems under various imaginary conditions, getting facts about insurance from agents or circulars in order that their problems may approximate to the actual.

459. Neil & Co., agents for the Westchester Insurance Co., insured the following risks for periods of 3 yr. at $1.30 per hundred. Their commission was 15% on the premiums, and they received from the insured a policy fee of $1 in each case, which they retained. How much did Neil & Co. earn and how much did they send to the insurance company?

Dwelling,	value	$ 1,800,	$\frac{3}{4}$ value taken
Store,	value	15,000,	$\frac{2}{3}$ value taken
Stock of goods,	value	17,000,	$\frac{1}{2}$ value taken
Opera house,	value	75,000,	$\frac{2}{3}$ value taken

460. Miss Otis bought from an insurance and annuity company a yearly annuity of $ 100, paying for it $ 1382.50. How much more or less would she receive from it than she paid for it if she lives 20 yr.? 7 yr.?

461. Mrs. Green owns a house from which she receives a monthly rental of $ 25. The insurance on it for the year 1898 was $ 8 and the taxes were $ 36.75. It was vacant three months. How much was her net income from it?

TAXES

462. It is necessary for all governments to tax the people to pay public expenses. Taxes upon property are calculated at a certain per cent of the assessed value.

At $1\frac{1}{3}\%$ how much are the taxes upon a piece of property worth $ 9000?

SUGGESTION TO TEACHER. Explain the duties of the assessor and tax gatherer. Procure copies of a part of an assessor's list of taxables and let pupils compute the taxes.

463. How much are the taxes upon Mr. Hudson's property which is assessed at $ 8000, where the rate of taxation is $1\frac{1}{4}\%$?

464. A fixed sum assessed, without regard to their property, upon male citizens who are at least 21 yr. of age, is called a **Poll Tax.**

If Mr. Hudson lived where the poll tax was $ 1.50, how much would all his taxes be?

465. Mr. Howe, who pays a poll tax of $ 2, owns property assessed at $ 6000, in a city where the rate of taxation is $.75 per $ 100, and $ 2500 in another city where the rate of taxation is $2\frac{1}{10}\%$. What are his taxes?

466. What are the taxes of Mr. Hearn who owns real estate assessed at $ 11,375, and other property valued at $ 2500, the rate of taxation being 1½%, and his poll tax $ 1?

467. Property is considered to be of two kinds: **Real Estate**, as lands, houses, stores, factories, mines, and other immovable property; and **Personal Property**, such as money, notes, furniture, and other property which can be carried from place to place.

Name a piece of real estate. Name different kinds of personal property.

468. Mrs. Kent owns real estate assessed at $ 5600, and personal property assessed at $ 1000. The rate of taxation is $ 1.20 on $ 100. What is the amount of her tax bill?

469. A penalty of a certain per cent of the amount of the tax is sometimes enforced if the taxes become delinquent; that is, if they are not paid at the time required. If Mrs. Kent allows her taxes to become delinquent, and the penalty is 10%, what will be the amount of her tax bill?

470. In a certain city the rate of taxation is $ 1.35 per $ 100, and the poll tax is $1. How much are the taxes of an adult male citizen whose real estate is valued by the assessor at $ 1540, and personal property at $ 300?

471. What is the tax of an adult female citizen whose real estate is valued at $ 1500, and personal property at $ 650?

472. Mention several things which are paid for with the money raised by taxation.

473. Some property, as churches, government bonds, and public property of all kinds, is exempt from taxation. Why are public school buildings not taxed?

474. A certain town raised $ 21,845 by taxation, a part of which was the assessment of 721 polls at $ 2 each. How much tax was raised from property?

475. Taxes to the amount of $ 24,704.35 were raised in the town of Nalasco, of which $ 23,456.35 was raised from property, and the rest from polls at $ 2 each. How many citizens paid poll taxes in the town?

476. In a town where there are 1236 polls assessed at $ 1.50 each, it was decided to raise $ 46,854 by taxation. How much must be raised from the property? If the property valuation of the whole town was $ 3,600,000, how much must each dollar's worth of property yield?

What would be the tax of each of the following residents of that town?

477. Mr. A., 31 yr. old, whose realty is $ 7000, personals $ 1375.

478. Mr. B., 19 yr. old, whose realty is $ 938, and who has $ 4500 worth of personal property, of which $ 3000 is in government bonds.

479. Mrs. C., whose realty is $ 7500, personal property $ 635.

480. Mr. D., 43 yr. old, who has no property.

481. Miss E., who has no real estate and $ 500 worth of personal property.

482. The money which defrays the public expenses of cities, counties, and states is raised by direct taxation upon property or person. Money for the expenses of the national government is raised by indirect taxation, of which there are two kinds, **Internal Revenue** and **Duties** or **Customs.**

The internal revenue is mostly derived from taxes on the manufacture of liquors and tobacco products and from the sale of stamps which the government requires to be placed upon certain legal documents and articles sold.

To defray the expenses of the war with Spain in 1898, a law was passed by Congress requiring among other provisions that a one-cent stamp should be affixed to every telegraphic message or express receipt, a two-cent stamp to every bank check,

sight draft, etc., two cents for every hundred dollars, or fractional part thereof, named in the face of time drafts, promissory notes, etc., and stamps of different values upon patent medicines, proprietary articles, insurance policies, contracts, leases, etc.

What should be the value of a stamp affixed to a promissory note for $2500? For $275? For $39.50?

483. A drug firm sold in one week 1216 bottles of patent medicines, each requiring a stamp whose value is $\frac{5}{8}$ of a cent, 1172 packages each requiring a stamp costing $1\frac{1}{4}$¢, and 298 packages each requiring a stamp costing $2\frac{1}{2}$¢. The firm sent 15 telegrams and 137 express packages. Fifty-one checks were given by the firm. How much revenue accrued to the government from the sale of stamps necessary for the business of that firm for that week?

484. The taxes levied by the government upon imported goods are called **Duties** or **Customs**. All goods which come into the country must be brought in at certain places called **Ports of Entry**. At these places the government maintains custom houses, with officers who collect the duties.

There are two kinds of duties, **Specific** and **Ad Valorem.**

A duty of a certain per cent of the amount at which the goods were invoiced in the country from which they were imported is called an **Ad Valorem Duty**. The Latin phrase ad valorem means "according to value."

Find the ad valorem duty of 100 yd. silk invoiced at $.50 per yd., duty 55%.

At the rates given, how much ad valorem duty would be paid by a firm of importers upon the following goods?

485. 50 yd. of silk, invoiced at $1.25 per yd., duty 55%.

486. 500 pieces of ribbon, 10 yd. in a piece, at 75¢ per yd., duty 40%.

487. 50 yd. of lace, at $2.25 per yd., duty 60%.

488. At 20%, what is the duty on 75 bales of wool, 400 lb. each, invoiced at 25¢ per pound?

489. At 25%, what is the duty on 500 boxes of raisins, each containing 40 lb., costing $6\frac{1}{2}$ cents per pound?

490. A duty levied upon a certain quantity of goods, without reference to their value, is called a **Specific Duty.**

If the specific duty is $2.25 per dozen pairs, how much is that duty on 600 pairs of gloves invoiced at 50¢ a pair? If they were invoiced at 75¢ a pair, what would be the specific duty?

491. Sometimes both specific and ad valorem duties are levied upon the same article.

What is the duty on 30 pieces of carpet, 25 yd. each, invoiced at $1.75 per yard, the specific duty being 25¢ per yard, and the ad valorem duty 40%?

492. What is the duty upon 800 lb. of cigars, invoiced at $5 per pound, which pay a specific duty of $4.50 per pound and 25% ad valorem?

493. An importation of silks from France was invoiced at 9324 fr. At 60% ad valorem, how much is the duty in American currency, $1 being considered equal to 5.18 fr.?

494. What duty is paid by an American importer upon 600 doz. pairs of gloves invoiced at 60 fr. per dozen, if there is a specific duty of $2 per dozen pairs and an ad valorem duty of 40%?

495. Persons are allowed to bring from abroad a limited amount of goods for their own use without paying duties upon them. An American lady brought home from Europe a silk dress pattern, upon which the duty was $31.75; $\frac{1}{2}$ doz. pairs of kid gloves, upon which the duty was $2.25 per dozen pairs; 20 yd. of lace worth $2 per yard, upon which the duty was 50% ad valorem; and 12 yd. Irish linen at 60¢ per yard, ad

valorem duty 35%. As these goods were for her own use, they were passed in duty free. How much less did the goods cost her than they would have cost had the duty been collected?

496. If an importer buys 700 yd. of velvet at $1.50 per yard, pays an ad valorem duty of 60%, and sells it at $4 per yard, how much does he gain?

497. How much is gained by an importer who buys 20 pieces of matting, 40 yd. in a piece, at $.10 per yard, pays a duty of 25%, pays for transportation $60, and sells the matting at 35¢ per yard?

498. A list of articles upon which duties must be paid, with the special duty upon each, is called a **Tariff.** Tariffs are changed from time to time by acts of Congress.

An importer brought through the custom house $80,000 worth of cut glass when the duty was 35% ad valorem. He sold $\frac{1}{2}$ of it at a profit of 25% upon invoice price plus the duty. The tariff upon glass was raised after his purchase to 60%. He sold the other half of his stock at a profit of 25% upon invoice price plus the new duty. How much more did he gain on the last half of his stock than on the first half?

499. A New York firm imported goods invoiced at $64,000, upon which there was a duty of $12\frac{1}{2}$% ad valorem. For how much must these goods be sold to give a profit of 20%?

500. Soon after those goods were bought, the duty on that class of goods was changed to 25% ad valorem. Another firm imported $64,000 worth of the same kind of goods under the new tariff, and sold their goods at a profit of 20%. How much did they receive? If the first firm sold their goods for the same amount as the second firm, how much more did they gain than the second firm gained?

501. Mr. Gilman imported $100,000 worth of goods, the duty upon which was 30% ad valorem. If, after he sold $\frac{1}{4}$ of them at a profit of 20%, this class of goods was put on the

free list, for how much could his competitors in business buy an amount of goods equal to what he had left on hand? If he sold the rest of his goods for 20% more than that sum, would he gain or lose on the whole transaction, and how much?

502. Make a problem in which an importer's business is injured or benefited by changes in the tariff.

MISCELLANEOUS EXERCISES

1. Resolve into prime factors 6750. 7920.

2. Find the g. c. d. of 235 and 685.

3. Find the l. c. m. of 8, 10, 12, 16, 18, 20.

4. Divide .012261 by 2.01.

5. $3\sqrt{64}+7\sqrt{81}=?$

6. $2\sqrt[3]{64}+4\sqrt[3]{125}=?$

7. If 1 qt. of nuts costs 11¢, how many bushels can be bought for $13.20?

8. Find the interest of $1240 for 5 yr. 9 mo. 27 da. at 3%.

9. A wheel of a bicycle is 7 ft. in circumference. How many times does the wheel turn in going 10 rd. 1 yd.?

10. John weighs 115 lb., and his cousin weighs 110 lb. John's weight is what per cent of the sum of their weights?

11. Write the following decimally and as common fractions in their lowest terms: 13%. 18%. $22\frac{1}{2}$%. 158%. 875%. $12\frac{1}{2}$%. $\frac{5}{25}$%. $\frac{5}{6}$%.

12. How much is 30% of 40 minus $16\frac{2}{3}$% of 66?

13. Find 11% of 24^2. $37\frac{1}{2}$% of 16^2. $62\frac{1}{2}$% of 64^2.

14. Find $12\frac{1}{2}$% of 12^3. $66\frac{2}{3}$% of 9^3. $87\frac{1}{2}$% of 12^3.

15. Square: $\frac{5}{9}$. .3. 1.2. $2\frac{1}{2}$. .06. $\frac{3\frac{1}{3}}{10}$.

16. Name four numbers between 100 and 200 that are perfect squares and give their square roots.

17. What number between 100 and 200 is a perfect cube?

18. Every prime number greater than 10 must end with either 1, 3, 7, or 9. Give the reason.

19. The arc AB is 24 in. long. BC is 50% longer than AB. CD is $33\frac{1}{3}$% longer than BC. DA is 25% shorter than DC. How long is the circumference? Diameter? Radius? Perimeter of the sector DOA? BOC? DOC? AOB?

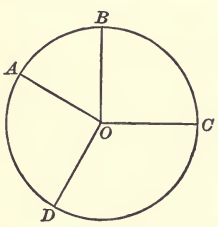


FIG. 1.

20. A circumference which is 65 in. long is divided into two arcs, the smaller arc of which is 13 in. long. The smaller arc is what per cent of the greater?

21. How many coins an inch in diameter could be placed in rows touching one another on a rectangle 4 in. by 3 in.? Represent.

22. When the hour hand of a clock is at 3, what per cent of one revolution around the clock face has it made since 12?

23. At 4 P.M. the time past noon is what per cent of the time before midnight?

24. The time past noon is what per cent of the time to midnight at 2 P.M.? 8 P.M.? 1.30 P.M.?

25. Thomas wished to add the fractions $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{8}$. He first multiplied each term of each fraction by the product of all the denominators except its own. How were the three fractions then expressed? He then added these fractions and reduced their sum to its lowest terms. Was his process correct? Can you show a better way to find the sum of these fractions?

26. Take Ex. 25, substituting the fractions, $\frac{1}{5}$, $\frac{1}{7}$, and $\frac{1}{11}$.

Fill blanks and solve:

27. A house worth —— dollars was insured for —— of its value, at —— per cent. What was the annual premium?

28. Mr. A. held a life insurance policy for $2000, on which he paid an annual premium of $52. He was insured March 1, 1890, and died June 1, 1900. How much more did his heirs receive than he had paid out in premiums?

29. A room 36 ft. long and 24 ft. wide is to be covered with carpet $\frac{3}{4}$ yd. wide, at $1.10 per yard. How much will it cost if the strips run lengthwise of the room and each strip is turned in 4 in.?

30. Advance the following goods 15% in price: Caps at 30¢, coats at $8, shoes at $1.25, gloves at 78¢, ties at 15¢.

31. A fisherman caught herring enough to fill 500 barrels. He sold 35% of the catch, and kept the rest for a rise in price. How many barrels of herring did he keep?

32. How many quarts of berries at $12\frac{1}{2}$¢ a quart would be required to pay for 9 yd. of cloth at $16\frac{1}{2}$¢ a yard?

33. Two men traveled from the same point, one east, $45\frac{5}{7}$ mi.; the other west, $92\frac{4}{5}$ mi. How far apart were they?

34. Two men started from the same point, and traveled in opposite directions. One man traveled at the rate of $7\frac{1}{3}$ mi. per hour, the other at the rate of $6\frac{3}{4}$ mi. per hour. How far apart were they at the end of 1 hr.? Of 3 hr.? Represent.

35. John's uncle showed him a half eagle one morning, and promised to give him at night 25% of all of it that was not spent. At night his uncle reported that 100% of the money had been spent, but he gave him 75¢ instead. How much more or less would John have received if his uncle had spent only 50% of the value of the half eagle? 40%?

36. Nine is how much greater per cent of 144 than of 288?

37. How much is gained on each tablet bought at the rate of $1 per dozen, and sold at 10¢ each?

38. Mr. Hall earned $125 in one month, which was 62½% of his earnings the next month. How much did he earn in both months?

39. An apple tree bore 21 bu. of apples, which was 87½% of what the tree next to it bore. What was the difference in the yield of the two trees?

40. An automobile started from New York, and ran 60 mi. the first day. On the next day its speed was 33⅓% greater than on the first day, and on the third day it was 25% greater than on the second. How far was the automobile from New York at the end of the third day?

41. What number plus 1% of itself equals 909? 2424?

42. Forty-five is 50% more than what number? 50% less than what number?

43. A man had $654 in bank. He drew out 33⅓% of it, and afterward drew out 25% of the remainder. How much had he left in bank?

44. A man sold a wagon for $180, and gained 25%. What was the cost of the wagon?

45. A man sold a wagon for $180 and lost 25%. What was the cost of the wagon?

46. Near the close of summer the price of goods costing $1.10 per yard was cut to 95¢ a yard. What per cent was lost?

47. Fifty yards of cloth were bought for $30. For what price per yard must they be sold to gain 25%?

48. A house valued at $8000 was insured for ⅘ of its value at 1¼%. What was the premium?

49. On the day before Christmas Mary counted at a certain corner 37 ladies who were carrying packages, and 13 who

had no packages. What per cent of the ladies that she counted had no packages?

50. Thirteen children were transferred from a class of 42. What per cent of them remained?

51. In making peach marmalade, Mrs. Harland boiled 4 lb. of peaches and 3 lb. of sugar in a quart of water. Each pint of water weighed a pound. If 1 pt. of the water evaporated in cooking, what per cent of the marmalade was sugar? Peaches?

52. How wide is a rectangle 20 cm. long and equal to $\frac{3}{5}$ of a square decimeter? Represent.

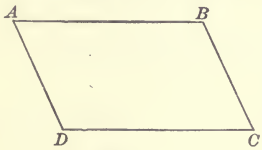


FIG. 2.

53. In the rhomboid $ABCD$ the line BC represents 10 ft. AB is 50% longer than BC. How long is the perimeter? What per cent of the perimeter is AD? DC?

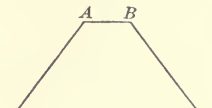


FIG. 3.

54. AB and DC are parallel. $DC = 48$ ft. $AB = 25\%$ of DC. $BC = 250\%$ of AB. $AD = 100\%$ of BC. Find the perimeter of the trapezoid.

55. A farm is in the shape of a trapezoid. The shorter parallel side is 16 rd. long. The longer parallel side is $12\frac{1}{2}\%$ longer. One of the non-parallel sides is 10 rd. and the other is 20% longer. Represent. Find the cost of fencing the farm at 75¢ per rod.

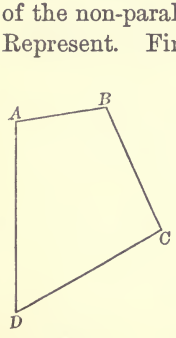


FIG. 4.

56. What is a trapezoid? How does it differ from a rhomboid?

57. A four-sided plane figure which has no two sides parallel is called a **Trapezium.**

Draw a trapezium.

58. How long is the perimeter of a trapezium of which the side AB is $3\frac{1}{4}$ in., the side BC $5\frac{7}{8}$ in., the side CD $6\frac{1}{2}$ in., and DA $7\frac{5}{8}$ in?

59. A garden is fenced in the form of a trapezium. One side is 4 rd. 3 yd. 2 ft. 8 in. long, another side is 5 rd. 1 ft. 10 in. long, another side is 4 rd. 5 yd. 4 in. long. The other side is 6 rd. 2 yd. 6 in. long. How long is the fence?

60. How long is the perimeter of a trapezium, the shortest side of which is 12 in. long, the next side 2 in. longer than the first, the next side 3 in. longer than the second, and the last 4 in. longer than the third?

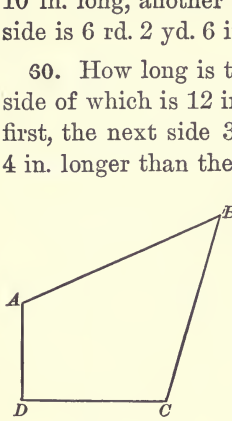


FIG. 5.

61. In the trapezium *ABCD*, *AD* represents 8 ft. *DC* represents 50% more than *AD*. *CB* represents $33\frac{1}{3}\%$ more than *DC*. *BA* represents $12\frac{1}{2}\%$ more than *CB*. How long is the perimeter?

62. John had a kite frame in the shape of a trapezium having two short equal sides and two long equal sides. If a long side was 3 ft. long and a short side $33\frac{1}{3}\%$ as long, what was the combined length of the sticks that made the frame, allowing $\frac{1}{2}$ an inch for lapping the sticks at each angle?

63. Plane figures bounded by four straight lines are called **Quadrilaterals.**

You have learned six different kinds of quadrilaterals. Draw one of each kind and write its name upon it.

64. How long is the perimeter of a rhombus whose sides are each 1.7 in.?

65. Find the perimeter of a rhomboid whose long sides are each 9.9 in. and whose short sides are each 5 in. less than a long side.

66. How long is the perimeter of a trapezoid if one of the parallel sides is 7.65 in., the other 8.45 in., and each of the non-parallel sides is 4.7 in.? Represent.

67. Plane figures bounded by straight lines are **Polygons.** Name four kinds of polygons.

68. What name is given to a polygon of 3 sides? 5 sides? 6 sides? 8 sides? 10 sides?

69. Is a sector a polygon? Explain.

70. Mr. K. bought 9 doz. pencils for $2.16. He sold them at $.03 apiece. What per cent was gained?

71. 500 sacks of coffee were bought for $300. At what price per sack must they be sold to gain 10%?

72. 350 bottles of ink were bought for $21. For how much per bottle must they be sold to gain $66\frac{2}{3}\%$?

73. 280 penknives cost $70. For how much apiece must they be sold to gain 25%?

74. A dealer paid $24 for 300 slates. In selling them he gained 50%. What was the selling price of each?

75. Two gross of handkerchiefs were bought for $28.80. At what price apiece must they be sold to gain 30%?

76. Mr. Fowler's salary was $1800 a year. Last year he paid $1175.50 for household expenses, $22.50 for life insurance, $15.75 for taxes, $178.25 for clothing, and $23.75 for incidentals. What per cent of his salary did he save?

77. A grocer sold 630 heads of cabbage, which was $66\frac{2}{3}\%$ of what he had. How many had he left?

78. It cost $15 to build a certain fence, and $10 to paint it. The cost of painting was what per cent of the whole cost?

79. A quart of water was added to 6 gallons of cider. What per cent of the mixture was water?

80. Eight pounds of river water, when distilled, furnished $7\frac{3}{4}$ lb. of pure water. What per cent was removed by distilling?

81. In making brown bread, Mrs. Goodwin mixed one cupful of white flour, one cupful of Graham flour, and four cupfuls of corn meal. What per cent of each was in the mixture?

82. Ten years ago Mr. D. paid $4000 for a house and lot, which has increased in value 18%. What is its present value?

83. By the sale of a horse, a man gained $10, which was $12\frac{1}{2}\%$ of what he gave for it. For how much did he sell it?

84. A house worth $4000 depreciates in value to $3280. What per cent does it depreciate?

85. If I buy 2000 bu. of wheat at 85 cents a bushel, and sell it for $340 more than I paid for it, what per cent shall I gain?

86. A boy buys papers for 2 cents each, and sells them at a gain of 150%. What price does he get for them?

87. A book agent receives a commission of 28% on all his sales. If he gets orders for 125 books at $2.50 each, how much does he gain?

88. If I send orders amounting to $28.75, getting a discount of 25% with 5% off for cash, how much money must I send?

89. About 80% of a human body is water. At that rate, if a man weighs 165 lb., how much of his body is water?

90. What per cent of 75 is each of the first 12 multiples of $6\frac{1}{4}$?

91. What per cent is gained or lost by buying goods at $.33\frac{1}{3}$ per yard and selling them at $.25? $.50? $.66\frac{2}{3}$? $.16\frac{2}{3}$? $.41\frac{2}{3}$?

92. What per cent is gained by buying apples at the rate of 3 for a cent and selling them at the rate of 2 for a cent?

93. Mr. Chapman built a house which cost $3600. He had $3250, and borrowed the rest June 1, 1895, giving his note at 6%. June 1, 1896, he paid the interest due and $100 of the principal. June 1, 1897, he paid the interest and $50 of the principal. How much was due June 1, 1898?

94. Mr. S. built a house for Mr. M., on which he made a profit of 25%. He received in payment $2000 and two lots

at $ 500 each. The lots depreciated in value $ 50 each before he sold them. How much was his real profit on the house? What per cent?

95. A wholesale dealer pays $ 260 for a car load of bananas containing 520 bunches, and sells them at $.60 a bunch. What per cent does he gain?

96. A retail dealer pays 60¢ for a bunch of bananas containing 8 doz. If he sells them at 15¢ a doz., what per cent does he gain? What per cent does he gain on the whole, if $\frac{1}{4}$ of them spoil before he sells them?

97. Mr. Malone, a traveling salesman, sold Mr. J. W. Smith of Salem, Ill., $ 360 worth of staples, and $ 170 worth of notions. He was allowed a commission of 2% on the staples, and 6% on the notions. How much commission did he receive?

98. Mr. Perry brings 49 bu. of wheat to the Melrose Mill to exchange for flour. If he gets 36 lb. of flour for every bushel of wheat, how many sacks of flour, each weighing 98 lb., will he get?

99. What number increased by 62½% of itself equals 2873?

100. Mr. Taylor bought 6 loads of hay, each weighing 1½ T., at $ 10 a ton. He sold all of it for $ 99. What was the gain per cent?

101. A farmer had a field 21 rd. square. Three rows of wire fencing were put around it, costing 2¢ a foot, 5% off for cash. If he paid cash, how much did the fence cost?

102. 33⅓% of the fence was blown down. The price of wire having risen 50%, and the cash discount being the same, how much will it cost to replace the fence if he pays cash?

103. Mr. Gibson's salary is $ 6000 a year. Last year he saved 20% of the first three months' salary, 40% of the next three months' salary, and 30% of the last six months' salary. **How much did he save during the year?**

104. W. H. Small & Co. bought 10 loads of hay, weighing 2700 lb. each, at $10 per ton, and sold it immediately to Mr. Knox at a profit of 20%. Mr. Knox gave in payment a note payable in 60 da. If it was discounted in bank at date of issue, at 7%, how much did Small & Co. gain by the transaction?

105. Arthur and Edward bought a paper route, paying $10 for it. Arthur put in $4, and Edward the remainder. What fractional part of the route belongs to Arthur? To what per cent of the profits is he entitled? In one week the earnings were $7.50. How much is each boy's share?

106. Mr. A. and Mr. B. hire a pasture for $50. Mr. A. puts in 5 cows, Mr. B. 15 cows. Mr. A.'s cows are what per cent of the whole number of cows? How much ought he to pay as his share of the cost of the pasture?

107. Mr. C. owns $\frac{5}{8}$ of a business, the profits of which last year were $16,000. What was his share of the profits?

108. Mr. Davis owns 51% of a business which year before last lacked $1000 of paying expenses. How much money was he obliged to advance in order to keep the business running?

109. Last year the receipts of the business were $1248.75 more than the expenses. How much did Mr. Davis receive from it?

110. If 10 hr. are considered a day's work and $2 a day's pay, what are the weekly earnings of a man who works 9 hr. on Monday, $8\frac{1}{2}$ hr. on Tuesday, 11 hr. on Wednesday, 7 hr. on Thursday, $6\frac{1}{2}$ hr. on Friday, and 5 hr. on Saturday?

111. What would be his weekly earnings for the same number of hours' work at $2.50 per day?

112. What would be his weekly earnings for the same number of hours' work at $2 a day, 8 hr. being considered a day's work?

113. At $2 a day for an 8-hour day, calculate the weekly earnings of each man in the following time sheet:

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
a Mr. Cox,	7	$8\frac{1}{2}$	$7\frac{3}{4}$	$6\frac{3}{4}$	7	$7\frac{1}{4}$
b Mr. Dow,	8	$8\frac{1}{2}$	6	$7\frac{1}{4}$	$7\frac{1}{2}$	6
c Mr. Lee,	$7\frac{1}{2}$	$8\frac{3}{4}$	9	9	$6\frac{1}{2}$	5
d Mr. Van,	$5\frac{1}{2}$	$6\frac{1}{4}$	8	$8\frac{3}{4}$	$7\frac{1}{2}$	6
e Mr. Gay,	7	6	$7\frac{1}{2}$	8	8	7

114. Julia saved money to buy a typewriter, the price of which was \$ 80. When she had saved \$ 48, what per cent of the price was lacking? When she had paid \$ 16 more, what per cent of the price was still unpaid?

115. Three boys bought a wagon, John paying \$ 1, James \$ 2, and Charles \$ 3. They sold the wagon for \$ 7.50. How much was each boy's share of the gain?

116. A man received a legacy of \$ 5000. He bought a house and lot with 40% of it, purchased \$ 1500 worth of bank stock, and invested the remainder in business. What per cent of it did he invest in business?

117. At \$ 1.50 per cord, how much will a man earn by sawing a pile of wood, 16 ft. long, 8 ft. wide, and 4 ft. high?

118. Find the amount of the following bill:

4 rms. sandpaper @ \$ 3.50, discounts 30% and 10%.

3 doz. packages tacks @ \$ 5.50, discounts 50% and 5%.

12 car jacks @ \$ 6.00, discounts 40%, 10%, and 5%.

2000 tin rivets, \$ 1.44, discounts 40% and 10%.

119. A man received a legacy. After investing $66\frac{2}{3}\%$ of it and spending 25% of it, he had left \$ 1728. How much was the legacy?

120. A man having \$ 4800 in a bank, drew out $12\frac{1}{2}\%$ of it and then deposited a sum which was 75% of what he had drawn out. How did his bank account stand then?

121. Mr. Boyd sold a watch for $45, which was 25% more than he gave for it. For how much must he have sold it to gain $66\frac{2}{3}\%$ on it?

122. Simplify

a	*b*	*c*	*d*
$\frac{\frac{3}{8}}{\frac{9}{16}}$	$\frac{\frac{1}{2} \text{ of } \frac{3}{7}}{\frac{5}{6} \text{ of } \frac{9}{10}}$	$\frac{1\frac{2}{3}}{1\frac{2}{9}}$	$\frac{\frac{1}{2}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}}$

123. How long is the circumference of a circle whose radius is $5\frac{1}{9}$ in.?

124. A burial lot in the form of a circle, 40 ft. in diameter, is inclosed by a fence. Find the cost of the fence at $.12\frac{1}{2}$ a foot.

125. A calf is tied to a tree by a rope 10 ft. long. The rope can slip around the trunk of the tree, which is 1 ft. in diameter. As the calf runs around the tree, how long is the circumference of the largest circle he can make?

126. Reduce 5 qt. 1.725 pt. to decimals of a bushel.

127. $14 \times (—) \times 2.3 \times \frac{5}{7} = 3.91$. Find the missing factor.

128. How many square centimeters are there on the surfaces of 7 l.?

129. In a park there is a circular fountain 35 ft. in diameter, bordered by a gravel walk 7 ft. wide. What is the distance around the outer edge of the walk?

130. A swimming pool is 60 ft. long, 20 ft. wide, and the water in it is 7 ft. deep. How many cubic feet of water are in it?

131. Draw a trapezoid, making one of the parallel sides 6 in. long, the other 8 in. long, and the shortest distance between them 4 in. Find the area of the trapezoid.

132. A farmer has a field in the shape of a trapezoid. One of the parallel sides is 90 rd. long, the other is 120 rd. long, and the shortest distance between them is 30 rd. How many acres in the field?

133. One of the sides of a rhomboid is 12 in., and a side adjacent to it is 8 in. Represent and tell what per cent of the perimeter each side is.

134. One of the parallel sides of a trapezoid is 10 in., which is $83\frac{1}{3}\%$ of the other parallel side. Each of the non-parallel sides is 9 in. Represent and find what per cent of the perimeter each side is.

135. A pupil drew a square and shaded 11% of it. If the area of the part shaded was 44 sq. in., what was the area of the whole square?

136. What per cent of the perimeter is each side of a rhombus? Of a regular hexagon? Of a regular octagon? Of a regular decagon?

137. A jeweler sold a watch for \$ 90, gaining 20%. What per cent would he have gained if he had sold it for \$ 100? \$ 110? \$ 120?

138. Mr. Baker bought \$ 100 worth of goods and sold $\frac{3}{5}$ of them for what $\frac{9}{10}$ of them cost. What per cent did he gain?

139. What per cent is gained or lost by buying \$ 100 worth of goods and selling $\frac{1}{4}$ of them for what $\frac{3}{5}$ of them cost?

140. An importer buys in France 1000 lb. of perfumery invoiced at \$ 2.00 per pound. Specific duty 60¢ per pound, ad valorem duty 45%. He sells it at \$ 5 per pound. How much does he gain and how much does the government gain by the transaction?

141. On Aug. 1, 1890, a man insured his life to the amount of \$15,000 in favor of his wife, paying an annual premium of \$ 29.30 per \$ 1000. He died Oct. 17, 1897. How much more than he had paid did his widow receive?

142. A farmer brought to market 24 doz. eggs, 21 chickens, and 20 lb. of butter. He sold $\frac{2}{3}$ of the chickens at 25¢ apiece, all the butter at 15¢ a pound, and 75% of the eggs at 10¢ a

dozen. How many eggs and chickens did he have left? He took groceries to the amount of $\frac{1}{3}$ of his sales. He bought 10 yd. of calico at 7¢ a yard, and a pair of shoes for $2.75. How much money had he left?

143. A boy made a chicken coop, using a bundle of laths which cost 15¢, and 5¢ worth of nails. He sold the coop for $.75. What was his gain per cent?

In the following shipments find the amount belonging to each railroad company:

144. From Trent's Landing, Ky., to Milwaukee, Wis., via A. & B., B. & E., and E. & M.: Empty kegs, 20,000 lb.; rate, 11¢ per hundredweight. The A. & B. receive 25% of the freight charges. Of the remainder, the B. & E. receive 45%, and the E. and M. 55%.

145. From Mather, Ill., to Cincinnati, Ohio, via M. W. & E. and A. & C.: 1 car hogs, 18,000 lb.; rate $20\frac{1}{2}$¢ per cwt. The M. W. & E. receive 25%, and the A. & C. the remainder.

146. From Leeds, Ky., to Rawley, Minn., via L. & H., E. & C., Chicago & St. Paul, St. P. & R.: $25\frac{1}{2}$ tons pig iron; rate, $2.75 per ton. 15¢ a ton goes to the Ohio Bridge Co.; the lines south of Chicago receive 55% of the remainder. The lines north of Chicago receive the balance. The L. & H. receives 25% of the amount paid to the southern lines, and the St. P. & R. 25% of the amount paid to the northern lines.

147. Mr. Ward bought a lot for $600 and built on it a house worth $2400. The property was assessed at $\frac{2}{3}$ its value, and the tax rate was $1\frac{1}{2}$%. The insurance was for $\frac{5}{6}$ of the value of the house, and the rate was 40¢ a hundred. He rented the house for a year at $30 a month. How much more did he receive the first year from his investment of $3000 than he would have received by putting it at interest at 6% and paying the tax of $1\frac{1}{2}$%?

148. The second year the house was vacant one month and required $10.65 worth of repairs. Insurance and taxes were

the same as the first year. How much more were the net receipts than 6% of the amount invested?

149. The third year the rent was raised to $35 a month. The repairs cost $48.75, and the house was vacant two months. Compare the net receipts with 6% on the investment.

150. Make problems showing the expenses and receipts of Mr. Ward's house for successive years.

151. At the end of 7 yr. Mr. Ward sold the house and lot for $4500. If he had received as rental during the 7 yr. $2000 more than the expenses of the house for that time, what average yearly per cent had he gained on his investment?

152. By selling a house for $1964, 20% was lost. What would have been the selling price if only 5% had been lost?

153. Room No. 5 in Baker School has 44 desks in it. If $77 was $87\frac{1}{2}\%$ of the cost of all of them, how much did one desk cost?

154. A bought 640 A. of woodland at $38 per acre. He sold the timber for $19,600, and the land for $13 per acre. Find the per cent of gain.

155. An insurance policy for $2200 cost $17.60. What was the rate of premium?

156. An agent for an oil company sells in three weeks $48,000 worth of oil at a commission of $1\frac{1}{2}\%$. If his expenses are $47.50 per week, how much are his net earnings for that time?

157. $62\frac{1}{2}\%$ of 200 = how many times $12\frac{1}{2}$? $6\frac{1}{4}$?

158. The sum of all the edges of a cube is 36 in. What is the volume of the cube?

CHAPTER VIII

BONDS AND STOCKS

BONDS

1. When a government or a private corporation is in need of money it sometimes borrows it and issues its bonds for the amount. A **Bond** is a promissory note issued under the seal of a government or a corporation.

Mr. A. has a bond issued by the United States government which promises to pay $1000 at a certain time, with interest at 4%. What is his yearly income from this bond?

2. In 1898 when the government wished to raise money to carry on the war with Spain, it issued bonds payable in 20 yr., with interest at 3%. People who wished to lend their money to the government made application for bonds. When their applications were granted, they sent to the Treasury Department of the United States the payment for the bonds, and received in return bonds for that amount. If you had sent $4000 asking for $500 bonds, how many bonds would you have received?

3. In September, 1898, Mr. Ross bought a $500 bond of the issue of 1898 and gave it to his daughter Julia. The bond contained these words:

The United States of America are indebted unto the bearer in the sum of Five Hundred Dollars. This bond is issued under authority of an Act of Congress entitled "An Act to provide ways and means to meet war expenditure," and is redeemable at the pleasure of the United States after the first day of August, 1908, and payable August 1, 1918, in coin with interest at the rate of three per centum per annum payable quarterly in coin on the first day of November, February, May, and August in each year. The principal and interest are exempt from all taxes or duties of the United States as well as from taxation in any form by or under State, municipal, or local authority.

When did the first payment of interest become due? How much was paid?

4. If Miss Ross keeps her bond until it becomes payable, how much interest will she receive? How many interest payments?

5. Mr. Ross has 85 $100 bonds, 375 $200 bonds, and 875 $500 bonds, all of the issue of 1898. What is his income each quarter from these bonds? What is the yearly income?

6. Many persons wish to invest their money in United States bonds. Can you see why?

7. As there was a great demand for the 1898 bonds, and the issue was limited, many people who applied for bonds could not get all they wanted from the government. They therefore tried to buy them from the holders, and the bonds soon rose in price. They were then said to be above par, or at a premium. When bonds are at a premium of 4%, a $100 bond costs $104. When bonds are at a premium of 4%, how much will 30 $100 bonds cost?

8. At 104, or at a premium of 4%, how many $100 bonds can be bought for $936? For $2600?

9. How much will 75 $100 bonds cost at 105?

10. Find the cost of 30 $500 bonds at 2% premium.

11. When bonds are offered at a lower price than their face value they are said to be below par, or at a discount. When bonds are at a discount of 2%, one $100 bond can be bought for $98. At 2% discount, how much would 50 $100 bonds cost?

12. At 98 how much must be paid for 20 $100 bonds?

13. At 98 how many $100 bonds can be bought for $1274? For $1666?

14. September 1, 1898, Mr. Kane bought at par a $5000 bond of the issue of 1898. He held it until March 1, 1899, and sold it at a premium of 7%. How much did he gain?

Observe that Mr. Kane would receive the interest that fell due while he held the bond.

In the following problems the value of a bond is $100, unless otherwise stated.

15. What is the value of 183 bonds of a city corporation at 92%? If the purchaser holds them long enough to realize two 4% interest payments on them, and then sells them at 90%, how much does he gain by the transaction?

16. In order to build a court house the county of Accalama issued 2500 bonds bearing 5% interest. One half of them were sold at par, and the rest at 102. How much money did the county receive for its bonds?

17. Mr. Harvey bought $15,000 worth of the Accalama bonds at par, and held them seven years, receiving his interest annually. At the end of the eighth year the county refused to pay the interest on them. The bonds fell to 70%, at which price Mr. Harvey sold them to Mr. Norton. Did Mr. Harvey gain or lose, and how much?

18. The next month after Mr. Norton bought the bonds, the county of Accalama redeemed them at 95%. How much did Mr. Norton gain by the transaction?

19. In order to raise the money to redeem the 5% bonds, the county of Accalama issued the same amount of bonds bearing 3% interest. The new bonds sold at 93. Mr. Norton bought 150 bonds at this price, held them 8 yr., and sold them at 97%. How much did he gain on this investment?

20. How much interest did the county save each year by substituting 3% bonds for 5% bonds?

21. Persons who buy and sell bonds for others are called brokers. They are paid a certain per cent on the par value of the bonds bought and sold. This percentage is called **Brokerage.**

If you were to pay a broker $\frac{1}{8}\%$ for buying 40 $100 bonds for you, how much brokerage would you pay? If the bonds were at par, how much would they cost you, including brokerage? If the bonds were below par would the brokerage be less?

22. Imagine yourself a broker receiving $\frac{1}{8}\%$ for buying or selling bonds for others. If you were to sell 70 $100 bonds and to buy 90 $500 bonds, how much brokerage should you receive?

23. How much brokerage should you receive if you sold 30 $100 bonds, 40 $1000 bonds, and a $5000 bond?

24. A broker sold 20 United States $100 bonds, 300 Cass County bonds, par value $50, and 200 City Improvement bonds, par value $25 each. His brokerage was $\frac{1}{8}\%$. To how much did it amount?

25. A broker bought for a client 40 railroad bonds at 87, and 40 United States bonds at 109. His brokerage was $\frac{1}{8}\%$. From which transaction did he receive the more brokerage, and how much more?

Observe that brokerage, premium, discount, and interest are all computed on the par value.

26. A broker bought for Mr. X. 20 bonds at 103, charging $\frac{1}{8}\%$ brokerage. How much did the bonds cost Mr. X.?

Since the market value of each bond was $103, and the brokerage on each bond was $\frac{1}{8}$ of a dollar, the purchasing price of each bond was $103\frac{1}{8}$. 20 bonds would cost 20 times $103\frac{1}{8}$, or $2062.50.

27. How much must be paid for 80 railroad bonds quoted at 77, brokerage $\frac{1}{8}\%$?

28. Through his broker Mr. S. invested $2619 in bonds at 109, paying $\frac{1}{8}\%$ brokerage. How many of these bonds did he buy?

29. Buying United States 4's at $111\frac{1}{2}$, and paying $\frac{1}{8}\%$ brokerage, Mr. S. invested $10,046.25. How many bonds did he receive? What was his annual income from them?

The expression "United States 4's" means United States bonds paying 4% interest.

30. If brokerage is $\frac{1}{8}\%$, how much money would be needed to make the following investments?

a 80 United States bonds at 105.

b 70 A. and X. Railroad bonds at 72.

c 19 Memphis bonds at 89.

31. Mr. X. owned 20 bonds. When they were quoted at 103, a broker sold them for him, charging him $\frac{1}{8}\%$. How much did he receive for the bonds?

In this case shall the brokerage be added to the market price of the bond, or subtracted from it? Why?

32. Mr. B. had 90 bonds issued by the M. and Q. R. R. Co. When they were quoted at 79, his broker sold them for him. How much did he receive, brokerage being $\frac{1}{8}\%$?

33. Mr. N. ordered the purchase of 90 shares at $87\frac{1}{2}$. When they had fallen to $86\frac{3}{4}$, he ordered their sale. Brokerage being $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling, how much did he lose?

34. Mr. James obtains 30 bonds quoted at 85, paying a broker $\frac{1}{8}\%$ for buying them. The same broker sells them for Mr. James at $86\frac{1}{2}$. Brokerage $\frac{1}{8}\%$. How much does Mr. James gain, and how much does the broker receive for his work?

35. Mr. A. bought 20 United States 4's at 108. How much did they cost him? How much interest did he receive from them each year?

36. Mr. A. invested \$7800 in bonds which were selling at 104. How many bonds did he buy? If they paid 3%, what yearly income would he receive from them?

37. When United States 4's were selling at $111\frac{1}{2}$ Mr. A. invested \$10,704 in them. How many bonds did he buy, and what income did he receive from them?

38. Find what yearly income can be derived from the following investments:

a $8240 invested in 5% bonds at 103.

b $6755 invested in 3% bonds at 96½.

c $1584 invested in 5% bonds at 99.

d $20,400 invested in 4% bonds at 102.

STOCKS

39. Across a certain river in Ohio there was formerly a toll bridge on which the fare for foot passengers was 15¢. John Smart, a schoolboy, thought it would be a profitable scheme to buy a $10 skiff, and to carry passengers across the river during his vacation, for 10¢ each. As he had not enough money to pay for the skiff, he formed a plan similar to a business enterprise called a stock company, in which his father was interested. John induced several of his friends to join him in buying the skiff. They agreed that John should row the passengers across the river every day in the week, except Sunday, and should retain in return for his labor 50¢ a day from the gross receipts. The rest of the money received was to be divided among the owners of the skiff in proportion to the amount each had invested. The first week 55 passengers were carried across. After deducting John's salary, how much remained to be divided among the owners of the boat? What per cent was that of the whole capital?

40. At that rate how much was received by Albert Blake, who had put in $5? By Edgar Howe, whose share of the capital was $3? By Fred Lee, whose investment was $1?

41. John had put in $1. How much did he receive from his investment and his salary?

42. Find the per cent of gain on capital, and the amount received by each boy at the end of the week in which 32 passengers were carried across. Of the week in which 69 passengers were carried across.

43. Through John's carelessness the boat was overturned one day, and the skiff route became unpopular. The receipts

for the week in which the accident occurred were only $2. How much did each boy have to contribute to pay John's salary of 50¢ a day?

44. The next week they sold the skiff for 60% of what it cost. What per cent of his investment should each boy receive from the sale? How much money?

SUGGESTION TO TEACHER. Before taking up the study of stocks each pupil should comprehend fully the principles involved in the problems about John and the skiff. Let pupils make similar problems by imagining different happenings to John and his companions.

45. As large business enterprises require more capital than is usually owned by one man, it is common for many persons to unite and form what is called a **Stock Company.** The money with which the company carries on business is called its **Capital.** Each member of the company is called a **Stockholder.**

That part of the earnings of a company which is divided among the stockholders is called a **Dividend.** Dividends are computed at a certain per cent on the par value of the capital.

The par value of A's stock is $5000. Find his dividend at 6%.

46. A company whose capital stock is $500,000 distributes $20,000 in dividends. What is the rate of dividend? What is the rate of dividend when it distributes $30,000?

47. Mr. Smart is a stockholder in a stock company called the Ohio Transportation Co., which runs a line of steamers. The capital stock is $100,000, and it is divided into 1000 shares of $100 each. Mr. Smart owns 10 shares; Mr. Howe, 20 shares; Mr. Blake, 500 shares; Mr. Lee, 70 shares; and the remaining shares are owned by others. In the first year of its existence the earnings of the company, after paying all the expenses, were $7000. What per cent of the capital were the earnings? How much should Mr. Howe receive? Mr. Blake? Mr. Lee?

48. Mr. Smart is the superintendent of the line of steamers, and receives a salary of $5000 a year. What was his income in the first year from his stock and his salary?

49. The next year the Ohio Transportation Co. paid a dividend of 9%. How much was received by each of the four men mentioned in Ex. 47?

50. How much would be received by each of those gentlemen if the dividend were 12%? 5%? 7½%?

51. When a company pays a large dividend, there arises a demand for its stock, and its shares sell above par, or at a premium.

When the stock of the Ohio Transportation Co. reached 104, Mr. Blake sold 400 shares of it. How much did he gain by the sale?

52. The next year the dividends of the Ohio Transportation Co. fell to 2%. How much dividend was received by Mr. Howe? Mr. Blake? Mr. Lee?

53. How much did Mr. Smart receive from the company that year?

54. The next year, through some unfortunate management, the company was unable to pay dividends. Instead, an assessment of 5% was made upon each share, in order to pay the running expenses of the business. How much was paid by Mr. Howe? Mr. Blake? Mr. Lee?

55. Mr. Smart's salary was lowered 10%. How much did he receive from the company that year?

56. The price of the shares of the Ohio Transportation Co. had fallen to 67. Mr. Blake bought 600 shares at that price. Mr. Smart, Mr. Howe, and Mr. Lee bought 50 shares each at the same price. At the end of the year the dividend was 10%. What was each man's dividend on all his stock?

Remember that the dividend is always reckoned on the par value — whatever may be the quotation in the market.

57. March 1, 1895, Mr. Reed bought 40 shares of A. & B. R. R. stock at 87. The shares paid a semiannual dividend of $3\frac{1}{2}\%$. He sold them March 1, 1896, at 88. How much did he receive in dividends? What was his profit from the advance in price?

In the following problems the par value of a share of stock is assumed to be $100.

58. If you were to receive 100 shares in a mining company which pays an average semiannual dividend of 4%, what would be your yearly income from those shares?

Suggestion to Teacher. Get blank certificates of stock. Let pupils form themselves into an imaginary stock company.

59. How much must be paid for 90 shares of A. & C. R. R. stock at $97\frac{1}{2}$? If these shares pay a semiannual dividend of $4\frac{1}{2}\%$, how much yearly income will be derived from them?

60. The A. & B. Belting Co., whose capital stock was $500,000, distributed $50,000 among its stockholders. What was the rate of dividend? How much was received by Mr. Smith, who owned 37 shares?

61. At the time the above dividend was declared, banks were paying 2% interest on long time deposits. Would the shares of the A. & B. Belting Co. be likely to be at par, at a premium, or at a discount?

62. How much must be paid for 70 shares of the A. & B. Belting Co., at 105?

63. Mr. A. bought 900 shares of the Unity Coal Mine at 47. He held them until he had received two semiannual dividends of 3%, two of $3\frac{1}{2}\%$, and three of 4%. He then sold the shares at 71. How much did he gain, including the dividends?

64. How much must be paid for 315 shares of the W. U. Telegraph stock at $137\frac{1}{2}$? If these shares yield a semiannual dividend of $11\frac{1}{2}\%$, what is the annual income from them?

65. How many shares of N. & St. L. stock quoted at 93 can be bought for $7440? If these shares pay an annual dividend of 5%, what is the annual income from them?

SUGGESTION. Since one share costs $93, how many shares will $7440 buy? On what is the dividend reckoned? What is the par value of the stock?

66. How many shares at 95 can be bought for $7600? What is the annual income from them if they pay 6% dividend?

67. Find annual income from $7254 invested in D. & H. R. R. stock at 78, the semiannual dividend being 4%?

68. The K. & X. R. R. Co. paid a dividend of 8% in January, and another of 7½% in July. What was the yearly income of a stockholder who owned 750 shares?

Find annual income from the following:

69. $22,464 invested in stocks at 108, which pay 12%.

70. $6965 invested in stocks at 99½, which pay 7%.

71. $12,390 invested in stocks at 88½, which pay 3½%.

72. Mr. A. bought a share of stock at 80, which paid 8%. What per cent did he gain on his investment?

What amount of dividend did Mr. A. receive? An $8 dividend is what per cent of an $80 investment?

Find what per cent is gained annually on the following investments:

73. Stocks bought at 50, paying 2%.

74. Stocks bought at 70, paying 3½%.

75. Bonds bought at 102½, paying 3%.

76. Stocks bought at 87½, paying 7%.

77. Stocks bought at 41¾, paying 2½%.

78. In the case of a person who does not have to pay brokerage, which pays the better per cent, and how much, 4% bonds at par or 6% shares at 90?

79. Mr. R. bought 75 U. S. 4's at 110 and 75 shares C. & L. R. R. stock at 90 without brokerage. The stocks paid 5% dividend. The bonds cost how much more than the stocks? How much more income did he receive from his stocks than from his bonds? What per cent did he make on each investment?

Brokerage being $\frac{1}{8}\%$ for buying and the same for selling, how much is gained or lost on:

80. 90 shares bought at 70, sold at 83?

81. 113 shares bought at 64, sold at $59\frac{1}{2}$?

82. 27 shares bought at 58, sold at $57\frac{3}{4}$?

83. 800 shares bought at $41\frac{3}{4}$, sold at $52\frac{1}{2}$?

84. 70 shares bought at $112\frac{1}{2}$, sold at 113?

85. 900 shares bought at 102, sold at $101\frac{3}{4}$?

MISCELLANEOUS EXERCISES

1. What is the largest prime number that can be expressed by three figures?

2. Resolve 54 into prime factors. What per cent of them are 3's?

3. If $\frac{4}{7}$ of the price of a ship is \$ 12,000, how much is the whole ship worth?

4. A man owning $\frac{5}{7}$ of an estate sells $\frac{2}{5}$ of his share for \$ 2400. At this rate, how much is the estate worth?

5. Jane is 8 yr. old, and Lucy 13. The sum of Jane's and Lucy's ages less 7 yr., is the age of Mary. How old is Mary?

6. A farmer had two fields of wheat; the first yielded 840 bu., which was $\frac{5}{12}$ of the amount yielded by the second. How many bushels did he get from both fields?

7. A man bought a firkin of butter for $ 17, a crock of lard for $ 8, and a barrel of flour for $ 9. To pay for them he needed $ 7.50 more than he had. How much money had he?

8. By what must 1.7 be multiplied to make 5.95? 6.46?

9. John rode $7\frac{7}{8}$ miles on his bicycle in one hour, $6\frac{7}{10}$ in the next hour, and $6\frac{4}{5}$ in the next. How far did he ride in all? How much farther in the first hour than in the second? Than in the third?

10. Harry walked 7.64 miles, and James walked twice as far. How far did they both walk?

11. A merchant bought a barrel of sugar for $ 28.50, and a barrel of flour for $ 7.50. He sold the two for $ 40. What per cent did he gain?

12. How many millimeters in the circumference of a circle whose diameter is 7 centimeters?

13. The circumference of a wheel is 2.6 m. How many times will it revolve in rolling 33.8 m.?

14. How many square decimeters in the surface of a stere?

15. How many steres of wood in a pile 17 m. long, 8 m. wide, and 2 m. high?

16. Find the cost of digging a cellar 7 m. long, 5 m. wide, and 2 m. deep, at 20 ¢ a stere.

17. Image a cubic centimeter of water. How much does it weigh? $5\frac{1}{2}$ liters of water weigh how many grams? Kilograms?

18. How many kilograms will 7 liters of alcohol weigh if alcohol is $\frac{4}{5}$ as heavy as water?

19. If ice weighs 94% as much as water, how many kilograms of ice are there in a block of ice 9 dm. long, 5 dm. wide, and 4 dm. high?

20. How many kilograms do 2 liters of mercury weigh, mercury being 13.5 times as heavy as water?

21. The product of two numbers is $\frac{5}{8}$. One of the numbers is $2\frac{1}{2}$. What is the other number?

22. A landowner divided $7\frac{1}{2}$ A. of land into city lots 55 ft. in front and 132 ft. deep, first taking out for streets and alleys 108,900 sq. ft. How many lots were there? He sold them at an average of $40 a front foot. The land had cost him $100 an acre 20 yr. before. He had paid an average of $300 a year in taxes upon it, and the expense of platting and selling it was $315. How much did he gain by holding the land?

23. How many bricks 8 in. by 4 in. will be required to pave a yard 168 ft. long and 60 ft. wide?

24. If Mr. A. were to lose $33\frac{1}{3}\%$ of his money, he would have $2000 left. How much money has he?

25. What is the value of a pile of wood 30 ft. long, 8 ft. wide, and 4 ft. high at $3.75 a cord?

26. $\frac{1}{2}$ of 8 is what per cent of $\frac{1}{2}$ of 20?

27. Mrs. A. has at interest $800 at 6% and $1000 at 5%. What is her yearly income from both investments?

28. If a piano which cost $260 is sold at $325, what per cent is gained?

29. What is the interest of $4270 from May 1, 1895, to Aug. 1, 1901, at 5%?

30. Mr. A. borrowed $7000 at 5%. At the end of each of the first two years he paid $1000. At the end of the third year he paid all that was due. How much did he pay?

31. A note for $782.50 payable in 60 da. with grace was discounted at 6%. What were the proceeds?

32. Mr. O. failed in business, owing $60,000 and having $30,000 with which to pay. What per cent of the amount could he pay? How much would a creditor receive to whom he owed $1800? A creditor to whom he owed $2456.65?

33. If by selling fruit at 9¢ a pound a grocer gains 12½%, how much will he gain by selling it at 11¢ a pound?

34. What per cent would a jeweler gain by selling a watch at $80, if by selling it at $75 he gains 50%?

35. A merchant bought goods at $1.60, marked them to sell at an advance of 37½%, and sold them at a reduction of 25% on the marked price. At what price were they sold and what per cent was gained on them?

36. Goods costing $864 were marked at an advance of 50% and sold at a discount of 16⅔% from the marked price. How much was gained on them? What per cent?

37. Find the net cost of a bill of goods amounting to $375.50 with discounts of 60%, 40%, and 5%.

38. Make a problem which involves trade discount.

39. An agent sold 250 bbl. of flour at $3.80 per barrel, commission 3%. What was his commission, and how much was sent to the owner of the flour?

40. A steamer valued at $750,000 was insured for ⅔ of its value at 1½%, in two companies, one company taking ¼ of the risk and the other the remainder. What was the amount of premium for each company?

41. Mr. A. has real estate assessed at $20,000 and personal property to the amount of $8224. He pays a poll tax of $1.50. What is the amount of his taxes when the rate of taxation is 37½ mills on a dollar?

42. What is the duty on 20 casks of wine, each cask containing 56 gal., invoiced at $2.35 per gallon, if 12½% is allowed for leakage and if there is an ad valorem duty of 45%?

43. Find the cost of 80 shares N. Y. C. R. R. stock at 112, brokerage, ⅛%.

44. How much must one pay for 65 U. S. 4's at 108, brokerage, ⅛%?

45. How much would a broker receive from selling 30 shares of mining stock at 91, brokerage, $\frac{1}{8}\%$?

46. How much does the owner receive from the sale of 95 shares of stock sold at 103, brokerage, $\frac{1}{8}\%$?

Brokerage being $\frac{1}{8}\%$, for buying and for selling, how much is gained or lost by

47. Buying 72 shares at 89 and selling them at 90?

48. Buying 40 shares at $71\frac{1}{2}$ and selling them at 70?

49. Buying 35 shares at 59 and selling them at $58\frac{1}{2}$?

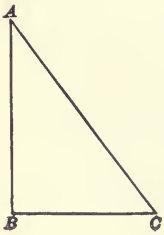
50. At $57\frac{1}{8}$ how many shares of stock can be bought for \$7156.25, brokerage, $\frac{1}{8}\%$? If they pay a semiannual dividend of 4%, what income is derived from them?

51. What income is derived from \$15,300 invested in stocks at $95\frac{1}{2}$, brokerage, $\frac{1}{8}\%$, if the stocks pay a semiannual dividend of $3\frac{1}{2}\%$?

52. A blacksmith's price for shoeing a horse was 50¢ a shoe, but he allowed a discount of 10% to any person bringing him at one time horses enough to require 10 or more shoes. Mr. Boyd's horses were shod by the blacksmith one day at a cost of \$13.50. How many shoes were put on?

53. A room 18 ft. by 15 ft. was covered with matting 1 yd. wide, at a cost of \$10.50. To lay the matting cost 5¢ a yard. What was the price per yard of the matting?

54. The Troy Edge Tool Works sold 12 doz. sledge hammers, weighing 5 lb. each, at 10¢ per lb., and 9 doz. hammers at 50¢ apiece. Discount, 45%. Make out the bill.



55. A triangle whose sides are all unequal is called a **Scalene Triangle.**

Draw a right triangle whose base is 3 in. and altitude, 4 in. Is it scalene? Give reasons for your answer.

56. Construct an equilateral triangle. An isosceles triangle. A scalene triangle.

57. Find the perimeter of a scalene triangle, of which one side is $4\frac{4}{5}$ in., another side is $6\frac{2}{3}$ in., and the third side is $8\frac{5}{6}$ in.

58. Find perimeter of the scalene triangle of which the side AB is 5 in., BC is $2\frac{1}{2}$ in. longer than AB, and CA is $3\frac{1}{2}$ in. longer than BC. Represent.

59. How long is the perimeter of the triangle ABC, when AB is 12 in., BC is $33\frac{1}{3}\%$ longer than AB, and CA is 25% longer than BC?

60. Tell how a line is bisected. In the same way bisect an arc.

61. If the base of an isosceles triangle is 16 in., and if each of the other sides equals $87\frac{1}{2}\%$ of the base, how long is the perimeter?

62. At $\$3\frac{1}{2}$ per day for board, how many days can a man board at the seashore for \$28?

63. A man had three lots, each containing $6\frac{1}{4}$ A., which he redivided into building lots of $\frac{5}{8}$ of an acre each. How many building lots did he have?

64. Mrs. A. wishes to cover the floor of a room 16 ft. long and 12 ft. wide with ingrain carpet 1 yd. wide. The carpet will cut to the best advantage if the strips are laid lengthwise. One pattern requires that the shortest possible strips shall be 16 ft. 8 in. long. This carpet costs 73¢ per yard. Another pattern requires that the strips shall be only 16 ft. 2 in. long, but it costs 75¢ a yard. What is the difference in the cost of the carpets?

65. How much more will it cost to cover a floor 21 ft. long and 18 ft. wide with Brussels carpet 27 in. wide at \$1.25 per yard than with a yard wide ingrain at 85¢, if the Brussels requires only 1 in. to be turned in at the end of the strips and the ingrain requires 6 in.? The strips run lengthwise.

CHAPTER IX

LITERAL QUANTITIES

1. If $x = 10$, how many days in x weeks? How many minutes in x hours? How many cents in x dollars?

2. If $x = 48$, how many yards in x ft.? Years in x mo.? Pounds in x oz.? Pecks in x qt.? Gallons in x qt.?

3. If b represents 3, how much is 3 times b or $3b$?

4. If $a = 8$, how much is $3a$? $\frac{1}{4}a$ or $\frac{a}{4}$? $2\frac{1}{4}$ times a?

5. If $x = 18$, how much is $.5x$? $.7x$? $\frac{5}{6}x$? $33\frac{1}{3}\%$ of x?

6. If $a = 12$, how much is 25% of a? $16\frac{2}{3}\%$ of a? $12\frac{1}{2}\%$ of a?

7. When $x = 20$, how much is $\frac{x}{10}$? $\frac{x}{30}$? $\frac{x}{5}$? $\frac{3x}{5}$?

8. x dollars $+ y$ dollars = how many dollars, when $x = 8$ and $y = 7$.

9. If $a = 10$ and $b = 3$, how much is $a + b$? $3a + 5b$? $5a - 4b$? $2a + 3b$? $6b - a$? $8b - 2a$? $a \div b$? $b \div a$?

10. Give some values to x and y that will make the following equations true: $x + y = 19$. $x + y = 15$. $x - y = 1$.

11. If an orange costs 3¢, how much will x oranges cost, when $x = 10$? When $x = 4$?

12. How many oranges in x doz. when $x = 7$?

13. If $x = 25$, how many weeks in the number of days represented by $x + 3$? $x + 10$? $x - 4$? $2x - 1$?

14. If we let x stand for 35, how many gallons are there in the number of quarts that are represented by $x+1$?

15. When x represents 14, how many feet in the number of yards represented by $x+1$? $x+3$?

The expression ab means a times b, just as $4\,b$ means 4 times b.

16. If $a=7$ and $b=5$, how much is ab? Which is the greater, 7 times 5 or 5 times 7? a times b or b times a?

17. If $a=11$ and $b=2$, how much is $3\,ab$? $3\,ba$?

18. How much will x apples cost at $y\,¢$ a piece, if $x=4$ and $y=2$? If $x=8$ and $y=3$?

19. If we represent the cost of one apple by y and a number of apples by x, how shall we represent the cost of them all?

20. Give some values to x and y that will make the following equations true. $xy=48$. $xy=35$. $xy=70$. $xy=98$.

21. When $x=50$ and $y=2$, how many oranges can be bought for $x\,¢$, if one orange costs $y\,¢$? How many when $x=30$ and $y=5$?

22. If we represent the cost of one orange by y and the cost of a number of oranges by x, how shall we represent the number of oranges?

23. If we represent the cost of one orange by x and the cost of a number of oranges by y, how shall we represent the number of oranges?

24. Give some values to x and y that will make the following equations true:

$$\frac{x}{y}=7. \qquad \frac{y}{x}=4. \qquad \frac{x}{y}=6. \qquad \frac{y}{x}=9. \qquad \frac{x}{x}=3.$$

25. $3(5+4)=?$

This expression means "3 times the sum of 5 and 4." Quantities inclosed in a parenthesis are to be considered as one quantity.

26. $2(7+2)=?$ $2(7-2)=?$ $8(6+4)=?$ $\frac{(7+3)}{5}=?$ $\frac{(8+4)}{6}=?$ $(9-3)^2=?$ $(8-5)^2=?$

27. If $a=7$ and $b=2$, how much is $2(a+b)$? $(a+b)^2$? $(a-b)^2$? $2(a-b)$? $\frac{(a+b)}{3}$? $\frac{(a-b)}{5}$?

28. When $a=4$ and $b=3$, how much is $5(a+b)$?
$2(a-b)$? $a(a+b)$? $b(a+b)$? $(a+b)^2$? $(a-b)^2$?

29. When $t=10$ and $u=3$, how much is
$3t+5u$? $u(2t+u)$? $t^2+2tu+u^2$? $(t+u)^2$?

30. If x is 3, then $9x-5x=$ what number?

Finish the following equations, supposing x to equal 7:

31. $2x+3x=$ 33. $25-3x=$ 35. $36+2x=$
32. $5x-3x=$ 34. $4x+11x=$ 36. $44-5x=$

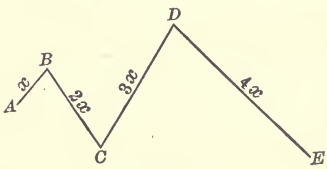


FIG. 1.

37. In the broken line $ABCDE$ the part BC is twice as long as AB, CD is 3 times as long as AB, and DE is 4 times as long as AB. How long is the entire line if x represents 7 in.? 2 ft.? 5 yd.?

38. Draw a broken line consisting of two parts in which one part is 4 times as long as the other. Let x stand for the length of the smaller part. What will represent the length of the other part? How long is the entire line if $x=2$ in.? 8 in.?

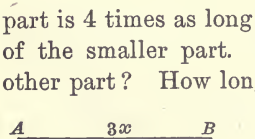


FIG. 2.

39. How long is the perimeter of the rhomboid $ABCD$ if $x=5$ in.? If $x=1\frac{1}{2}$ ft.?

40. Draw a rhomboid, making a longer

side twice as long as either of its adjacent sides. Mark a short side x and the other sides accordingly. How long would the perimeter of the rhomboid be if $x = 8$ in.? 1.1 in.? $2\frac{1}{2}$ in.?

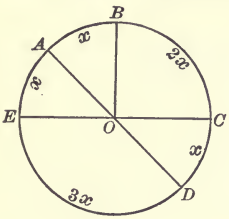


FIG. 3.

41. In the circle whose center is O, the arcs are in the ratios represented in Fig. 3. How long is the circumference if $x = 7$ in.? $3\frac{1}{2}$ ft.?

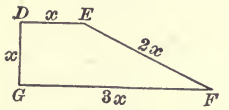


FIG. 4.

42. How long is the perimeter of the trapezoid $DEFG$, if $x = 10$ in.? $3\frac{6}{7}$ in.?

43. $8a - 6a + 3a =$ how many a's?

$8a - 6a = 2a$; $2a + 3a = 5a$.

Express in one term:

44. $9b - 7b + 4b$.

45. $17c + 2c - 8c$.

46. $16x - 4x + 8x - 5x$.

47. $21x + 2x - 5x - 13x$.

48. $40y - 13y + 8y - 7y$.

49. $24x + 13x - 2x + 8x$.

50. $48x - 3x - 11x - 20x$.

51. $39x - 14x + 11x + 13x$.

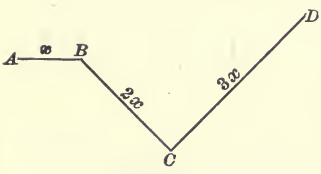


FIG. 5.

52. In the broken line, $ABCD$, the part BC is twice as long as AB, and CD is 3 times as long as AB. The entire line is how many times as long as AB?

If we know the length of the entire line, we may find by equations the length of each part. If the entire line is 12 in. long, we have,

$$x + 2x + 3x = 12 \text{ in.}$$

then $6x = 12$ in.

and $x = 2$ in., length of AB.

and $2x = 4$ in., length of BC.

and $3x = 6$ in., length of CD.

53. Find the length of each part of the broken line represented in Fig. 5, if that line is 30 in. long. 54 in. long.

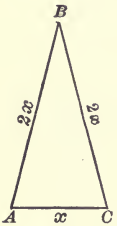


FIG. 6.

54. In the isosceles triangle, *ABC*, each of the equal sides is twice the base. The perimeter is 45 in. How long is each side?

55. Construct an isosceles triangle, in which each of the equal sides is 3 times the base. Let $x =$ the base, and find how long each side would be if the perimeter were 56 in. 105 in. 147 in.

56. In the trapezoid, *ABCD*, the non-parallel sides are equal, the upper base is 3 times as long as either of its adjacent sides, and the lower base is 4 times as long as either of its adjacent sides. How long is each side, if the perimeter is 45 in.? 153 in.?

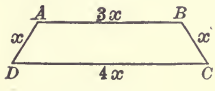


FIG. 7.

57. Turn to Fig. 4, page 299, and find the length of each side of the trapezoid, if its perimeter is 42 in. 9 ft. 4 in.

58. In the trapezium, *ABCD*, the sides have to one another the ratios expressed in Fig. 8. How long would each side be if the perimeter were 80 in.? 75 in.?

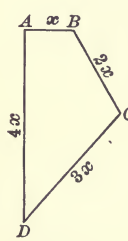


FIG. 8.

59. Mr. Morton spent some money on Monday, 3 times as much on Tuesday, and 5 times as much on Wednesday. If he spent $36 in all, how much did he spend each day?

Let $x =$ the number of dollars spent on Monday.

60. John has 4 times as many marbles as James, and they both have 75 marbles. How many has each?

61. The sum of two numbers is 21, and one of them is 6 times the other. What are the numbers?

62. Ella's mother is 3 times as old as Ella. Her father is 4 times as old as Ella. The sum of all their ages is 96 years. How old is each?

63. I am thinking of two numbers, one of which is 5 times the other. Their sum is 18. Find the numbers.

64. Class Exercise. —— may think of two numbers, one of which is a multiple of the other. He may give the sum and the ratio of these numbers and the class may find them.

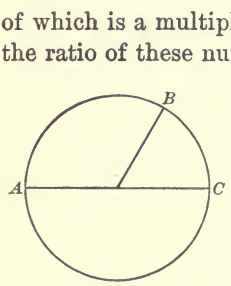


Fig. 9.

65. The circumference represented is 54 ft. The arc AB is twice the arc BC, and the arc CA is three times the arc BC. How long is each arc?

66. Turn to Fig. 3, page 299. Find the length of each arc when the circumference is 96 ft. When it is 8 ft. 4 in.

67. Ida set out a number of geraniums, twice as many roses as geraniums, and three times as many pansies as roses. There were 27 plants in all. How many were there of each?

Let x = the number of geraniums,

then $2x$ = " " " roses,

and $6x$ = " " " pansies.

$$x + 2x + 6x = 27.$$

68. There are three numbers whose sum is 80. The second is 3 times the first, and the last is 4 times the second. Find them.

69. Ida, Frank, and Henry paid $ 10 to have a tennis court prepared. Frank gave three times as much as Ida, and Henry gave twice as much as Frank. How much did each give?

70. On the day that Ruth Owen was 22 years old she received a bunch of roses consisting of one rose for each year of her life. There were twice as many pink roses as red roses,

and four times as many white roses as pink roses. How many were there of each?

71. Make similar problems.

72. A certain number plus itself equals 320. What is the number

$$x + x = 320.$$

73. Find the number which added to itself equals 258. 237.

74. A certain number plus twice itself equals 396. Find the number.

75. Find the number which added to twice itself equals 297.

76. Find the number which added to four times itself equals 195. 275. 300. 177.

77. Separate 18 into two parts, one of which is 8 times the other.

$$x + 8x = 18.$$

78. Separate 30 into two parts, one of which is 5 times the other. One of which is 9 times the other. One of which is 14 times the other.

79. Separate 24 into two parts, one of which is twice the other. 3 times the other. 5 times the other.

80. Class Exercise. —— may name a number which he can separate into two parts whose ratio is a whole number. He may give the ratio of those parts. The class may find the parts.

81. Two brothers, Messrs. Arthur and Philip Owen, paid $ 18,000 for a piece of land. Mr. Arthur Owen paid 5 times as much as his brother. How much did each pay?

82. What number is that to which, if 4 times itself, and 6 times itself be added the sum is 77?

83. A farmer sold a cow and a pig for $ 30, receiving 9 times as much for the cow as for the pig. What was the price of each?

84. Separate 60 into three parts such that the second is 4 times the first, and the third is 5 times the first.

85. What number added to six times itself equals 147?

86. Albert has 3 times as many marbles as James. Roy has as many marbles as both the other boys have. They all have 72 marbles. How many has each?

87. Thomas caught 3 fish. The largest fish weighed as much as the other two. One of those weighed twice the other. The weight of all was 6 lb. How much did each weigh?

88. One hundred can be separated into 3 integers, of which the second is 4 times the first, and the third is equal to the sum of the first and second. What are the numbers?

$$x + 4x + (x + 4x) = 100.$$

89. In the same way separate 120. 150. 600.

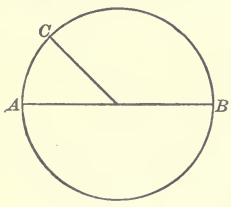


FIG. 10.

90. AB is a diameter. The arc BC is 3 times the arc AC. The arc BA is 72 in. How long is the arc AC? BC? The circumference?

91. If the circumference of the circle in Fig. 10 were 112 in., how long would be the arc AC? AB?

92. Seven times a certain number, minus three times that number, equals 24. What is the number?

Let x = the number,

then $7x - 3x = 24.$

93. Six times a certain number − 4 times that number = 10. What is the number?

94. If a certain number is multiplied by 7, and if the same number is also multiplied by 5, the difference between those products is 16. Find the number.

95. The difference between the 8th multiple and the 5th multiple of a certain number is 21. Find the number.

96. Class Exercise. —— may think of a number and of two of its multiples. He may tell the class which multiples they are and the difference between them. The class may find the number.

97. John picked 3 times as many quarts of berries as his sister picked. He picked 8 more quarts than she did. How many quarts did each pick?

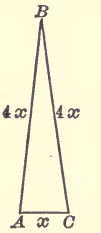


Fig. 11.

98. Mr. Bond drew a sum of money from the bank on Friday, and five times as much on Saturday. He drew $ 128 more on Saturday than on the previous day. How much did he draw each day?

99. The sides of the triangle ABC are in the ratios expressed in Fig. 11. The sum of AB and BC is 77 in. more than AC. How long is each side?

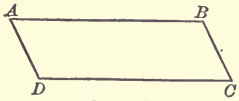


Fig. 12.

100. In the rhomboid $ABCD$, the side AB is three times the side BC, and it is 10 in. longer than BC. How long is the perimeter?

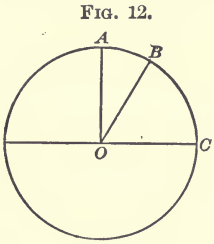


Fig. 13.

101. The arc BC is twice as long as the arc AB, and it is 8 in. longer than AB. How long is AB? BC? AOC is a right angle formed by radii. How long is the circumference? The diameter?



Fig. 14.

102. In the trapezium $ABCD$, $AB = BC$ and $CD = AD$. $AD = 3$ times AB, and it is 6 in. longer than AB. Find the perimeter of the trapezium.

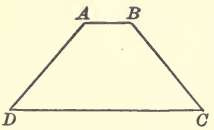


FIG. 15.

103. In the trapezoid $ABCD$, the side DC equals 4 times AB, and is 9 in. longer than AB. If each of the non-parallel sides is 7 in., how long is the perimeter?

104. If John's shoes cost twice as much as his hat, and they both cost \$ 3.60, how much will each cost?

In solving this problem it will be more convenient to let x equal the number of dollars that John's hat costs than to let x equal the number of dollars that his shoes cost. Do you see why?

105. What number subtracted from 3 times itself gives for a remainder 14? 26? 32?

106. Mary has twice as many books as Alice, and together they have 36 books. How many has each?

107. John and William together have eighty marbles, and John has 7 times as many marbles as William. How many has each?

108. Albert walks 3 times as far east from a certain point as John walks west from the same point. They are then 80 ft. apart. How far does each walk?

109. A pole 12 ft. long is sunk in the water so that the part below the surface is 3 times as long as the part above. How much is below the surface?

110. A tree 60 ft. high is broken so that the part which has fallen down is 5 times as long as that which remains standing. Find height of the stump.

111. Mr. Colton had a sum of money at interest at 5% and a sum twice as large at 6%. In all he had \$ 600 at interest. Find how much he had at each rate and his yearly income from both principals.

112. $3x = 8 + 7$. Find the value of x.

113. If we have the equation "$4x + 7x = 52 + 3$," how is the equation "$11x = 55$" obtained from it?

Uniting quantities of the same kind on the same side of the equation is called **collecting the terms.**

Find value of the literal quantity in each of the following:

114. $2x + 5x - 7x + 8x = 60 + 5 + 4 + 3.$

115. $5x + 3x = 9 + 5 + 10.$

116. $6y - 4y = 21 - 6 - 5.$

117. $9x + 2x = 50 - 4 + 20.$

118. $18y - 5y + 7y = 30 - 10 + 24 + 36.$

119. The terms written before the sign of equality form the first or left-hand member of the equation. Those written after the sign of equality form the second or right-hand member of the equation.

How many terms are there in the second member of the equation in Ex. 118? In the left-hand member?

120. The number which shows how many times the literal quantity is taken is called the **Coefficient** of that quantity. In the expression $11x$, 11 is the coefficient.

Supply missing coefficients in the following equations, assuming that $x = 3$. $?x = 24$. $?x = 21$. $x + ?x = 24$.

121. Make an equation containing a literal quantity whose coefficient is 5.

122. If $a = b$, is it true that $a + 7 = b + 7$? Illustrate.

An equation is like a pair of scales and the sign of equality is like the beam of the scales. If a pound is added to one side, what must be added to the other side, in order to keep it balanced? If the amount on one side is doubled, what must be done to keep the balance?

123. Add 7 to both members of the equation $40 = 40$. Is the resulting equation true?

124. If 7 is added to one member of an equation and 9 to the other member, is the resulting statement true? Illustrate.

125. Write an equation. Subtract the same quantity from both members of it and show whether or not the members are still equal.

126. Multiply both members of the equation $8 = 8$ by the same quantity. Are the members still equal?

127. If both members of an equation are divided by the same quantity, how is the equation affected? Illustrate.

128. If $a = 5$, is it true that $a^2 = 25$? What has been done to each member of the original equation?

129. Illustrate by numbers the truth of the statement, "If the same operation is performed upon each member of an equation the members are still equal."

130. Find the value of x when $x - 2 = 6$.

SOLUTION

$x - 2 = 6.$

Adding 2 to each member $x - 2 + 2 = 6 + 2.$

Hence, $x = 6 + 2.$

or $x = 8.$

What was the purpose of adding 2 to the left-hand member? To the right-hand member?

Compare the first and third equations in the above solution. It will be seen that in the first equation "2" is written in the left-hand member and has the minus sign, while in the third equation "2" is in the right-hand member and has the plus sign.

131. Find the value of x when $x + 3 = 12$.

SOLUTION

$x + 3 = 12.$

Subtracting 3 from each member $x + 3 - 3 = 12 - 3.$

Hence, $x = 12 - 3.$

or $x = 9.$

Why should we here subtract, instead of adding 3 to each member?

Compare the first and the third equations. What change has been made in the first to produce the third?

132. Changing a quantity from one side of an equation to the other is called **transposing** the quantity. Illustrate.

133. Study the solutions of Exs. 130 and 131 until you see the truth of the following principle:

A quantity may be transposed from one side of an equation to the other if the sign prefixed to the quantity is changed from plus to minus or from minus to plus.

When no sign is prefixed, the plus sign is understood.

134. In the equation $2x+7=28-x$, if x is transposed what sign will it have? What sign will 7 have if transposed?

What is the purpose of transposing quantities?

It may help you to remember to transpose correctly if you repeat "When I change the side I change the sign."

Find the values of the literal quantities:

135. $x-7=23$.

136. $x-8=21$.

137. $5x-5=50$.

138. $5y-4=35$.

139. $3x+7=25$.

140. $7x-3=67$.

141. $4y+3=51$.

142. $12x+7=67$.

143. Class Exercise. —— may think of some number, call it x, and make an equation like the above for the class to solve.

144. Finding the value of the literal quantity in an equation is called **solving** the equation.

Solve $12x+7=8x+31$.

Transposing, we have $12x-8x=31-7$.

Collecting the terms, we have $4x=24$.

Dividing, we have $x=6$.

Solve the equations:

145. $5x+9=3x+17$.

146. $10x+8=32-2x$.

147. $7y-9=5y+7$.

148. $4y+20=y+29$.

149. $9x-7x+4=x+8$.

150. $6x+7-3x=2x+19$.

151. $15y-21=14y+37$.

152. $15x+9-12x=25-2x$.

153. $15+7x=30-8x$.

154. $23x-24=48-11x$.

155. Complete the equation $4x+?=35$, when $x=8$.

156. Complete the equation $7y+5=30+?$, when $y=9$.

157. If $x=10$, is the equation $5x+7=54$ true?

158. Substituting the value of the unknown quantity in an equation and thus proving the truth of the equation is called **verifying** the equation.

Solve and verify the equation $x+9=15$.

Solve and verify:

159. $x+7=21$.

160. $3x-5=19$.

161. $5x=x+36$.

162. $7x-2x+8=33$.

163. $5x-3x+21=x+34$.

164. $11x+1=x+91$.

165. One of two numbers is 3 more than twice the other. Their sum is 15. What are the numbers?

Let x = the less number,
then $2x+3$ = the greater number,
then $x+2x+3=15$.

166. There are two numbers whose sum is 17. The greater is 2 more than 4 times the less. Find the numbers.

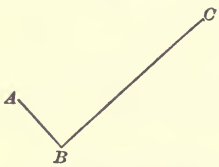


FIG. 16.

167. In the broken line ABC, BC represents a distance which is 7 ft. more than twice AB. If the entire line represents 31 ft., how much does AB represent? BC?

168. Each of the equal sides of an isosceles triangle is 4 ft. longer than the base. The perimeter is 29 ft. How long is each side? Represent.

169. Fred is seven years older than his brother, and the sum of their ages is 23 years. How old is each?

170. Class Exercise. —— may think of the ages of two persons, and tell the class the sum of those ages and the difference between them. The class may find the ages.

171. Mr. Lee has a watch which is worth $20 more than the chain. The watch and chain together are worth $50. How much is each worth?

172. Two boys bought a skiff for $8. The older boy gave $2 more than the younger. How much did each give?

173. A bootblack earned 30¢ more on Tuesday than on Monday. His earnings for the two days were $1.70. How much did he earn on each of these days?

174. A farmer, who had 100 acres of corn and wheat, had 20 acres more of wheat than of corn. How many acres of corn had he? How many acres of wheat?

175. The sum of two numbers is 72. The greater is 8 more than the less. Find the numbers.

176. The sum of two numbers is 90, and the greater is 26 more than twice the less. Find the numbers.

177. A traveled north from the Chicago post office, and B traveled south from that point. When they were 50 mi. apart, A had traveled 10 mi. more than B. How far was each from the Chicago post office?

178. The length of a rectangular lot is 70 ft. more than its width. Its perimeter is 220 ft. Represent and find the length and the area.

179. Edwin went fishing. If he had caught 10 times as many fish as he did catch and 40 fish more, he would have had 100 fish. How many did he catch?

180. Find three numbers such that the first is 10 more than the second, the second is 5 more than the third, and their sum is 47.

Let x = the smallest or third number,

then $x + 5$ = the second number,

and $x + 5 + 10$ = the first number.

$$x + (x + 5) + (x + 5 + 10) = 47.$$

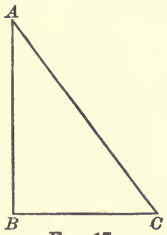


FIG. 17.

181. The perimeter of the triangle which ABC represents is 36 in. The side AB is 3 in. longer than the side BC, and the side AC is 3 in. longer than the side AB. Find length of each side.

Represent the following:

182. In the scalene triangle ABC, the side AB is 12 in. longer than the side AC. The side AC is 8 in. longer than the side BC. The perimeter is 73 in. Find each side.

183. The side XY of the triangle XYZ is 11 in. longer than the side YZ, and the side XZ is 17 in. longer than the side YZ. The perimeter is 88 in. Find each side.

184. In the triangle DEF, the side DE lacks 8 in. of being twice as long as the side EF. The side DF lacks 17 in. of being three times as long as EF. The perimeter is 65 in. Find each side.

185. 4 times a certain number = that number + 21. Find the number.

186. Separate 27 into two parts such that the greater is 9 more than the less.

187. There were four brothers, the sum of whose ages was 32 yr. Each boy was 2 yr. older than his next younger brother. How old was each?

188. If 3 yr. were subtracted from 4 times John's age, the remainder would equal his father's age, which is 45 yr. How old is John?

189. 32 boys voted for the president of their club. John received 6 more votes than the other candidate. How many votes did each candidate receive?

190. Mr. A pays $13 more in taxes than Mr. B. Mr. C pays $7 more than Mr. B. Mr. D pays $8 more than Mr. C. They all pay $69. How much does each pay?

191. An importer received three shipments of goods from Germany. The duty on the second shipment was $3000 more than on the first, and the duty on the last was $2500 more than on the second. The duty on all the shipments amounted to $9500. How much was paid on each?

192. In a game of football the successful team scored 3 times as many points as the other. The difference in the scores was 18. What was the score of each team?

193. Make a problem to be solved by equations.

194. The profits of a farm in 3 yr. were $2300. The profits for the second year were $100 more than for the first year. The profits for the third year were $300 less than for the second year. Find the profits for each year.

195. Mr. Rowe gave 3 notes to a collector, who collected $7 more on the second note than on the first, and on the third $3 less than on the second. The sum of the collections was $40. How much was each?

196. The senior partner in a firm has $20,000 more in the business than the junior partner. The whole capital is $80,000. What is the capital of each partner?

197. Mr. A owes Mrs. B $21 more than he owes Mr. C. Both debts amount to $231. How much is his debt to Mr. C? If he pays $7 a week to Mrs. B, in how many weeks will he have paid his debt to her?

198. A chord AB divides a circumference into two arcs, the greater of which is 30 ft. longer than twice the less. The circumference is 120 ft. How long is each arc?

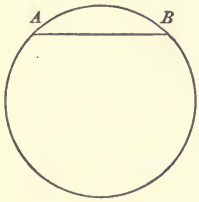


FIG. 18.

199. Multiplying all the terms of an equation by the same number has what effect upon the equation? Illustrate with numbers.

200. The expression $\frac{x}{3}$ means $\frac{1}{3}$ of x; $\frac{2x}{3}$ means $\frac{2}{3}$ of x. If you multiply both terms of the fraction $\frac{x}{3}$ by 3, to what integral expression is the result equal? Why?

201. Multiply by the same number both terms of the equation $\frac{x}{3} = 7$ and solve the equation.

202. Multiplying the terms of a fractional equation by a quantity that causes the terms to become integral is called **clearing the equation of fractions.** How did you clear of fractions the equation in Ex. 201?

203. Solve $\frac{x}{3} + \frac{x}{4} = 7$.

Multiplying all the terms by 3 we have $x + \frac{3x}{4} = 21$. Multiplying all the terms of that equation by 4 we have $4x + 3x = 84$. It would have been a shorter process to multiply all the terms by 12 at once instead of by the 3 and 4 separately. Hence we use the method given below.

204. Solve Ex. 203 by the following rule:

To clear an equation of fractions —

Multiply each term of the equation by the least common multiple of the denominators.

205. Clear of fractions the equation $\frac{x}{6} + \frac{x}{4} = 5$.

Multiplying each term by 12, the l. c. m. of 6 and 4, we have

$\frac{x}{6} \times 12 = 2x \quad \frac{x}{4} \times 12 = 3x \quad 5 \times 12 = 60.$ Hence, $2x + 3x = 60$.

Solve:

206. $\frac{x}{5}+\frac{x}{7}=24$. 207. $\frac{x}{6}+\frac{x}{9}=60$. 208. $\frac{2x}{3}+\frac{4x}{15}=28$.

209. $\frac{5x}{7}+\frac{3x}{5}=19\frac{5}{7}$. 215. $3x+\frac{2x}{3}=85-\frac{2x}{9}$.

210. $2x+\frac{x}{5}+\frac{x}{3}=43\frac{1}{15}$. 216. $3x+\frac{5x}{8}-\frac{4x}{9}=85$.

211. $\frac{x}{8}+\frac{x}{3}-5=\frac{1}{3}$. 217. $\frac{2x}{5}+\frac{7x}{10}=x+5$.

212. $\frac{x}{7}+\frac{x}{2}-\frac{x}{3}=4\frac{1}{3}$. 218. $\frac{x}{5}+4=\frac{x}{7}+6$.

213. $x+\frac{x}{5}=29-\frac{x}{4}$. 219. $\frac{3x}{4}-\frac{2x}{7}=\frac{x}{14}+11$.

214. $2x+\frac{x}{3}+\frac{5x}{6}=47$. 220. $\frac{9x}{7}-4=\frac{8x}{11}+39$.

221. One fifth of a certain number $+\frac{1}{7}$ of it $=24$. Find the number.

222. Find a number such that if $\frac{1}{2}$ of it and $\frac{1}{4}$ of it are added to it, the sum will be 28.

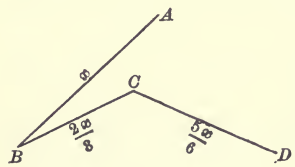


Fig. 19.

223. The parts of the broken line $ABCD$ are in the ratios given in Fig. 19. How long is each part if the entire line is 30 in.? 75 in.? $7\frac{1}{2}$ in.?

224. A circumference which is 1 yd. in length is divided into 2 arcs, one of which is $\frac{4}{5}$ of the other. How long is each arc? Represent.

225. Find a number such that if 15 is subtracted from 3 times the number, the remainder will be $2\frac{1}{2}$ times the original number.

226. The perimeter of a given isosceles triangle is 286 ft., and the base is $\frac{4}{11}$ of one of the equal sides. Find the length of its sides.

227. The perimeter of a rectangle is 1254 ft., and the width is $\frac{7}{12}$ of the length. Find width, length, and area.

228. A certain number $+ 2\frac{1}{2}$ times itself $+ 7 = 37$. Find the number.

229. Draw a right-angled scalene triangle. If the altitude were $1\frac{7}{8}$ times the base, the hypotenuse $2\frac{1}{8}$ times the base, and the perimeter 40 in., how long would each side be?

230. Separate 45 into two parts, one of which is $\frac{7}{8}$ of the other.

Let x = the greater number.

231. 42 is the sum of two numbers whose ratio is $\frac{3}{4}$. What are they?

232. Separate 90 into two parts whose ratio is $\frac{7}{8}$.

233. Class Exercise. —— may give the sum of two numbers whose ratio is a fraction, and the class may find the numbers.

234. How would you divide 75¢ between two boys, giving one boy $\frac{7}{8}$ as much as the other?

235. The session of a certain school is $4\frac{1}{2}$ hr. a day. How many hours and minutes are given to recitation if the recitation periods take $\frac{2}{3}$ as much time as that devoted to other purposes?

236. There are three numbers whose sum is 108. The first is $\frac{4}{5}$ of the second, and the third is twice the first. Find the numbers.

Let x = the second number.

237. $\frac{1}{7}$ of a certain number minus $\frac{1}{3}$ of it $= 2$. What is the number?

238. There are three numbers whose sum is 84. The second number is $1\frac{2}{7}$ times the first, and the third number is $\frac{4}{3}$ of the second. Find each number.

239. The profits of a business during its second year were $1\frac{1}{8}$ times the profits during its first year, and the profits for the

third year were $1\frac{1}{6}$ times those for the second year. The profits for the 3 yr. were \$ 8250. Find the profits for each year.

240. Draw a trapezoid making the upper base $\frac{1}{2}$ the lower base. If one of the non-parallel sides is $\frac{1}{3}$ of the lower base, the other non-parallel side $\frac{5}{12}$ of the lower base, and the perimeter of the trapezoid is 54 in., how long is each side?

241. How long is each side of a rhomboid whose perimeter is 14 ft. 6 in. and whose short sides are each $\frac{5}{6}$ as long as a long side?

MISCELLANEOUS EXERCISES

1. 7% of 7% of \$ 825 = ?

2. A lawyer collected \$ 1275 for a client. He charged 10% for collecting. He gave 60% of his fee to his wife. How much money was received by the client? By the lawyer? By his wife?

3. Mr. and Mrs. Shaw and two children took a trip on a lake steamer. The fare was \$ 9.00, children half price. Meals on the steamer were \$ 1.00 each. The family took supper, breakfast, and dinner on board, and paid \$ 5.00 for a stateroom. What was the cost of the trip?

4. Find the g. c. d. of the first composite odd number after 39 and the first composite odd number after 57.

5. How much is gained by buying a \$ 500 bond at 105, keeping it until 2 yr. interest at 3% has been received, and selling it at 109?

6. In a certain city, the highest temperature in July was 100°. The highest temperature in December was 75°. The difference in temperature was what per cent of the highest temperature in July? In December?

7. Solve $9x - 25 + 3x = 7x - 5$.

8. Solve $21x - 20 = 7x - 15x + 67$.

9. Three times a certain number equals 148 minus the number. What is the number?

10. What number doubled and increased by 4 equals 188?

11. If Mary were 15 years older than twice her present age, she would be as old as her cousin, who is 37 years old. How old is Mary?

12. Find a number such that the difference between $\frac{1}{8}$ of it and $\frac{1}{3}$ of it is 2.

13. The perimeter of a rhomboid is 70 dm. Its long sides are each 5 dm. longer than the sum of its short sides. Find the length of each side.

14. Class Exercise. ——may write an equation having 90 for its second member and a prime number for the coefficient of the unknown quantity. The class may solve the equation.

15. Solve $.6x = 120$.

Clear the equation of fractions by multiplying each term by the denominator of the decimal .6.

16. Solve $.06x = 24$, $.03x = \$240$, 8% of $x = 32.64$.

17. Solve $.7x = 2800$, $.016x = 32$, $4\%\ x = 48$.

18. Solve $.9x = 540$, $11\%\ x = 33$, $.012x = 720$.

19. An agent who charged 7% for collecting a sum of money, received \$ 210 as his commission. How much did he collect?

Let x equal the number of dollars collected, then $.07x$ equal 210.

20. What amount must be collected that the fee for collecting it may be \$70 when the rate is 5%? 7%? 2%? 10%? 8%?

21. Express in terms of x the interest of \$ x for 1 yr. 6 mo. at 6%.

The interest of \$1 for 1 yr. 6 mo. at 6% is 9¢. The interest of \$ x is x times 9¢ or $9x$¢.

22. Express in terms of x the interest of \$ x at 6% for 2 yr. 6 mo. 6 da. For 5 yr. 8 mo. 12 da. For 7 yr. 10 mo. 24 da.

23. What principal will gain \$ 157.50 in 3 yr. 6 mo. at 6%?

Let x = the number of dollars in the principal. \$1 in 3 yr. 6 mo. at 6% will gain \$.21. x dollars will gain x times \$.21 or \$.21x. Then $\$.21x = \$157.50$.

24. By similar reasoning find the principal which will gain $19.75 in 2 yr. 3 mo. at 6%. When you have found it, see if the interest upon it at 6% for 2 yr. 3 mo. is $19.75.

SUGGESTION TO TEACHER. Pupils should prove these problems until they realize that each of them is merely a reversed case of the ordinary problem in which the interest is required to be found.

Find the principal which will gain:

25. $240 in 3 yr. at 5%.

26. $360 in 4 yr. 6 mo. at 6%.

27. $780 in 5 yr. at 8%.

28. $175 in 6 yr. 3 mo. at 4%.

29. $200 in 3 yr. 2 mo. 15 da. at 8%.

30. $250 in 2 yr. 8 mo. at 6%.

31. In what time will $500 gain $34 at 6%?

Let x = the number of years. The interest of $500 at 6% for 1 yr. is $30. For x yr. the interest will be x times $30 or $30x$ dollars. Hence $30x = 34$ and $x = 1\frac{2}{15}$ yr. or 1 yr. 1 mo. 18 da.

In what time will:

32. $560 gain $106.40 at 8%?

33. $750 gain $192 at 6%?

34. $187.50 gain $37.50 at 5%?

35. $65 gain $2.60 at 6%?

36. $216 gain $122.22 at 10%?

37. At what per cent will $400 gain $35 in $2\frac{1}{2}$ yr.

Let x = the number of per cent. The interest of $400 for $2\frac{1}{2}$ yr. at 1% is $10. At x% the interest will be x times $10 or $10x$ dollars. Hence $10x = 35$ and $x = 3\frac{1}{2}$%.

At what per cent will:

38. $700 earn $63 in 2 yr. 3 mo.?

39. $600 earn $45 in 1 yr. 6 mo.?

40. $225 earn $49.50 in 2 yr. 9 mo.?

41. $500 earn $105 in 7 yr.?

42. $600 earn $125 in 8 yr. 4 mo.?

43. What principal will amount to $532 in 3 yr. 8 mo. at 9%.

Let x = the number of dollars in the principal. The amount of $1 for 3 yr. 8 mo. at 9% = $1.33; the amount of x dollars = $1.33x$ dollars.

44. What principal will amount to $94.50 in 2 yr. 6 mo. at 5%?

45. $155 in 3 yr. 4 mo. at 6%?

46. $85 in 1 yr. 8 mo. at 8%?

47. $168.36 in 2 yr. 5 mo. at 7%?

Find the missing term in the following:

	Prin.	Rate	Time	Int.
48.	$600	5%	$2\frac{1}{2}$ yr.	x
49.	$400	x	3 yr.	$48
50.	x	7%	$1\frac{1}{2}$ yr.	$31.50
51.	$500	6%	x	$50

52. A man wishes to set aside a sum of money, the interest of which will furnish his daughter a yearly income of $1000. If 6% can be obtained for it, how much shall he invest for her? How much would be necessary to invest if only $3\frac{1}{2}$% could be obtained for it?

53. Which is the greater price for an article, $100 cash or $108 due in 1 yr., without interest, when the customary interest is 8%. Why?

54. The **Present Worth** of a sum of money due at a given time is that smaller sum of money which, when put at interest at the usual rate, will amount to the given sum in the given time.

What is the present worth of $770 due in 2 yr., when money is worth 5%?

The above question really asks, "What sum put at interest at 5% will in 2 yr. amount to $770?"

55. When money is worth 6%, what is the present worth of $224, due 2 yr. hence?

When you have found the present worth, prove your work by computing the interest upon it for the given time and rate to see whether the principal and interest will amount to $224.

56. The difference between the present worth and the amount due at maturity is called the **True Discount.**

Supposing money to be worth 6%, find the present worth and the true discount of $4720, due 3 yr. hence.

True discount must be distinguished from bank discount, which is merely the simple interest on the face of the note.

Find present worth and true discount of:

57. $199.80, due 1 yr. 10 mo. hence.

58. $307.50, due 5 mo. hence.

59. $143.75, due 2 yr. 6 mo. hence.

60. There are two ways of finding the true discount after the present worth is known. What are they?

61. Class Exercise. —— may think of a sum of money and find how much it will be worth at a given future time, the customary rate of interest being —%. He may report to the class the amount of that sum, and the time and rate, and the class may find the original sum.

Find the difference between the bank discount and the true discount of the following, the rate of interest being 4%:

62. $856.00, due in 1 yr. 9 mo.

63. $817.50, due in 2 yr. 3 mo.

64. $712.50, due in 3 yr. 6 mo.

65. $626.00, due in 1 yr. 1 mo.

66. $987.00, due in 2 yr. 5 mo.

67. Resolve 18 into two factors, one of which is a perfect square.

68. Resolve 108 into two factors, one of which is the largest possible square.

69. Find the area of a right triangle whose base is 20 in. and altitude 1 ft.

70. Find the area of a trapezoid whose upper base is 10 in., lower base 14 in., and altitude 5 in.

71. Find the interest of $1000 for 3 yr. at 6%.

72. If Mr. Brown puts $1000 at interest at 6%, how much interest will be due him at the end of one year? If, instead of collecting this interest, he adds it to the principal, and loans the $1060 for a second year at 6%, what will be the second year's interest? If this second year's interest, $63.60, is added to the second principal, $1060, and the sum, $1123.60, is put at interest for the third year, what will be the amount at the end of that year?

73. How much greater is that amount than the original $1000? That increase is the **Compound Interest.** How much greater is the compound interest than the ordinary simple interest of $1000 for 3 yr. at 6%?

74. Find by the method shown in Exs. 72 and 73 the compound interest of $1000 for 4 yr. at 10%.

75. Money put at compound interest gains more rapidly as the number of years increases. Can you see why?

76. One dollar put at compound interest at 7% will amount in 25 yr. to $5.427. To how much will $2000 amount in that time at that rate of compound interest? How much of that sum is interest? Find the simple interest of $2000 for 25 yr. at 7%, and find how much less it is than the compound interest.

77. On his eighteenth birthday John Smith deposited $100 in a savings bank, which paid 4% compound interest. He did the same on his next two birthdays. How much had he to his credit in that bank on his twenty-first birthday?

CHAPTER X

INVOLUTION AND EVOLUTION

1. What is meant by the power of a number? Illustrate.

2. Raise to the 3d power 19. $\frac{1}{2}$. .3. $2\frac{1}{2}$. 1.2. .000007.

3. The process of raising a quantity to a higher power is called **Involution.**

Involve 7^4. $.2^5$. $(4\frac{1}{2})^3$. $.6^3$. $(1.1)^4$. $(\frac{1}{3})^2$.

4. $9^3 : 3^5 = ?$ $8^3 : 2^7 = ?$ $6^4 : 4^7 = ?$

5. When $a = 2$, how much is $3a^5$?

6. Give quickly the squares of the first 12 numbers.

7. Give the square of 20. 30. 40. 80. 70.

8. Give quickly the cubes of the first 12 numbers.

9. Give the cube of 20. 40. 60. 80. 50.

10. Name a perfect square that is a factor of 72. Of 50.

11. Name a perfect cube that is a factor of 250. Of 24.

12. What is meant by the root of a number? Illustrate.

13. The process of finding any root of a given quantity is called **Evolution.** It is the exact opposite of involution.

What is the square root of 121? Of 144?

14. Draw a square, and illustrate the following statement:

The number of units of length in one side of a square is the square root of the number of corresponding units of square measure in the area of the square.

15. Copy Fig. 1, making $ABCD$ a 4-inch square, and AE and CG each 2 in. How many square inches in the square, $ABCD$? In $HFKB$? In the rectangle, $BKCG$? In the rectangle $EHBA$? How many square inches in all the additions to the square $ABCD$?

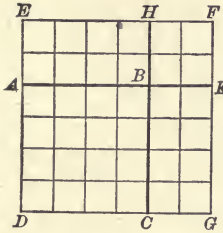


FIG. 1.

16. How long is one side of a square which contains 4 sq. ft. Represent it. Draw the additions which would be needed to make the figure represent a square yard.

17. Draw a square which contains 81 sq. cm. Add squares to it until it is a square decimeter. How many square centimeters are there in the two rectangles and little square which were added?

18. How many square inches are there in the two rectangles and the small square which, when added to a 5-inch square, will change it to an 8-inch square?

19. How many square inches are there in the two rectangles and small square which, when added to a 10-inch square, will change it to a 14-inch square?

20. A 10-inch square is changed to a 15-inch square by additions on two sides. How wide is each addition? How many square inches are there in the sum of all the additions?

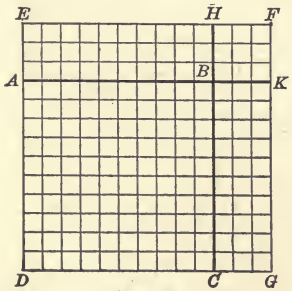


FIG. 2.

21. If the square $ABCD$ in Fig. 2 contains 100 sq. in., how long will the line DC be? If the sum of all the additions is 69 sq. in., how long will the line CG be?

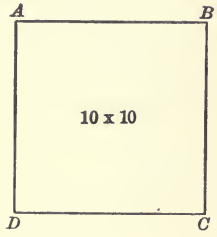


FIG. 3.

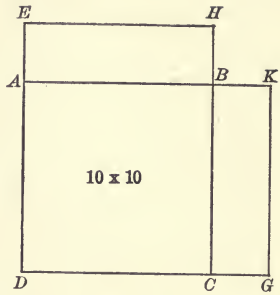


FIG. 4.

SOLUTION. CG in Fig. 4 represents the width of the additions which will change the square $ABCD$ (Fig. 3) to the square $EFGD$ (Fig. 2).

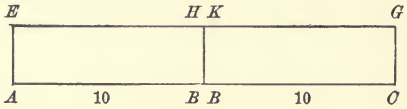


FIG. 5.

When the two rectangles in Fig. 4 are placed side by side as in Fig. 5, they form a rectangle whose length is 10 ft. + 10 ft., or 20 ft. We know that its area plus the area of the square needed to complete Fig. 4 is 69 sq. ft. We wish to know its width. Dividing 69, the number of square feet in all the additions, by 20, the number of feet in the base of the two rectangles, we have the quotient 3, which shows that the probable width of the rectangles is 3 ft., and that a side of the small square needed to complete Fig. 4 is 3 ft.

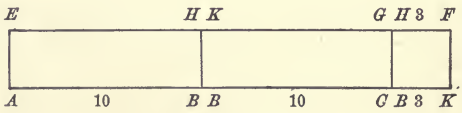


FIG. 6.

When the small square is added to the sum of the rectangles as in Fig. 6, a rectangle is formed whose length is 10 ft. + 10 ft. + 3 ft., or 23 ft. Assuming that its width is 3 ft., its area is 69 sq. ft., which exactly equals all the square feet which were to be added to the 10-inch square.

22. How many square feet would be represented by Fig. 4 if the square were completed? How long would one side of it be? What, then, is the square root of 169?

23. A lot 30 ft. square was increased by the addition of 61 sq. ft. The additions were made on two sides, and in such a way that the lot when increased was also in the form of a square. Represent and make additions as in the solution of Ex. 21. How wide was each addition? How long was a side of the square after the additions were made? How many square feet were in the completed square? What is the square root of 961?

24. A square contains 100 sq. in. If 96 sq. in. are added to it, as in Fig. 2, how wide will the addition be? How long will one side of the whole square be? What is the square root of 196?

25. How wide must be the additions that will change a square containing 400 sq. in. to a square containing 441 sq. in? What is the length of one side of a square containing 441 sq. in.? What is the square root of 441?

26. If you had 576 sq. ft. of boards to be arranged in the form of a square, how long would one side of the square be?

First make a square containing 400 sq. ft., then apply the remaining 176 sq. ft. on two sides.

27. Find in the same way the side of a square containing 625 sq. in. What is the square root of 625?

28. $1^2 = 1$. $9^2 = 81$. $10^2 = 100$. $99^2 = 9801$.

How many figures are there in the expression of the square of a number less than 10? In the expression of the square of a number greater than 9 and less than 100?

To find how many figures there are in the integral root of a given number separate the number into groups of two figures each by placing an arc over the units' and tens' figures, and also over each succeeding group or part of a group, as $\overbrace{12544}$. The number of arcs equals the number of figures in the root.

29. How many figures are there in the integral root of a number whose expression takes three places? Six places? Nine places?

30. When $t = 40$ and $u = 5$, how much is $(t + u)^2$? $t^2 + 2tu + u^2$?

31. When $t = 30$ and $u = 2$, how much is $(t + u)^2$? $t^2 + 2tu + u^2$?

32. Find the square of 25 in terms of its tens and units.

$25 = 2$ tens and 5 units.

$20 + 5$

$20 + 5$

$(20 \times 5) + 5^2$

$20^2 + (20 \times 5)$

$20^2 + 2\,(20 \times 5) + 5^2$

Beginning at the right, and multiplying units by units, we have 5×5, expressed 5^2. Multiplying tens by units we have (20×5). Multiplying 5 units by 20 we have the equivalent of another (20×5), which is written under the first. 20×20 is expressed 20^2. The sum of all these products is $20^2 + 2$ times $(20 \times 5) + 5^2$.

33. Is there any difference between the value of 25^2 and of $20^2 + 2\,(20 \times 5) + 5^2$?

34. Illustrate the following principle:

The square of any number consisting of tens and units is equal to the square of the tens plus twice the product of the tens and units plus the square of the units.

This principle may be expressed by the following formula: $(t + u)^2 = t^2 + 2tu + u^2$. By the use of this formula we may readily find the square root of any number.

35. Find the square root of 5329.

$t^2 + 2tu + u^2 = \overbrace{5329}$ $\boxed{73}$

$t^2 = 4900$

$2t + u = 143)\ 429$

$u\,(2t + u) = \underline{429}$

Beginning at the right and separating 5329 into groups of two figures each, we find that the root will consist of tens and units. We find that 4900 is the largest square of a multiple of ten that 5300 includes. Here, $4900 = t^2$, and $t = 70$. We write the 7 as the tens' figure of the root. Subtracting t^2, or 4900, from 5329, we have remaining 429, which equals $2tu + u^2$. As t equals 70, $2t$ equals 140. We wish to find the value of u. We use 140 as a trial divisor, and place the quotient 3 as the unit figure of the root. We add the 3 to

140 to find the true divisor. Multiplying the true divisor 143 by the quotient 3 and subtracting the product from 429, we find that there is no remainder. Hence the number 5329 is a perfect square, and 73 is the square root.

NOTE TO TEACHER. It is suggested that when this explanation is fully understood, the process be shortened by adopting the following method for the work, after the tens' figure of the root has been found, and its square subtracted from the left hand period.

"Double the root already found and place the result as a trial divisor. Cover the right-hand figure of the dividend, and see how many times the trial divisor is contained in what remains. Place the quotient beside the tens' figure as the next figure of the root, and annex it also to the trial divisor, thus making the true divisor. Multiply the true divisor by the quotient and subtract the product from the dividend," etc.

36. Find the square root of 1225.

Extract the square root of the following:

37. 625	43. 1764	49. 2809
38. 484	44. 9216	50. 2025
39. 289	45. 6889	51. 1089
40. 1681	46. 2601	52. 6561
41. 4356	47. 8649	53. 1156
42. 2209	48. 5184	54. 5041

55. Find the square root of 324.

Although 2 is contained in 22 eleven times, yet nothing larger than 9 can be used for a root figure.

56. $\sqrt{7921} = ?$ $\sqrt{2401} = ?$ $\sqrt{3481} = ?$

57. Of what number is 15 the square root? 20?

58. Give a perfect square whose root is a prime number.

59. CLASS EXERCISE. —— may give to the class the square of a number of two places. The class may find the number.

60. Find the square root of 12544.

There are three figures in the root. After finding the hundreds' and tens' figures double the root already found and proceed as before.

Find the square root of the following numbers:

61. 245,025	66. 249,001	71. 511,225
62. 375,769	67. 546,121	72. 811,801
63. 784,996	68. 56,169	73. 674,041
64. 776,161	69. 23,409	74. 70,756
65. 938,961	70. 529,984	75. 119,716

76. How long is a side of a square whose area is 1681 sq. in.? 961 sq. cm.? 729 sq. ft.? 1024 sq. m.?

77. How long is the perimeter of a square whose area is 1089 sq. in.? 8836 sq. ft.? 5476 sq. cm.? 3969 sq. dm.?

78. A square field contains 3600 sq. rd. How much will it cost to fence it at 80¢ a rod?

79. At $.60 a rod how much will it cost to fence a square field containing 1849 sq. rd? 1369 sq. rd?

80. Find the square root of $\frac{4}{9}$. Of $\frac{81}{121}$.

81. Find the square root of: $\frac{121}{196}$. $\frac{49}{484}$. $\frac{9}{169}$. $\frac{81}{729}$.

82. Reduce $1\frac{7}{9}$ to an improper fraction and find its square root.

83. Find the square root of: $2\frac{7}{9}$. $4\frac{21}{25}$. $3\frac{1}{16}$. $2\frac{2}{49}$. $6\frac{19}{25}$.

84. Class Exercise. —— may give to the class the square of a small mixed number. The class may find the number.

85. Find the square root of the decimal .000625.

In dividing a decimal into groups, begin at the decimal point and group to the right as $.\overbrace{000}\overbrace{625}$. There will be as many decimal places in the root as there are groups in the original decimal.

86. Find the square root of: .0049. .0081. .0025. .000036.

87. Find the square root of: .000529. .000121. 29.16.

88. Give one of the two equal numbers whose product is: 50.41. 60.84. 94.2841. 38.3161.

89. Which is greater, 124 or 124.0000?

90. Find the approximate square root of 124 to two places of decimals.

Annex naughts to 124.

91. Find to two places of decimals the square root of: 27. 24. 85. 1.35. 2.7. 3.81. 4.09.

92. Find to one decimal place the length of the side of a square that contains 45 sq. in. 75 sq. in. 95 sq. in.

93. Find value of x, when $x^2 = .014641$. When $x^2 = .3969$.

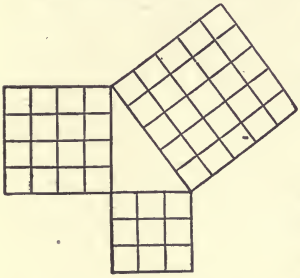


FIG. 7.

94. Draw a right triangle making the base 3 in. and altitude 4 in. If your drawing is correct how long will the hypotenuse be?

95. Construct a square on each side of the triangle as in Fig. 7 and compare the square of the hypotenuse with the sum of the squares of the shorter sides.

96. Geometry shows the reasons for the following fact, called the "Pythagorean Theorem" after Pythagoras, the Greek philosopher who gave it to the world.

In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

How long is the hypotenuse of a right triangle whose altitude is 12 in. and base 5 in.?

Find the hypotenuse of each of the following triangles:

	Base.	altitude.		Base.	Altitude.
97.	7	24	100.	15	20
98.	8	15	101.	60	11
99.	40	9	102.	24	45

103. A traveled east 120 miles and B south 160 miles from the same point. How far apart were they then?

104. How long is the diagonal of a rectangle which is 24 in. long and 10 in. wide?

105. How long is the longest straight line that can be drawn on a blackboard 12 ft. long and 5 ft. wide?

106. If your schoolroom were 36 ft. long and 27 ft. wide, what would be the length of the longest straight line that could be drawn upon the floor?

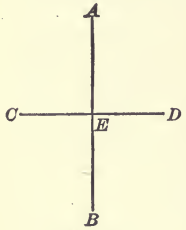


FIG. 8.

107. Draw a vertical line AB 8 in. long, and mark the middle point E. Draw CD, a horizontal line, 6 in. long, whose middle point is also E. Draw AD, DB, BC, and CA. What kind of a figure do they outline? How long is its perimeter?

108. The diagonals of every rhombus bisect each other at right angles. If the diagonals of the rhombus you have drawn were 16 in., and 12 in., how long would a side of the rhombus be?

109. How long is the perimeter of a rhombus whose long diagonal is 2 ft., and short diagonal $1\frac{1}{2}$ ft.?

110. A gate 8 ft. long and 6 ft. wide has a diagonal crossbar. How long a piece of wood was required for the crossbar?

111. A window is 12 ft. from the ground. How long a ladder is required to reach it if the foot of the ladder is 16 ft. from the foundation of the house?

112. John's kite is caught in the top of a tree 69 ft. high, and John, who is standing 156 ft. from the tree, holds the kite by the end of the string. His hand is 4 ft. from the ground. Represent the conditions of this problem and find the length of the string.

113. How long is the perimeter of a right triangle whose base is 21 ft. and altitude 72 ft.?

114. In a right triangle whose base is 80 cm. and altitude 18 cm. the hypotenuse is how much less than the sum of the other two sides?

115. What distance is saved by walking diagonally across a vacant lot 300 ft. long and 160 ft. wide instead of walking along two sides of it?

116. By taking the diagonal path across a rectangular lot instead of walking between the same points along the edge of the lot, what fractional part of the distance is saved when the lot is 42 ft. long and 40 ft. wide? 75 ft. long, 40 ft. wide? 80 ft. long, 60 ft. wide?

117. Since the square of the hypotenuse equals the sum of the squares of the shorter sides, the square of either short side is equal to the square of the hypotenuse minus the square of the third side. Let b stand for base, a for altitude, and h for hypotenuse. Show how the following equations are derived:

$$b^2 + a^2 = h^2$$
$$b^2 = h^2 - a^2$$
$$a^2 = h^2 - b^2$$

Use the equations of Ex. 117 in finding the missing side of triangles which have the following measurements.

	Base.	Alt.	Hyp.		Base.	Alt.	Hyp.
118.	18	x	30	**121.**	48	x	50
119.	x	12	26	**122.**	x	72	75
120.	7	12	x	**123.**	x	9	20

124. The center pole of a round tent is 18 ft. high. A rope stretches from a point 3 ft. from the top of the pole to a point on the ground 36 ft. from the base of the pole. How long is the rope?

125. Draw a right-angled isosceles triangle. If each of the equal sides were 9 in., how long would the hypotenuse be?

126. How long is the diagonal of a square each of whose sides is 10 ft.?

127. How long is the diagonal of a square whose area is 64 sq. ft.?

128. The diagonal of a square is 15 in., how long is one side of the square?

$$x^2 + x^2 = 15^2$$

129. A chimney 60 ft. high casts a shadow 144 ft. long. What is the distance from the end of the shadow to the top of the chimney?

130. Try to find an integer whose square is equal to the sum of the squares of two other integers. With the numbers make a problem about finding one of the sides of a right triangle.

The following is an easy way of finding three integers, the square of one of which is equal to the sum of the squares of the other two. Take any two unequal numbers. The sum of their squares will represent the hypotenuse of a right triangle, the difference of their squares one of its sides, and twice their product the other side. Taking 2 and 1,

$$2^2 + 1^2 = 5, \quad 2^2 - 1^2 = 3, \quad \text{and } 2\,(2 \times 1) = 4.$$

131. Find three integers which may represent the length of the three sides of a right triangle, by combining 3 and 1 as directed in the previous note. Combine 4 and 1, 3 and 2, 7 and 1, 5 and 2. Prove your work.

In the following problems, when the square root is not exact, extract the root to one decimal place.

132. In a room 20 ft. long, 15 ft. wide and 10 ft. high, a beetle crossed the floor diagonally and then crawled up to the ceiling. How far did he travel? What was his shortest distance then from the point at which he started?

Below are given the dimensions of several rooms. Find distances from an upper corner to the opposite lower corner.

	Length.	Width.	Height.
133.	24 ft.	18 ft.	9 ft.
134.	17 ft.	15 ft.	10 ft.
135.	25 ft.	24 ft.	11 ft.

136. A tank 8 ft. long and 6 ft. wide has water in it to the depth of 5 ft. How long is the longest straight stick that can be wholly in the water if the stick is pointed at both ends?

MISCELLANEOUS EXERCISES

1. A party of excursionists, consisting of 9 adults and 4 children, went from St. Louis to Washington, D. C., and returned. Round trip tickets were $25.45, children half price. The party occupied 7 berths in the sleeping car in going, and the same number in returning. The price of each berth was $4.75 each way. Meals in the dining car cost $1.00 each. The party left St. Louis at 4 P.M., on Monday, and arrived in Washington in time for breakfast Wednesday morning. During the trip the whole party took each meal in the dining car. They took an equal number of meals on the cars on their return trip. What were the expenses of the whole party on the trip?

2. Multiply nine hundredths by three ten-thousandths and divide the product by three hundred seventy-five hundred-thousandths. Then multiply the quotient by six hundredths and subtract the result from seven thousand eight hundred fifty.

3. What is the l. c. m. of the three prime numbers next after 19?

4. How is the least common multiple of several prime numbers found?

5. The l. c. m. of 6, 8, and 12 is what per cent of the fourth multiple of $12\frac{1}{2}$?

6. What is the largest integer that will exactly divide 8, 12, and 16?

7. The g. c. d. of 18 and 24 is what per cent of the third multiple of $8\frac{1}{3}$?

8. Find five numbers between 1 and 101, each of which can be resolved into two factors, both of which are perfect squares whose roots are greater than 1.

9. The divisor of a certain number is 189, the quotient is 22, and the remainder is $14\frac{2}{3}$. What is the number?

10. Find the interest of $17.35 for 2 mo. 3 da., at $\frac{1}{2}\%$ per month.

11. Mr. Rowe bought a bill of goods amounting to $500, receiving a discount of 40% and 60%. How much did he pay?

12. Mr. R. sold $150 worth of goods to Mr. W., taking Mr. W.'s note for the amount on 3 mo. time. The note was discounted at 8%. How much did Mr. R. receive?

13. Mr. A. bought 120 U. S. 3's at 102, brokerage $\frac{1}{8}$%. These bonds are non-taxable. How much more or less would he gain from them each year, than from the same amount of money loaned at 6% in a locality where the tax rate is 37 mills on a dollar?

14. Mr. N. bought 1000 shares of Gas and Electric Light Co. stock at 105. He kept the shares until a semi-annual dividend of $7\frac{1}{2}$% and another of 9% had been paid, and sold them at 110. How much did he gain?

15. Mr. Walker bought a horse for $75 and sold it at a gain of $33\frac{1}{3}$%. He took a note for the amount, due in one year, and had the note discounted at 10%. What per cent did Mr. Walker realize?

16. A boy had 80% of a dollar, spent 80% of what he had, and lost 50% of what remained. What per cent of the dollar did he then have?

17. A lawyer having a debt of $1346.50 to collect, compromised by taking 80%. His fee was 5% of the amount collected. What was his fee and how much should he return to his client?

18. A farmer sold 25% of a tract of land containing 120 A., at $.50 a square rod. How much did he receive for it?

19. The largest bell in the world is in Moscow. It weighs 216 T. If 77% of it is copper, and the rest tin, how many tons of each are in the bell?

20. A bell in Burmah weighs 117 T. If it contains the same proportion of tin as the Moscow bell, how much tin is there in it?

21. A bell in Pekin containing the same proportions of copper and tin weighs 53 T. How much copper is there in it?

22. 25 bu. of lime were bought for $6.25. At what price per peck must it be sold to gain $66\frac{2}{3}\%$?

23. In a war the Brazilians lost 43,365 men, which was 35% of their army. How many were left in the army?

24. Mr. C.'s agent in N. Y. bought for him a bill of goods amounting to $2575, charging him a commission of $3\frac{1}{2}\%$. How much must Mr. C. remit to pay all expenses?

25. An agent's commission at 3% upon a sale was $99. For how much was the property sold? How much did the owner receive?

26. How much must an importer pay as duty on 7 casks of wine, each containing 42 gal., 2% being allowed for leakage, and there being a specific duty of $1.25 per gallon?

27. 3 gal. 1 qt. 2 gi. is $16\frac{2}{3}\%$ of how much?

28. A sold goods for $70, making a profit of $16\frac{2}{3}\%$. What per cent would he have made by selling them for $72?

29. After retaining 3% for selling my potatoes, my agent sends me $523.80. For how much did he sell them?

30. The smaller of two numbers is 359.7 and the difference is $28\frac{1}{2}$. What is the larger number?

31. Multiply the square of $\frac{1}{5}$ by the reciprocal of $\frac{1}{5}$.

32. Multiply $\frac{2}{3}$ by the square of the reciprocal of $\frac{2}{3}$.

33. Multiply $(\frac{3}{4})^2$ by the cube of the reciprocal of $\frac{3}{4}$.

34. How many times is the square of 5 contained in the square of 10?

35. How many times is the square of 6 contained in the square of 12?

36. How many times is the square of any number contained in the square of twice that number? Illustrate.

37. How many times is the cube of 3 contained in the cube of 6? The cube of 5 in the cube of 10?

38. How many times is the cube of any number contained in the cube of twice that number? Illustrate.

39. How many times does a cube of a number contain the cube of half that number? Illustrate.

40. Make a true equation about the number 10 with two terms in each member. Let one of the coefficients be 7.

Solve:

41. $8x - 5x - 4 = 31$.

42. $9x - 5x + 30 = 58$.

43. $3x + 8x + 6x - 5 + 2x - 8 = 31 - x$.

44. What number increased by 5 times itself equals 42?

45. Each of the sides of an isosceles triangle is $\frac{5}{7}$ as long as the base. The perimeter is 34 in. How long is each side?

46. The perimeter of a trapezium is 54 in. The first side is $\frac{3}{4}$ as long as the second, the third is $1\frac{1}{4}$ times the second, the fourth is $1\frac{1}{2}$ times the second. Find the length of each side.

47. What principal will gain \$ 108 in 3 yr. at 5%?

48. In what time will \$ 800 gain \$ 108 at 6%?

49. At what rate will \$ 900 gain \$ 78 in 2 yr. 9 mo.

50. What principal will amount to \$ 629.20 in 3 yr. 6 mo. at 6%?

51. What is the present worth of \$ 635.60 due in 2 yr. 3 mo., when money is worth 6%?

52. What number is that to which if its $\frac{1}{3}$ and its $\frac{1}{2}$ be added the sum is 33?

53. From a rectangular field 20 rd. long and 16 rd. wide, a man bought a square lot which was $\frac{4}{5}$ of the field. What was the cost of fencing the lot at \$ 4.95 per rod?

54. In the Webster School building there is a flight of stairs in which each step is 6 in. high and 10 in wide, each step projecting 2 in. over the edge. There are 13 steps in the flight. Find the length of the handrail.

55. How much would it cost to carpet the same flight of stairs with carpet at \$.90 per yard, allowing 4 in. for turning under at top, the same at the bottom, and 3 in. for each turn at the edge of the steps?

56. How long is the perimeter of a right triangle whose base is 60 in. and altitude 61 in.?

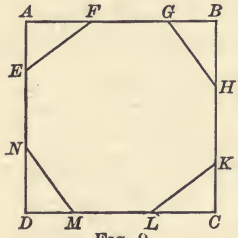


FIG. 9.

57. *ABCD* is a 12-inch square. *AF*, *BH*, *CL*, and *DN* are each 4 in. *EA*, *GB*, *KC*, and *MD* are each 3 in. How long is the perimeter of the irregular octagon *EFGHKLMN*? What is its area?

58. The diagonal of a square is 12 in. Find its perimeter.

59. The diagonal of a rectangle is 51 in., and the width of the rectangle is 24 in. Find the area and the perimeter of the rectangle.

60. How long is a quadrant of a circumference whose diameter is 21 in.?

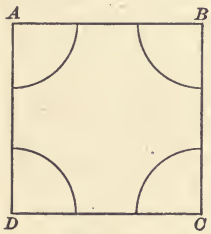


FIG. 10.

61. *ABCD* is a 10-inch square. With the vertex of each angle as a center and with a radius of $3\frac{1}{3}$ in. an arc is drawn. How long is the perimeter of each sector formed by the arc and the parts of the sides of the square? How long is the perimeter of the irregular figure left after the sectors are cut from the square?

62. A chord is drawn across a circle in such a way as to divide the circumference into two arcs, one of which is four times the other. If the circumference is 85 cm., how long is each arc?

When a body falls from an elevated place, if it is not hindered by the air or other obstructions, it falls $16\frac{1}{12}$ ft. in the first second, and 3 times as far in the second second. In the third second of time it falls 5 times as far as in the first second. In the fourth second it falls 7 times as far as in the first second, and so on, the ratio of the distance passed over in the first second to the distance passed over in any given second being equal to the ratio of 1 to the corresponding odd number.

63. Develop the following table to the fifth second of time:

Distance passed over in 1st sec. equals $16\frac{1}{2}$ ft.

Distance passed over in 2d sec. equals 3 times $16\frac{1}{2}$ ft.

Distance passed over in 3d sec. equals 5 times $16\frac{1}{2}$ ft.

64. Through what distance will a falling body pass in the fourth second of its fall? In the fifth?

65. How far does such a body fall in the first three seconds? In the next three seconds?

66. 1024 sq. ft. can be arranged either as a perfect square or as a rectangle 64 ft. long, or 128 ft. long, or 256 ft. long. What would be the length of the perimeter in each case?

67. 1296 sq. ft. can be arranged either as a perfect square or as a rectangle 72 ft. long, or 216 ft. long, or 432 ft. long, or 648 ft. long. Find the length of the perimeter in each case.

It will be seen that when a given area is arranged in the form of a square, it has a shorter perimeter than when it is arranged in the shape of any other rectangle, and that as the ratio of the length to the width increases, the perimeter of the figure also increases. Illustrate this fact.

68. A farmer has a rectangular field 160 rd. long and 40 rd. wide, which he wishes to fence and divide by fencing into 4 equal lots. If the fencing costs \$.60 a rod, what will be the difference between the expense of running his dividing fences parallel with the long sides of the field, and the expense of running them parallel with the short sides of the field?

69. Mrs. Wood has a rectangular hall 35 ft. long, and 14 ft. wide, the floor of which is laid with parquetry flooring. The border is $3\frac{1}{2}$ ft. wide. At \$1.25 per square yard, what is the cost of the floor surface inside the border?

70. The floor of Mrs. Wood's sitting room, which is square, is covered with the same kind of flooring, and has a border of the same width. The surface inside the border in the sitting room is equal in area to the surface inside the border of the hall floor. If the cost of the border is \$2.75 per linear yard,

measured around the edge of the room, how much more will the border in the hall cost than that in the sitting room?

71. Find the area of the largest rectangle that can be inclosed by a line 48 in. long.

72. In a box, each of whose inside measurements is 9 in., there were packed blocks enough to make 3 cubes. One was a 6 in. cube, another was an inch cube. How long was the edge of the third cube?

73. How many cubic inches are there in a cube whose edge is $\frac{5}{12}$ of a foot?

74. A pan in the shape of a rectangular prism, 11 in. long, and 7 in. wide, was out doors during a rain storm. After the storm the pan was found to contain a gallon of water. How deep was the water in the pan?

231 cu. in. = 1 gal.

75. Extract the square root of 2704. Of 60516. Of $\frac{36}{121}$. Of $\frac{441}{729}$. Of 44.89.

76. How much is x when $3x^2 = 75$? 432? 243?

77. If a rectangular lot is twice as long as it is wide, and its width is represented by x, how is its area represented? If the area is 72 sq. in., what are the dimensions of the lot? Represent.

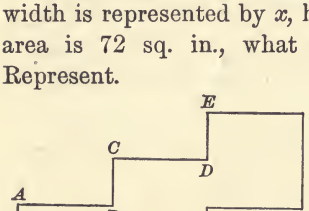


FIG. 11.

78. Figure 11 is composed of three equal squares so placed that $BC = BF$ and $DE = DH$. If the area of the whole figure is 147 sq. in., how long is its perimeter?

79. A rectangular lot is 5 times as long as it is wide, and it contains 80 sq. rd. How many trees can be placed on its edge, the distance from center to center of each tree being 1 rod? Represent.

80. How long is the radius of the largest circle that can be drawn on a piece of paper 8 in. long and 6 in. wide?

CHAPTER XI

PROPORTION

1. What number has the same ratio to 6 that 1 has to 3? That 2 has to 3? That 4 has to 3?

2. Supply the missing terms:

$$\frac{3}{5}=\frac{x}{10} \qquad \frac{5}{9}=\frac{x}{27} \qquad \frac{x}{4}=\frac{5}{10} \qquad \frac{7}{15}=\frac{x}{90}.$$

3. Which of the following statements are untrue?

$3:6=5:10.$ $\quad 8:4=12:6.$ $\quad 9:3=7:5.$ $\quad 1:3=3:15.$

4. An equation which states the equality of two ratios is called a **Proportion.**

Write a proportion.

5. Write a proportion with the terms 4, 6, 8 and 12, and show that it is true.

6. Can you write more than one true proportion with the terms 4, 6, 8, and 12?

7. Write a proportion with the terms 15, 20, 10, and 30. With 4, 24, 7, and 42. With 7, 9, 21, and 27.

8. Substitute values for x and y that will make true proportions.

$10:5=x:y.$ $\quad 8:12=x:y.$ $\quad \frac{9}{27}=\frac{x}{y}.$ $\quad 80:40=x:y.$

9. The terms of each ratio form a **Couplet** of which the first term is called the **Antecedent** and the second term the **Consequent.**

In the proportion $8:16=9:18$, which are greater, the antecedents or consequents?

10. Write a proportion in which the antecedent of the first couplet is 12 and the antecedent of the second couplet is 8.

11. Write a proportion in which the consequent of the first couplet is 10 and the consequent of the second couplet is 30.

12. Can you write a true proportion in which the antecedent of the first couplet is greater than its consequent and the antecedent of the second couplet is less than its consequent? Illustrate and explain.

13. Of how many terms must a proportion consist?

14. The first and fourth, or outside terms of a proportion, are called the **Extremes**. The second and third, or inner terms, are called the **Means**.

In the proportion $10 : 5 = 14 : 7$ find the product of the means and the product of the extremes and compare them.

15. Write some proportions and compare the product of the means with the product of the extremes until you see the truth of the following principle:

In a proportion the product of the means equals the product of the extremes.

When three terms of a proportion are given the other term is easily found by this principle, as:

$3 : 6 = 25 : x$. The product of the means is $3x$. The product of the extremes is 6×25. As these products are equal we have the equation $3x = 6 \times 25$.

$$\frac{\overset{2}{\cancel{6}} \times 25}{\cancel{3}} = 50 \; Ans.$$

Find the missing term in each of the following proportions:

16. $27 : 40 = 9 : x$.

17. $24 : 3 = 48 : x$.

18. $25 : x = 35 : 7$.

19. $144 : x = 12 : 1$.

20. $9 : 6 = x : 12$.

21. $11 : 77 = x : 42$.

A double colon is sometimes used between the ratios instead of the sign of equality, as $6 : 4 :: 3 : 2$. This is read 6 is to 4 as 3 is to 2.

22. Find the value of x in the proportion $x : 9 :: 10 : 18$.

Find the values of x:

23. $x : 36 :: 3 : 12$.

24. $x : 40 :: 18 : 80$.

25. $21 : x :: 25 : 8\frac{1}{3}$.

26. $28 : 35 :: 16 : x$.

27. $6\frac{2}{3} : 33\frac{1}{3} :: 12 : x$.

28. $7\frac{1}{2} : 22\frac{1}{2} :: 5 : x$.

29. $3\frac{1}{4} : 7\frac{1}{3} :: 3 : x$.

30. $2\frac{1}{2} : 10 :: 8\frac{1}{3} : x$.

31. $6\frac{1}{4} : 25 :: 12\frac{1}{2} : x$.

32. $6\frac{1}{4} : 12\frac{1}{2} :: 75 : x$.

33. $16\frac{2}{3} : 33\frac{1}{3} :: 66\frac{2}{3} : x$.

34. $5\frac{1}{2} : x :: 6 : 12$.

35. $3\frac{1}{3} : 50 :: x : 9$.

36. $16\frac{2}{3} : 12\frac{1}{2} :: x : 4$.

37. What is the ratio between 30 min. and 10 hr.? Between 3 lb. and 1 lb. 8 oz.?

Observe that a ratio is only possible between quantities of the same denomination.

Substitute numbers for x and y that will make the following proportions true:

38. $x : 2 :: 12 : y$.

39. $x : 8 :: 8 : y$.

40. $x : 6 :: 3 : y$.

41. $x : 7 :: 7 : y$.

42. A proportion in which the means are equal is called a **Mean Proportion,** and the number which each mean represents is called a **Mean Proportional** between the other two numbers.

Write a proportion in which each of the means is 6.

43. In the mean proportion $1 : 4 = 4 : 16$, what number is the mean proportional? In the proportion $2 : 6 = 6 : 18$, 6 is a mean proportional between what numbers?

Find value of x:

44. $3 : x :: x : 12$.

45. $1 : x :: x : 121$.

46. $2 : x :: x : 8$.

47. $2 : x :: x : 98$.

48. $7 : x :: x : 28$.

49. $3 : x :: x : 27$.

50. Write several proportions in which each of the means is 12.

51. If 3 hats cost \$11, how much will 6 hats cost?

Let x = the number of dollars paid for 6 hats. The greater the number of hats the greater the number of dollars paid for them. 3 hats are to 6 hats as \$11 (the price of 3 hats) are to x dollars (the price of 6 hats) or $3 : 6 :: 11 : x$.

52. If 5 hats cost \$7, how much will 10 hats cost?

In each problem it is assumed that the articles considered are of the same kind and of equal value.

53. If 8 apples cost 15¢, how much will 16 apples cost?

54. If 9 apples cost 17¢, how much will 3 apples cost?

55. If 10 oranges cost 53¢, how much will 5 oranges cost?

56. If 7 hats cost \$25, how much will 21 hats cost?

57. If 3 vases cost \$26, how much will 5 vases cost?

58. If 9 pairs of opera glasses cost \$36, how much will 11 pairs cost?

59. If $5\frac{1}{2}$ yd. of lace cost \$17, how much will $16\frac{1}{2}$ yd. cost?

60. If 10 yd. of silk cost \$23, how much will $13\frac{1}{3}$ yd. cost?

61. If 80 yd. of cloth cost \$375, how much will $5\frac{3}{7}$ yd. cost?

62. If 20 yd. of calico cost \$1.35, how much will $6\frac{3}{4}$ yd. cost?

63. If 15 yd. of jet cost \$7.50, how much will $8\frac{1}{3}$ yd. cost?

64. In the proportion $5 : 10 :: 12 : 24$, if both terms of the first couplet are divided by 5, is the proportion still true? By what number may each term of the second couplet be divided without destroying the proportion?

65. If 9 men can earn \$23 in a day, how much can 18 men earn?

66. If 15 men earn \$37 in a day, how much will 25 men earn?

67. If 30 men earn \$175 in a day, how much will 20 men earn?

68. When $19 are paid for 8 hats, how much will 24 hats cost?

69. If a train runs 85 mi. in 3 hr., how far will it run in 21 hr.?

All problems in proportion can be solved by analysis, as "If a train runs 85 mi. in 3 hr., in 21 hr. it will run $\frac{21}{3}$ times 85 mi., or 595 mi."

Solve the following problems by analysis as well as by proportion:

70. John rode on his bicycle 5 mi. in 45 min. At the same rate, how far would he ride in 90 min.?

Find the cost of 10 articles of the same kind:

71. When 3 fans cost $3.50.

72. When 6 maps cost $.25.

73. When 8 tops cost $.25.

74. When 4 bags cost $.75.

75. When 6 caps cost $1.

76. When 4 books cost $5.

77. Mr. A. collected $125 in the first four days of the week. During the rest of the week he collected at the same rate. How much did he collect in the whole week?

78. From 80 A. of land Mr. Porter gathered 1680 bu. of corn. At the same rate, how many bushels could he gather from 167 A. of corn land?

79. Polygons which have the same shape are said to be **Similar.** Their corresponding angles are equal, and their corresponding sides are proportional.

There are two similar rectangles. The larger rectangle is 18 in. long and 12 in. wide. The smaller rectangle is 9 in. long. How wide is it? Represent.

80. Draw a rectangle 8 in. long and 2 in. wide. Draw a similar rectangle 4 in. long. Find the ratio of the perimeters of the rectangles.

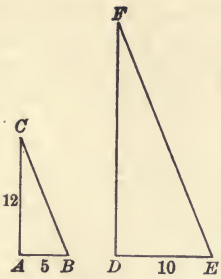


FIG. 1.

81. The right triangles *ABC* and *DEF* are similar. Find the side *BC*. Then find by proportion the sides *DF* and *FE* of the larger triangle. Find the ratio of the perimeters of the triangles.

82. Find the hypotenuse of a right triangle whose shorter sides are 7 in. and 24 in. Find the length of each side of a similar triangle whose shortest side is 21 in. Find the ratio of the perimeters.

83. The perimeter of an isosceles triangle, whose base is 5 in., is 19 in. How long is each of the equal sides? Find the length of each side of a similar triangle whose base is 20 in. Represent. Find ratio of perimeters.

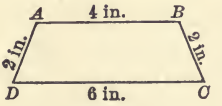


FIG. 2.

84. Find the length of each side of a trapezoid similar to *ABCD*, but larger, each of the non-parallel sides of the larger trapezoid being 8 in. long. What is the ratio of the perimeter of the greater trapezoid to the perimeter of the less?

Observe that the ratio of the perimeters of two similar polygons is the same as that of any pair of their corresponding sides.

85. There are two similar trapeziums. The perimeter of the smaller is 18 in., and its shortest side is 3 in. Find the perimeter of the greater trapezium, its shortest side being 15 in. Represent.

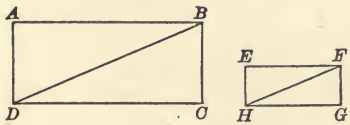


FIG. 3.

86. The rectangle *ABCD* is twice as long as the similar rectangle *EFGH*, and all the corresponding lines of the two rectangles are proportional. If *AB* is 24 in., *EF* 12 in., and *DB* 26 in., how long is *HF*? If *BC* is 10 in., how long is *FG*? Find the area of each rectangle and the ratio of their areas.

87. There are two similar rectangles, whose perimeters are respectively 36 in. and 9 in. If the base of the larger rectangle is 11 in., how long is the base of the smaller rectangle? If the altitude of the larger rectangle is 7 in., what is the area of the smaller rectangle?

88. The upper base of a certain trapezoid is 5 in. Its perimeter is 17 in. How long is the upper base of a similar trapezoid whose perimeter is 51 in.? If the altitude of the smaller trapezoid is 4 in., what is the altitude of the larger trapezoid?

89. There are two similar rhomboids, a base of one being 12 in., and a corresponding side of the other 6 in. If the altitude of the larger rhomboid is 4 in., what is the altitude of the smaller? Represent.

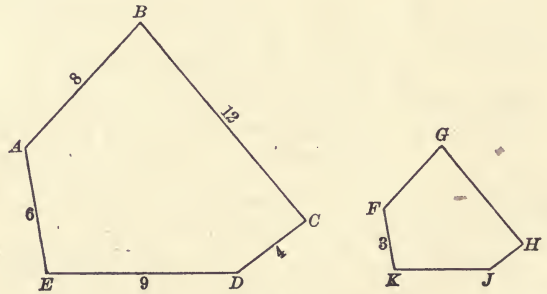


FIG. 4.

90. The pentagon *FGHJK* is similar to the pentagon *ABCDE*. Find the length of the sides which are unmarked.

91. The perimeter of an irregular hexagon is 330 in. Its longest side is 72 in. Find the length of the longest side of a similar irregular hexagon, whose perimeter is 55 in.

92. If 6 men can do a piece of work in 15 da., how long will it take 2 men to do the work?

In cases like the foregoing the proportion is inverse. The less the number of men employed, the greater the number of days required. 6 men : 2 men, *not* as 15 da. : x da., but as x da. : 15 da. We have, therefore, the proportion

men men da. da.
6 : 2 : : x : 15

We know that in this proportion x stands for a number greater than 15 because 6, the antecedent of the first couplet, is greater than 2, the consequent of that couplet, and the ratio of x to 15 is the same.

93. If 30 men can do a piece of work in 40 da., how many men are required to do the work in 20 da.? In 10 da.? In 5 da.? In 8 da.? In 4 da.?

In solving a problem by proportion it is necessary first to determine whether the proportion is inverse or direct. In the following problems write the word "Direct" or "Inverse" at the beginning of each solution.

94. A pole 40 ft. high casts a shadow 8 ft. long. How long is the shadow of a 10-ft. pole at the same time and place? Of a 20-ft. pole? Of a 16-ft. pole?

95. Twelve days are required for a piece of work, if the men work 10 hr. a day. How many days will be required if they work 8 hr. a day? 9 hr. a day? 6 hr. a day?

96. The taxes upon a piece of property valued at $7800 are $195. At the same rate, what is the amount of taxes upon property valued at $5600?

97. When a factory runs 18 hr. a day, a piece of work is finished in 20 da. How many hours a day must the factory run to finish the work in 15 da.? In 24 da.? In 30 da.?

98. Goods costing $700 are sold for $800. At the same rate what is the selling price of goods costing $1050? $1750?

99. If 63 gal. of vinegar cost $12.60, how much will 7 gal. cost? 9 gal.? 23 gal.? 10 gal.?

100. From a debt of $8000, Mr. A. collects $7000. At the same rate how much will he collect from a debt of $4800? Of $1600? Of $560? Of $60?

101. If a 5¢ loaf of bread weighs 12 oz. when flour is $6 per barrel, how much should it weigh when flour is $4 per barrel? $8 per barrel? $3 per barrel?

102. If it costs $60 to cover a floor with carpet costing $1.25 per yard, how much would it cost to cover it with carpet at $2 per yard? $1.75 per yard? $1 per yard?

103. The carpet which covers a room 20 ft. long costs $88.60. What would be the cost of covering with carpet of the same quality a room of the same width but 5 ft. longer? 4 ft. shorter?

104. A stock company paid a semiannual dividend of 5%. Mr. B.'s dividends were $800. How much would he have received if the rate had been 7%? 9%? 10%? 12½%?

105. The amount of duty upon a certain importation was $216.64. The rate was 40% ad valorem. If the rate had been 55% ad valorem what amount of duty would have been paid?

106. Mr. L.'s yearly income from bonds paying 3% is $1275. What would be his yearly income from those bonds if they were 5% bonds?

107. Mr. K. has a sum of money invested at 6%. 12 years' interest on that sum paid for his house and lot. How many years' interest would have been sufficient to pay for the house and lot if the rate of interest had been 8%?

PROPORTIONAL PARTS

108. John and Harry hired a boat to go fishing. John paid 30¢. Harry paid ⅔ as much as John. How much did both pay? What part of the whole expense did each pay? They sold their fish for $1. How ought the money to be divided?

109. If 20¢ are divided between two boys so that the first boy's share is to the second boy's share as 2 is to 3, how much will each boy receive?

If 20 were divided into 5 equal parts, ought not the first boy to have 2 of these parts, or $\frac{2}{5}$ of the whole, and the second boy 3 of the parts or $\frac{3}{5}$ of the whole?

110. Separate 27 into 2 numbers in the ratio of 4 to 5.

Combining 4 and 5 we have 9 equal parts to which 27 is equal. Each number equals how many 9ths of 27?

111. Divide 12 in the ratio of 5 to 1. In the same way divide 36. 54. 84.

112. How would you divide 14¢ between 2 boys, giving one boy 6 times as much as the other?

113. How must \$99 be divided among 3 men that their shares may be in the proportion of 1, 3, and 5?

114. Three men are to receive \$54 for papering a house. The first man has worked 5 da., the next 6 da. and the next 7 da. How much shall each receive?

115. The bill for the labor of shingling a house is \$48. How shall that be divided among the three workmen A, B, and C, if A has worked 6 da., B 4 da., and C 2 da.?

116. Two painters received \$51 for painting a house. The first worked 10 da., and the other worked 7 da. How ought the money to be divided?

117. Mr. A., Mr. B., and Mr. C. sent some poor children to spend a week in the country. Mr. A. paid for 5 children, Mr. B. for 8 children, and Mr. C. for 11 children. The whole expense was \$54.72. How much did each man pay?

118. In a pasture there were 40 cows of which Mr. D. owned 7 cows, Mr. E. 21 cows, and Mr. F. the remainder. If the cost of pasturage was \$70 a month, how much did each owner of the cows pay each month?

119. Two men contract to do a piece of work for \$122.50. The work is finished in 20 days, but one man is unable to work for 5 days. How shall the amount paid be divided?

120. A and B sent in a bid for the plumbing of a house for $ 150, but afterwards reduced the price $ 14. The work was finished in 16 da. after its commencement. They hired a helper at $ 2.50 per day who worked every day. A worked every day except one, and B was absent 7 da. How shall the amount be divided?

121. Gunpowder is made of 75 parts of saltpeter, 15 of charcoal, and 10 of sulphur. How many pounds of each are there in $1\frac{1}{2}$ T. of gunpowder?

122. Two young men hired a boat for $ 1.25. One invited two friends, and the other invited one friend to go in the boat with them. How should the young men divide the expense?

123. Divide 150 into 3 parts in the ratio of 3, 4, and 8.

124. Mrs. A. had three children, one 5 yr., one 7 yr., and one 13 yr. old. She proposed to divide a bag of nuts among the children in the ratio of their ages, if the eldest would tell correctly how they should be divided. In that case, how many nuts would each child receive, if there were 75 nuts in the bag?

125. Mr. Allen and Mr. Ward formed a partnership, Mr. Allen putting in $ 1200, and Mr. Ward $ 2400. They gained $ 2700 the first year. How ought the money to be divided?

126. The next year Mr. Ward put in $ 600 more, and the profit was $ 3000. Find each man's share.

127. The next year Mr. Allen took out $ 200, and Mr. Ward put in $ 1000. The profit was $ 3500. As Mr. Allen's share of the profits was so small and he was giving much time to the business he was paid $ 500 from the profits. How much did each man receive?

128. Two brothers bought a $ 5000 United States bond at 105, the younger brother furnishing $\frac{3}{4}$ as much of the money as the elder. How much did each pay?

129. Mr. A. and Mr. B. hired a pasture for $76. Mr. A. put in 8 cows for 9 weeks, and Mr. B. 10 cows for 8 weeks. How much ought each man to pay?

130. CLASS EXERCISE. —— may make a problem in which a number of dollars are unequally divided among a number of persons, and the class may solve it.

MISCELLANEOUS EXERCISES

1. $9^2 : 3^2 = 6^2 : ?$

2. $7\sqrt{529} = ?$ $4\sqrt{1521} = ?$ $8\sqrt{1369} - 6\sqrt{1156} = ?$

3. $\sqrt{2025} : \sqrt{1225} = ?$

4. $\sqrt{3025} : \sqrt{4225} :: 11 : ?$

5. $\sqrt{1764} : \sqrt{2401} :: \sqrt{2304} : ?$

6. $4\sqrt{196} : 7\sqrt{576} :: 3\sqrt{784} : ?$

7. $8^3 : 16 :: 4^3 : ?$

8. $\sqrt{1^3 + 2^3 + 3^3 + 4^3 + 5^3} = ?$

9. $\sqrt{729} : 3\sqrt{2916} :: \sqrt{1296} : ?$

10. How many cubic inches in a cube whose surface contains 96 sq. in.

11. How many acres of land in a road 10 mi. long and 55 ft. wide?

Use cancellation.

12. A lake, whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot of ice weighs 920 oz.

13. A room 10 ft. high contains 30,000 cu. ft. How much will it cost to carpet it at 75¢ per square yard?

14. Sunset Park contains 115 A. of land. How much is it worth at $.12\frac{1}{2}$ a square foot?

15 A man bought a cow and paid $20.25 cash, which was 90% of the cost. How much did the cow cost?

16. Change .15 and .025 to common fractions in their lowest terms.

17. Reduce to lowest terms $\frac{8\,3\,0}{9\,9\,0}$.

18. Reduce 5 to a fraction having 11 for a denominator.

19. Change .37 and .0016 to decimals having common denominators.

20. $\dfrac{7.5 \times .25 \times 3.6 \times .18}{.009 \times .08 \times 5 \times .125 \times .3} = ?$

21. Divide the product of 13.5 and 1.8 by 8.1.

22. A farmer sold $8\frac{1}{4}$ cords of wood at $\$5$ per cord, and received an equal amount of money from selling apples at $\$1.25$ per bushel. How many bushels did he sell?

23. Mr. Brown bought $47\frac{1}{2}$ T. of coal at $\$6\frac{2}{5}$ a ton. He paid cash $\$175$, and gave his note for the balance payable in 3 mo. with grace. What were the proceeds of the note discounted at 6%?

24. Make out and receipt a bill for the following: May 1, 1898, James Bentry of Ft. Wayne, Ind., bought of H. A. Cook & Son, 16 lb. of tea at $\$.85$ per pound, 36 lb. of coffee at $\$.18\frac{3}{4}$ per pound, 8 packages of macaroni at $\$.12\frac{1}{2}$ per package, 3 gal. strawberries at $\$.25$ per gallon, 25 loaves of bread at $\$.05$ per loaf.

25. Find the l.c.m. of 18, 24, 15, 30.

26. Find the prime factors of 342.

27. The sum of two numbers is 219.5, and one of the numbers is 96.875. What is the other number?

28. If a table costs as much as two chairs, and five chairs cost $\$43.75$, how much does the table cost?

29. Draw a representation of a cord of wood, marking its dimensions.

30. What per cent is gained by buying peanuts at $\$2$ a bushel and selling them at $5\not{¢}$ a pint?

31. How much will it cost to fence a rectangular lot 25 rd. long, containing 425 sq. rd., if the fence costs 20¢ a foot?

32. The dimensions of a bin are 1 ft. 3 in. by 3 ft. 4 in. by 9 ft. 4 in. How many bushels of wheat will it hold?

There are 2150.4 cu. in. in a bushel.

33. At 15¢ a square yard, how much will it cost to paint the walls and ceiling of a room 36 ft. long, 24 ft. wide, and 12 ft. high, having a baseboard 9 in. high. No allowance for openings.

34. When 5 children had left a class, 75% of the class remained. How many children belonged to the class at first?

35. If envelopes are bought at the rate of 15 cents for a package of 25 and sold for a cent apiece, what per cent is gained?

36. From a box containing a dozen packages of envelopes, 100 envelopes were used. What per cent of the number was left?

37. After a battle, the number of soldiers who answered to their names at roll call was only 612, which was 60% of the number that went into battle. How many went into battle?

38. There were 72 bananas in a bunch that cost 85¢. $12\frac{1}{2}\%$ of them were sold at 3¢ apiece, $33\frac{1}{3}\%$ at 15¢ a dozen, and $8\frac{1}{3}\%$ of them were spoiled. The rest were sold at 2¢ each. What was the gain on the whole bunch?

39. From a school of 48 pupils, $8\frac{1}{3}\%$ were absent on a rainy day. 25% of those present went out of the room to a recitation. How many remained in the room?

40. A bar of fresh soap weighed 3 lb. 6 oz. When dry, it weighed $33\frac{1}{3}\%$ less. How much did it weigh then?

41. Walter bought a knife for 50¢, and exchanged it for a school book worth 67¢. What per cent did he gain?

42. A jeweler bought some pins at the rate of $24.00 a dozen. The cost of each was $66\frac{2}{3}\%$ of the price for which he sold them. What was the selling price of each pin? What per cent of profit did he make?

43. A shoe dealer sold a pair of shoes for $4.00, gaining $33\frac{1}{3}\%$. How much did they cost?

44. On the 4th of July Alfred had half a dollar and James had 30 cents. They put their money together, and spent 30% of it for firecrackers, 25% of the remainder for candy, and the rest for lemonade. How much did they spend for each?

45. John's arithmetic when new cost $.60. When he had finished the study of arithmetic, he sold the book for $.40. What was the per cent of reduction?

46. Out of 3 dozen trees that a gardener set out, only 24 trees lived. What per cent died?

47. Cranberries sold at 15¢ a quart brought a gain of 20%. How much did they cost per bushel?

48. A man who had $675 spent 2% of it for a coat, and put $66\frac{2}{3}\%$ of the remainder in bank. Find price of coat, amount of money in the bank, and money left.

49. A housekeeper used $8\frac{1}{3}$ lb. from a bag of flour leaving $66\frac{2}{3}\%$ of it. How many pounds were left?

50. 20% of Mr. Ward's farm is planted in corn and 50% in wheat. The rest, which is 36 acres, is pasture land. How many acres are there of corn? Of wheat?

51. Mr. A. bought a horse for $350 and sold it to Mr. B. at a gain of 10%. Mr. B. sold it to Mr. C. at a gain of 5%. How much more did Mr. C. pay for the horse than Mr. A. paid?

52. 100 yd. of carpet which cost $60 to manufacture were sold by the manufacturer to the wholesale dealer at a profit of $33\frac{1}{3}\%$. The wholesale dealer sold them at a profit of 25%.

The retail dealer sold them to a customer at a profit of 25%. What was the retail price per yard? How much more per yard did the customer pay than the manufacturer received?

53. A dealer lost $16\frac{2}{3}\%$ by selling goods for $\$500$. What was the cost?

54. A boy sold his bicycle for $\$31.25$, gaining 5%. How much did the bicycle cost him?

55. By selling a bicycle for $\$40$ a dealer gained $33\frac{1}{3}\%$. How much would he have gained by selling it for $\$50$?

56. $2\frac{1}{2}$ is what per cent of $7\frac{1}{2}$? 25? $8\frac{1}{3}$? $16\frac{2}{3}$?

57. What per cent is gained by buying berries at the rate of 24 qt. for a dollar and selling them at the rate of 4 qt. for 25¢?

58. What per cent is gained by buying fans at the rate of $\$1$ a dozen and selling them at 10¢ apiece?

59. Mr. X. bought a stock of groceries for $\$1200$. He sold $\frac{1}{4}$ of them at a profit of $33\frac{1}{3}\%$, $\frac{1}{3}$ of them at a profit of 25%, and lost 10% on the remainder. Did he gain or lose on the whole, and how much? What per cent?

60. A lawyer collected 60% of a debt of $\$2400$, receiving 10% for collecting. He afterward collected 75% of the remainder, for which he was paid at the same rate. How much of the debt was paid to the creditor, how much did the lawyer receive, and how much did the debtor fail to pay?

61. A collector who received 5% for his services earned in one day $\$17.50$. How much did he collect?

62. A store valued at $\$25{,}000$ was insured for $\frac{4}{5}$ of its value at $\frac{5}{8}\%$. What was the premium?

63. The premium for insuring a house for $\frac{4}{5}$ of its value at 1% was $\$16$. What was the value of the house?

64. The premium for insuring a building for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$ was $\$60$. What was the value of the building?

65. Mr. S. has real estate assessed at $8000 in a city where the tax rate is $.033 on the dollar, and the poll tax is $2. How much are his taxes?

66. The taxes of Mr. T., who lives in the same city, are $233, of which $2 is his poll tax. For how much is his property assessed?

67. What is the duty on 750 yd. of cloth invoiced at 5 francs a yard, a franc being $19\frac{3}{10}¢$, the duty being 30% ad valorem?

68. An importer paid $7000 in duties upon an importation. The duty was 25% ad valorem. How many dollars' worth of goods did he import?

69. A man failing in business paid $.40 on the dollar. He owed Gay & Co. $8756, W. H. Reed $10,857, the First National Bank $5000, and Hall & Co. $4221. How much did each receive?

70. A man failed in business owing $24,000, and having assets $12,000. What per cent of his debts could he pay? How much would a creditor receive to whom he owed $1785.50?

71. How many cents on a dollar can each of the bankrupts in the following list pay?

	Liabilities	Assets		Liabilities	Assets
Mr. Low,	$84,000	$46,200	Mr. Dow,	$125,600	$37,680
Mr. Van,	$96,800	$33,880	Mr. May,	$242,850	$97,140

72. Mr. Wright is a creditor of Mr. Low to the amount of $7000. Mr. Van owes Mr. Wright $6400. Mr. Dow owes him $475, and Mr. May owes him $2821. How much will he receive in the settlement of those debts?

73. Mr. L. paid $800 for a lot, and $\frac{7}{8}$ as much for another lot. He sold the higher priced lot at a gain of $62\frac{1}{2}$%, and the cheaper lot at a loss of 10%. How much did he gain on the whole investment? What per cent?

74. Goods invoiced at $837 were discounted 30% and 20%. What was the net price?

75. The list price of a bill of hardware was $90. The discounts were 60% and 25%. The goods were sold at 40% below list price. What per cent was gained?

76. After a discount of 30% had been taken off, the net price of some goods was $140. What was the list price?

77. A merchant was offered a bill of goods for $1000, with 10%, 15%, and 5% off for cash. He offered $800 cash for the goods, which was accepted. Who, if either, was the loser, the buyer or the seller? How much?

Solve the equations:

78. $9x - 12 = 8x - 5$.

79. $8x - 24 - 4x + 16 = 32$.

80. $21x - 35 - 8x + 14 = 2x + 89$.

81. $2x - \frac{19}{4} = \frac{3x}{4} + 4$.

82. 4 times a certain number + 5 times the number + 10 = 82. What is the number?

83. An agent sold a sewing machine for $50 and sent to the company $10 less than 4 times the amount of his own commission. How much was his commission? How much did he send to the company?

84. In three camps there were 4000 soldiers. In the second camp there were 3 times as many men as in the first camp, and in the third camp 4 times as many as in the first camp. How many soldiers in each camp?

85. The perimeter of an isosceles triangle is 32 inches. Each of the equal sides is 7 inches longer than the base. Find length of each side.

86. Find a number which when multiplied by 7 and divided by 8 gives 42 for a quotient.

87. The perimeter of a rectangle is 110 cm. Its length is 15 cm. greater than its width. Find width and length. How many square centimeters in each of the triangles formed in it by its diagonal?

88. Two squares, one containing 144 sq. in. and the other 121 sq. in., are placed side by side so that their base lines form one continuous line. Represent and find perimeter of the surface which they cover.

89. Divide 39 into two parts such that one part shall be 12 times the other.

90. John caught three times as many fish as James. William caught twice as many as John and James both caught. They all caught 120 fish. How many fish did each catch?

91. Tom, Fred, and Will whitewashed both sides of the fence around a circular lot 60 ft. in circumference. Fred whitewashed three times as much as Tom, Will whitewashed four times as much as Tom. How many feet of the fence did each whitewash?

92. The circumference of a given circle is 20 in. How long is the circumference of a circle whose radius is 4 times as long as the radius of the given circle?

93. A merchant bought dress goods at 60¢ a yard and marked them to be sold at an advance of $33\frac{1}{3}\%$. They were sold at a reduction of $12\frac{1}{2}\%$ from the marked price. What was the selling price? What per cent was gained?

94. Class Exercise. —— may name a cost price for goods, a marking price, and a selling price, which is a certain per cent of reduction from the marked price. The class may find the per cent of gain or loss on the goods.

95. The marked price of some silk was $1.60 per yard. It was sold at a reduction of 5% from the marked price. The selling price was 27¢ more than the cost price. What was the per cent of gain?

96. When lace marked $1.50 per yard was sold at a reduction of $16\frac{2}{3}\%$ from the marked price, a gain of $.25 per yard was made. Find cost price and per cent of gain.

97. A coat marked at $7.50 was sold at a reduction of $33\frac{1}{3}\%$. The selling price was 125% of the cost price. Find cost.

98. A suit of clothes marked $25 was sold at a reduction of 20% from the marked price. If the selling price was 25% above the cost price what was the cost price?

99. Goods marked at 40¢ per yard were sold at a reduction of $12\frac{1}{2}\%$ from the marked price. The selling price was 75% above cost. Find the cost.

100. Some goods were sold for $.80, which was 25% above the cost price. How much did they cost? The selling price was 20% below the marking price. How were they marked?

101. Find the cost and the marked price of goods sold at $.75, which was a reduction of 25% from their marked price and an advance of 50% upon their cost.

102. Find the cost and the marked price of goods sold at $1.20, the selling price being an advance of 25% upon cost and a reduction of 25% from the marked price.

103. Class Exercise. ——may suppose himself to be a merchant buying goods at a certain price and marking them to sell at any price he may select. Let him lower the marked price by a certain per cent and then find the per cent of gain or loss which the selling price is on cost price. Let him give the selling price to the class and tell them what per cent that is below the marked price and above or below the cost price. Let the class find the marked price and the cost price.

104. How shall goods that cost 80¢ per yard be marked that a reduction of 10% from the marked price may be made and the goods sold at a gain of $12\frac{1}{2}\%$?

SOLUTION. Let x = the number of dollars in the marked price. Since a reduction of 10% from the marked price is to be made, the actual selling price is .90 x. The actual selling price is 80¢ + 12½% of 80¢, which is 90¢. Therefore

$$.90\,x = 90¢$$

Clearing of fractions, $$90\,x = 9000¢$$

$$x = \$1.00$$

Prove this and the following problems.

105. I marked goods which cost me \$ 1.44 per yard so that I could deduct 10% from the marked price and still make 15% profit. What was the marked price?

106. How shall I mark goods that cost \$ 72, so as to deduct 10% from the marking price and yet gain 12½%?

107. A buys goods for \$ 12. He wishes to make 33⅓% after discounting 20% from the marked price. How shall he mark them?

108. How shall goods costing 75¢ be marked that 10% may be deducted from the marked price and the goods still be sold at a profit of 20%?

109. How shall goods that cost \$ 1.20 be marked that a discount of 25% from the marked price may be made and the goods sold at a profit of 25%?

110. After a merchant had marked goods that cost 90¢, a clerk by mistake sold them at a reduction of 40% from the marked price. This selling price was 20% below cost. What was the marked price?

111. The list price of some goods is \$ 1.60. A merchant buys them at a discount of 40%, marks them to be sold at a profit of 25% on the cost price, and discounts them to the customer 10% from marked price. How much does the merchant gain? What per cent?

112. The list price of granite soup kettles is \$ 1.20 and the discounts are 50% and 30%. If the merchant who buys them

marks them at an advance of $16\frac{2}{3}\%$ on list price, and discounts them to his customer 40% from marked price, how much does he gain on each? What per cent?

113. A merchant buys knives at a discount of 80% and 10% from list price, which is \$ 18 per dozen. He marks them at a price 40% below the list price, and sells a knife to a boy at a reduction of 10% from the marked price. How much and what per cent does he gain on that sale?

114. Find the length of the hypotenuse of a right triangle whose base is 40 ft. and altitude 75 ft.

115. The hypotenuse of a right triangle whose base is 40 ft. and altitude 42 ft. is how much less than the sum of the other two sides?

116. If the hypotenuse of a right triangle is 95 ft. and altitude 57 ft., how long is the base?

117. How long is the perimeter of a right triangle whose base is 42 ft. and hypotenuse 150 ft.?

118. How long is the longest stick that can be carried through a doorway 6 ft. high and $2\frac{1}{2}$ ft. wide, the stick being sharply pointed at both ends?

119. The sum of all the edges of a cube was 132 in. What was the volume of the cube?

120. What is the volume of a cube if the area of all its faces is 384 sq. in.?

Find the missing term in each of the following proportions:

121. $6 : 8 = 54 : x$.

122. $5 : 3 = x : 21$.

123. $7 : x = 35 : 45$.

124. $x : 61 = 2 : 122$.

125. $\frac{1}{2} : \frac{3}{4} = 6 : x$.

126. $\frac{5}{6} : \frac{3}{4} = 40 : x$.

127. $2\frac{1}{2} : 17\frac{1}{2} = 3 : x$.

128. $8\frac{1}{2} : 35 = x : 20$.

129. $3\frac{3}{4} : 75 = x : 24$.

130. $\frac{7}{8} : \frac{1}{2}$ of $\frac{3}{4} = 42 : x$.

Solve by proportion and by analysis:

131. If 7 T. of hay cost $ 77, how much will 5 T. cost?

132. If 10 men earn $ 25 in one day, how much will 3 men and a boy earn provided the boy receives half as much as a man?

133. If a man travels 124 mi. in 4 da., how far will he travel in 9 da.?

134. The perimeter of a right triangle is 8 ft. The hypotenuse is 3 ft. 4 in. and the base 2 ft. 8 in. Find the sides of a similar triangle whose base is 3 ft. 8 in.

135. The circumference of a given circle is 12.78 in. Find the circumference of a circle the radius of which is $\frac{1}{4}$ that of the given circle.

136. If 8 bbl. of flour can be made from 40 bu. of wheat, how many barrels of flour can be made from 70 bu. of wheat?

137. If 75 United States bonds can be bought for $ 7725, how much will 60 of the same bonds cost? At 3% what will be the income from the 60 bonds?

138. If the grocer's bill for a family of 5 persons is $ 15 per week, what will be that of a family of 7 persons at the same rate?

139. $ 40 pays the board of 10 persons for a week at a farmhouse. If the price of board were doubled, how many persons could obtain board for the same time for the same amount?

140. If 15 men can do a piece of work in 40 da., how long will it take 12 men to do it?

141. If 30 men can do a piece of work in 16 da., how many men can do the same work in 15 da.?

142. Mr. A. holds stock in a company which last year paid 12% dividends. His dividends were $ 192. If this year the rate of dividend is 9%, how much will he receive from that investment?

143. Mrs. C. has a yearly income of $ 1876 from an investment which pays 8%. If it paid 6%, how much would she receive from that investment?

144. When Mr. D.'s house is rented at $ 40 a month the rent is 8% of the cost of the house. What per cent of the cost of the house does he receive when the tenant pays $ 35 a month?

145. A quantity of wheat was shipped 1400 mi. For the first 800 mi. the charges were $ 60. For the rest of the distance the rate per mile was twice as great. What was the cost of transportation for the whole distance?

146. How would you divide $ 90 among three persons in the ratios of 2, 3, and 4?

147. A and B formed a partnership, A's capital being $\frac{1}{4}$ as much as B's. Their profits were $ 8000 the first year. How much ought each to receive?

148. Mr. Ball owned twice as many shares in a mining company as his brother. The sum of their dividends was $ 1800. How much dividend should each receive?

149. Three heirs, James, Lucy, and Henry, own a farm which rents for $ 2100 a year. James owns 3 times as many acres as Lucy, and Lucy owns twice as many as Henry. How shall the rent be divided?

150. The distance from Alta to Reed's Crossing is 84 mi. by the A. & M. R.R., and from Reed's Crossing to Doane 96 mi. by the C. & D. R.R. The freight charges on some merchandise carried from Alta to Doane are $ 29.16. How much shall each railroad receive from it?

151. The freight charge on merchandise shipped 824 mi. over three railroad lines is $ 206. To how much is the A. & X. R.R. entitled which carried it 378 mi.? What sum belongs to the C. & Y. R.R. which carried it 212 mi.? To the third railroad?

At 5% what is the interest of

152. $725 from May 9, 1892, to Sept. 11, 1899?

153. $638 from June 17, 1873, to May 21, 1880?

154. $525 from Aug. 12, 1884, to June 17, 1893?

155. $631 from Jan. 21, 1887, to June 20, 1892?

156. $375 from Oct. 13, 1899, to Feb. 3, 1905?

157. How long is the side of a square whose area is equal to that of a rectangle 36 in. long and 4 in. wide? The perimeter of the square equals what per cent of the perimeter of the oblong?

158. How long is the side of a square whose area is equal to that of a rectangle whose length is 343 ft. and width 7 ft.? The perimeter of the square equals what per cent of the perimeter of the oblong?

159. A agreed to dig potatoes for B, taking $\frac{1}{5}$ of the potatoes for his pay. He dug 5 bu. of potatoes and set them aside for his employer. The next bushel he set aside for himself, and so continued. When he had dug 600 bu., how many bushels that belonged to him had he failed to get because of his ignorance of ratio?

160. Two paper hangers finished papering a house. They were to receive $22.60 for their work. The first man was to receive $1.40 more than the other; they divided the $22.60 into two equal parts; then the second man gave the first man $1.40. Was the division right? If not, how may it be corrected?

161. Find the sum of the squares of two numbers, as 9 and 7. Find the difference of their squares. Add the difference of their squares to the sum of their squares. Divide the number thus found by 2, extract the square root, and the result will be the greater number. Take several pairs of unequal numbers and see if this holds true.

162. $\sqrt{1^3+2^3}=?$ $\sqrt{1^3+2^3+3^3}=?$ $\sqrt{1^3+2^3+3^3+4^3}=?$

163. Find the square root of the sum of the first 5 perfect cubes. Of the first 6 perfect cubes.

164. Mr. Wilson is building a house which has a roof 45 ft. long with rafters 15 ft. long. How many shingles will be used to cover the two sides of the roof if 900 shingles 4 in. wide and exposed $4\frac{1}{2}$ in. to the weather are required to cover 1 square 10 ft. in dimensions? How much will the shingles cost at \$2.75 a thousand? At $3\frac{1}{4}¢$ per pound, how much will the shingle nails cost if 5 lb. are required for each square? How much must be paid to the carpenter for shingling the roof at \$1.25 per square? What is the entire cost of shingling the roof?

165. Think of a number, multiply it by 6, add 9, divide the result by 3, subtract 3, divide the remainder by 2, and you will have the original number. Try this with several numbers, and then try to find why this is true whatever the original number may be.

166. What is the ratio of the perimeter of an equilateral triangle a side of which is 3 in. to the perimeter of an equilateral triangle a side of which is 6 in.?

167. Take Ex. 166 substituting "hexagon" for "triangle."

168. Take Ex. 166 substituting "pentagon" for "triangle."

169. How many right angles can a trapezium have? A trapezoid? A rhomboid?

CHAPTER XII

MEASUREMENTS AND CONSTRUCTIONS

LINES AND SURFACES

1. A plane figure bounded by straight lines is called a **Polygon.**

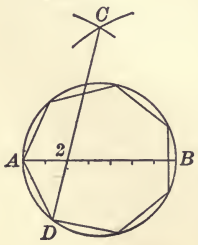
Draw a polygon of five sides.

A polygon of three sides is a **Triangle**; of four sides, a **Quadrilateral**; of five sides, a **Pentagon**; of six sides, a **Hexagon**; of seven sides, a **Heptagon**; of eight sides, an **Octagon**; of nine sides, a **Nonagon**; of ten sides, a **Decagon**; of twelve sides, a **Dodecagon.**

2. A polygon having all its sides equal and all its angles equal is called a **Regular Polygon.**

Draw a regular polygon.

Polygons which approximate very closely to regular polygons may be constructed by a method of which the following construction of an approximately regular heptagon is an illustration: Draw AB 7 units long, and mark the divisions of units. Draw a circle of which AB is the diameter. With A as a center and AB as a radius draw an arc. With the same radius and with B as a center draw an arc intersecting the first arc at C. Draw $C2$ passing through the second division of the diameter, and prolong it until it meets the circumference at D. AD will lie seven times consecutively as a chord.



A polygon of any required number of sides may be constructed in the same way by making the number of divisions in the diameter equal to the required number of sides. The line *CD* must always pass through the second division of the diameter.

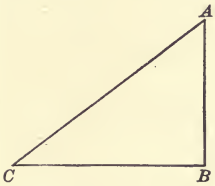
3. Draw a heptagon by the method described in the note and change it to a seven-pointed star. In the same way draw a five-pointed star and a nine-pointed star.

4. Triangles are classified with regard to their sides as, **Equilateral,** having three equal sides; **Isosceles,** having two equal sides; and **Scalene,** having no two sides equal.

Construct an equilateral triangle, an isosceles triangle, and a scalene triangle, each having a perimeter of 15 in.

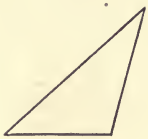
5. Triangles are classified with regard to their angles as **Right** triangles, **Obtuse-angled** triangles, and **Acute-angled** triangles.

Draw a triangle containing a right angle.



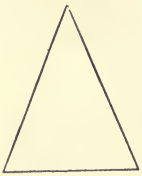
6. A **Right** triangle is one that has a right angle.

Draw a scalene right triangle. An isosceles right triangle.



7. An **Obtuse-angled** triangle is a triangle that has an obtuse angle.

Draw a scalene obtuse-angled triangle. An isosceles obtuse-angled triangle.



8. An **Acute-angled** triangle is a triangle in which all the angles are acute.

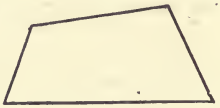
Draw a scalene obtuse-angled triangle. An isosceles acute-angled triangle.

9. Into what kind of triangles is a rhombus divided by its long diagonal? By its short diagonal?

10. Quadrilaterals are of three kinds, trapeziums, trapezoids, and parallelograms:

Quadrilaterals.

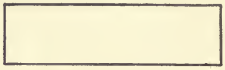
- Trapezium. No parallel sides.



- Trapezoid. Two parallel sides.



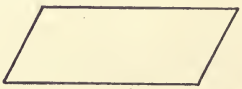
- Parallelogram. Opposite sides parallel.
 - Rectangle. All angles right angles.



 - Square. Equilateral rectangle.



 - Rhomboid. Angles oblique.



 - Rhombus. Equilateral rhomboid.



Draw four kinds of parallelograms and write the name of each upon it.

11. State the difference between a square and any other rectangle. Between a rectangle and a rhomboid. Between a square and a rhombus. Between a rhombus and any other rhomboid. Between a trapezoid and a trapezium. Between a trapezium and a rectangle.

12. Find the area of a rectangle whose base is 7 in. and altitude 4 in. Find the area of a right triangle having the same base and altitude.

13. Represent a trapezoid whose lower base is 8 in. and upper base 6 in., the bases being 4 in. apart. Find the area of the trapezoid.

14. The parallel sides of a field of trapezoidal shape are 120 rd. and 80 rd. long; they are 90 rd. apart. How many acres are there in the field?

15. What is the altitude of a trapezoid whose area is 825 sq. ft. and whose bases are 60 and 90 ft.?

Let x = the number of feet in the altitude.

16. Which is greater, and how much, a trapezoid whose parallel sides are 10 in. and 4 in., and altitude 5 in., or a trapezoid of the same altitude whose parallel sides are 8 in. and 6 in.? Represent.

17. The sum of the parallel sides of a trapezoid is 16 in. and the altitude is 3 in. What is the area?

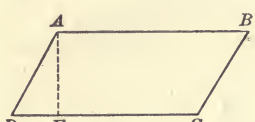


FIG. 1.

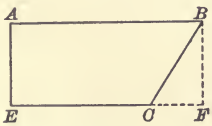


FIG. 2.

18. Draw and cut out the rhomboid $ABCD$. Draw AE perpendicular to DC (Fig. 1). Cut off the triangle ADE and place it on the other side of the rhomboid so that AD and BC unite (Fig. 2). The figure thus formed will be a rectangle having the same base and altitude as the rhomboid. If the base DC of the rhomboid is 7 in. and altitude AE 3 in., what is the area of the rhomboid?

19. Draw and cut out a rectangle 6 in. by 4 in. Cut and arrange its parts into a rhomboid of equal area. If you had another rectangle 6 in. by 4 in., could you make another rhomboid equal to the first but of different shape? Explain.

20. Make problems to illustrate the following rule:

To find the area of a rhomboid —

Multiply the base by the altitude.

21. Find the area of a rhomboid whose longer sides are 9 in. apart, and are each 48 in. long.

Supply the missing numbers in the measurements of the following rhomboids:

	Base	Alt.	Area
22.	40 in.	5 in.	x sq. in.
23.	x in.	10 in.	60 sq. in.
24.	25 in.	x in.	175 sq. in.
25.	13.8 in.	x in.	55.2 sq. in.
26	x in.	$25\frac{1}{2}$ in.	10.2 sq. in.
27.	5 ft.	x in.	2 sq. ft. 12 sq. in.

28. The perimeter of a certain rhomboid is 60 in. The base is 20 in. The altitude is 6 in. Represent and find area.

29. A base of a rhomboid is 5 in. longer than the altitude, which is 16 in. What is the area of the rhomboid?

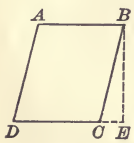


FIG. 3.

30. The altitude of a rhomboid is sometimes represented by a line that falls outside the rhomboid. *BE*, the altitude, is the perpendicular distance between the side *AB* and the opposite side produced. If *DC* is 7 in. and *BE* 8 in., what is the area of the rhomboid *ABCD*?



FIG. 4.

31. *LB* and *ED* are parallel. Which is the greater rhomboid, *GFKL* or *GFBC*? Explain.

32. Reproduce *LB* and *ED*, making them 5 in. apart. Let *GF* be 4 in. Draw several rhomboids whose base is *GF* and whose side parallel to the base is a part of *LB*. What is the area of each?

33. A farmer who had a rectangular lot 300 ft. long and 80 ft. wide, sold a strip of it to a railway company for a right of way. The agreement was that, beginning at the southwest corner of his lot, the width of the strip should be measured off 80 ft. along the southern boundary of the lot, that the strip should thence cross the lot, bounded by straight parallel lines, and that the railroad company should pay 5¢ a square foot for it. How much did the farmer receive for the land?

34. The shaded part of Fig. 5 shows the portion of land that the farmer intended to sell. The shaded part of Fig. 6 shows the portion of land that the company bought. *AD* being 80 ft., is the strip in accordance with the agreement?

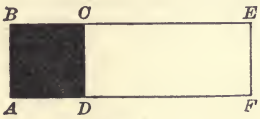


FIG. 5.



FIG. 6.

35. The farmer, thinking that the company had taken more land than belonged to it, consulted a lawyer, who proved to him that the two strips were equal, by drawing diagrams like Fig. 5 and Fig. 6, and showing him that if the two triangles *AHB* and *DCG* in Fig. 6 were cut out and applied to the rectangle *CEFD* in Fig. 5, they would exactly equal it. How could you determine whether or not the strips were equal?

36. Construct a rhomboid, cut and rearrange its parts in such a way as to make a trapezoid of equal area. A rectangle of equal area.

37. What name is given to a rectangle whose base and altitude are equal?

38. Construct a rhomboid whose base and altitude are each 3 in. Cut and rearrange its parts into a square.

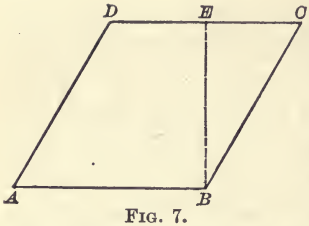


FIG. 7.

39. Find the area of the rhombus $ABCD$ if AB is 8 in. and EB 7 in.

40. Find the area of a rhombus whose perimeter is 4 ft. 4 in., and whose altitude is 6 in. less than one side.

41. Find the area of a rhombus whose perimeter is 40 in. and whose altitude is $\frac{3}{5}$ of a side.

42. Is a rhombus a regular polygon? Give reasons for your answer.

43. Draw a square whose sides are each 2 in. long, and a rhombus whose sides are each 2 in. long. Which has the greater altitude? The greater area?

44. Can you draw a rhombus in which the base and altitude are equal? Explain.

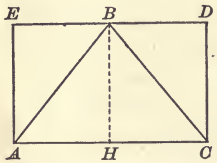


FIG. 8.

45. If the rectangle $AEDC$ were 8 in. long and 5 in. wide, what would be the area of the triangle ABC? Give reasons.

46. What would be the area of ABC if $AEDC$ were 9 in. long and 4 in. wide? 50 in. long and 18 in. wide? 6 ft. 3 in. long and 1 ft. wide?

47. Draw a figure and show the truth of the following statement:

The area of a triangle is equal to one half the product of its base and altitude.

48. Find the area of a triangle whose base is 18 in. and altitude 5 in.

49. Find the area of a triangle whose base is 17 in., and whose altitude is 5 in. greater than the base.

50. What is the area of a triangle whose base is 2 ft. 6 in., and altitude $\frac{1}{2}$ as much?

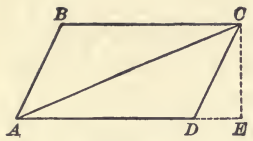


FIG. 9.

51. If AD were 10 in., and CE 5 in., what would be the area of the rhomboid $ABCD$? Of the triangle ADC? ABC?

52. If AD were 24 cm., and CE were half as long as AD, how many square centimeters would the rhomboid contain? Each triangle?

53. Either side of a triangle may be considered a base. The angle opposite the base is called the **Vertical Angle.** The **Altitude** is the perpendicular distance from the vertical angle to the line of the base.

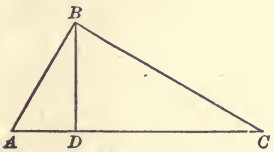


FIG. 10.

B is a right angle. If AC is considered the base, what angle is the vertical angle? What line is the altitude? If AB is the base, what angle is the vertical angle? What line is the altitude? If AB is 6 in., and BC is 10 in., what is the area of the triangle? Which is greater, $\frac{AB \times BC}{2}$ or $\frac{AC \times BD}{2}$?

The altitude of a triangle is sometimes represented by a line that falls outside the triangle.

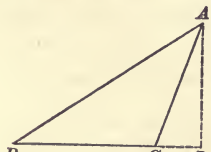


FIG. 11.

54. If BC is considered the base of the triangle ABC, what line is the altitude? If BC is 7 in., and AD 6 in., what is the area of the triangle?

55. Reproduce the triangle ABC, and draw a line to represent the altitude when BC is considered the base. When AB is the base. When CB is the base.

56. If CB were 10 in., and the corresponding altitude 6 in., what would be the area of the triangle? If AB were 12 in., how long would the corresponding altitude be?

57. The longest side of a triangular field is 120 rd. The perpendicular distance from the opposite corner to that side equals $33\frac{1}{3}\%$ of the side. Find the area of the field in square rods. In acres.

Find the missing measurements of the following triangles:

	Base	Alt.	Area		Base	Alt.	Area
58.	10 in.	7 in.	x sq. in.	63.	12 ft.	x ft.	36 sq. ft.
59.	6 in.	9 in.	x sq. in.	64.	13 ft.	x ft.	65 sq. ft.
60.	20 ft.	30 ft.	x sq. ft.	65.	x in.	10 in.	200 sq. in.
61.	60 yd.	40 yd.	x sq. yd.	66.	x ft.	50 ft.	750 sq. ft.
62.	15 rd.	13 rd.	x sq. rd.	67.	20 rd.	x rd.	300 sq. rd.

68. The base of a triangle is 45 in., and the altitude is twice the base. What is the area?

69. The altitude of a triangle is 14 in., and the base is 3 times the altitude. What is the area?

70. Find the area of a triangle whose base is 8 in. and altitude 9 in. How long is the side of a square whose area equals that of the triangle?

71. Find the perimeter of a square equal in area to a triangle whose base is 20 in., and altitude 10 in.

72. How wide is a rectangle 18 in. long, and equal in area to a triangle whose base is 12 in. and altitude 9 in.?

73. Draw an isosceles right triangle. If each of the equal sides were 12 in. long, what would be its area? If a rectangle equal in area to the triangle is 3 in. wide, what is its length?



FIG. 12.

74. By folding an isosceles triangle in such a way that the equal sides coincide, it will be seen that a line drawn from the vertical angle to the middle of the base divides the triangle into two equal right triangles. How long is the altitude AD of the isosceles triangle, of which the base CB is 10 in., and the equal sides are each 13 in.?

75. Find the altitude of an isosceles triangle whose base is 30 in., and whose equal sides are each 39 in. Find the area of the triangle.

76. Find the altitude of an isosceles triangle whose perimeter is 50 in. and base 16 in. Find the area.

77. The area of an isosceles triangle whose base is 18 in. is 108 sq. in. What is its altitude? The length of one of its equal sides? Its perimeter?

78. Given an isosceles triangle whose base is 40 in. and area 300 sq. in. Find its altitude. Find one of its equal sides. Find its perimeter.

79. Given an isosceles triangle whose altitude is 42 in. and area 1680 sq. in. Find the base. Find the length of each of the equal sides.

80. Find to one place of decimals the altitude of an equilateral triangle whose side is 10 in. Find its area.

81. Find the altitude and the area of an equilateral triangle whose side is 8 in.

82. Find the area of an equilateral triangle whose perimeter is 60 in.

83. Arrange 6 equilateral triangles so as to form a regular hexagon, and find the area of the hexagon, supposing each side of the triangles to be 12 in.

84. Construct a regular octagon, and draw a line from its center to the vertex of each angle.

85. Into how many and what kind of triangles is a regular octagon divided by lines drawn from its center to the extremities of its sides?

86. The distance from the center of a regular polygon to the middle point of one of its sides is called the **Apothem** of the polygon.

Draw a regular polygon and a line to show its apothem.

87. How does the apothem of a polygon compare with the altitude of one of the isosceles triangles into which a regular polygon may be divided by lines drawn from its center to the extremities of the sides?

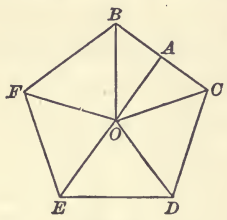


Fig. 13.

88. If a side of the regular pentagon *BCDEF* were 8 in., the apothem *OA* would be 5.44 in. What would be the area of the triangle *OBC*? Of the whole pentagon?

89. If a regular pentagon were cut into 5 equal isosceles triangles, and arranged as in Fig. 14, the sum of the bases of the triangles would equal what line?


Fig. 14.

90. Since every regular polygon may be divided into as many isosceles triangles as it has sides, we may find the area of a regular polygon by the following principle:

The area of a regular polygon is equal to one half the product of its perimeter and apothem.

What is the area of a regular pentagon if the perimeter is 80 in. and the apothem 10.88 in.?

91. The ratio of the apothem to the side of a regular pentagon = .68+; of a regular heptagon = 1.03+; of a regular octagon = 1.20+; of a regular decagon = 1.53+. The apothem of a regular hexagon can be easily found by the Pythagorean Theorem.

If the perimeter of a regular pentagon is 60 in., what is the apothem? The area?

92. Find the area of a regular decagon, one side of which is 8 in.

Given one side 14 in.:

93. Find the area of a regular pentagon.

94. Of a regular hexagon. **96.** Of a regular decagon.

95. Of a regular octagon. **97.** Of a regular heptagon.

98. What is the area of a flower bed in the shape of a regular hexagon whose perimeter is 96 ft.?

99. At $1.25 per square foot, what is the value of a park in the shape of a regular octagon, each side of which is 40 ft.?

100. In decorating a schoolroom 15 six-pointed stars were used. Each star was made by combining 12 triangles, each side of which was 4 in. Draw a figure to show how the stars were made. Find the cost of gilding all of them at 5¢ a square foot.

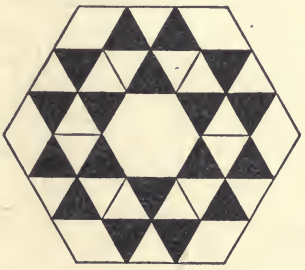


FIG. 15.

101. Figure 15 represents a "block" of patchwork from a quilt. Find the area of the whole block if the perimeter of the inner hexagon is 12 in.

102. What is the difference between the area of a square whose perimeter is 24 in. and the area of a regular hexagon whose perimeter is 24 in.?

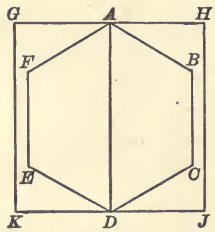


FIG. 16.

103. Construct the regular hexagon *ABCDEF*, each side being 4 in., and divide it into equal trapezoids by a line represented by *AD*. What is the area of each trapezoid?

104. Draw the square *GHJK*. What is its area? How long is the perimeter of the irregular figure *ABCDJH*? What is its area?

105. If the apothem of a regular pentagon is 20.4 in., how long is one side?

See table of ratios, p. 376.

106. Find the area of a regular decagon whose apothem is 1071 in.

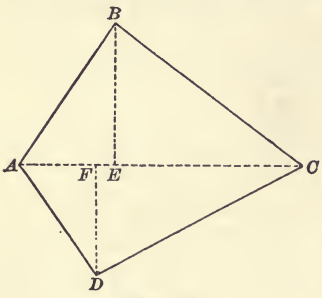


FIG. 17.

107. Find the area of a regular octagon whose apothem is 13.2 in.

108. Find the area of a regular heptagon whose apothem is 175.1 in.

109. Find the area of the trapezium *ABCD*, if the line *AC* is 8 in., *BE* perpendicular to *AC* 4 in., and *FD* perpendicular to *AC* 3 in.

In finding the area of an irregular figure it is customary to divide it into triangles or trapezoids and find the area of each part separately.

110. A surveyor found the area of a piece of land represented by the irregular pentagon *ABCDE*. The diagonal *AD* was 50 ch., of which *AH* was 10 ch. and *HF* 28 ch.

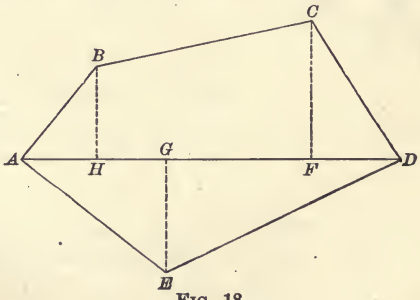


FIG. 18.

The perpendiculars *BH*, *CF*, and *EG* were respectively 12 ch., 18 ch., and 15 ch. A chain measures 4 rd. How many acres were there?

It will be observed that *HBCF* is a trapezoid and that the angles at *H* and at *F* are right angles.

111. CLASS EXERCISE. —— may draw an irregular polygon upon the board, and the class may show different ways of dividing it into triangles or trapezoids to find its area.

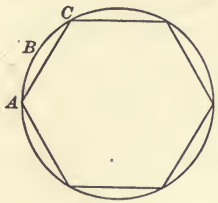


FIG. 19.

112. A polygon is said to be **inscribed** in a circle when the vertex of each angle of the polygon is in the circumference of the circle. The circle is said to be **circumscribed** about the polygon.

Inscribe a regular hexagon in a circle as in Fig. 19.

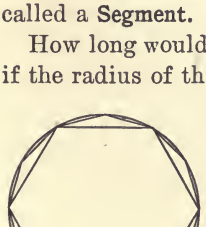
113. Can a rhombus be inscribed in a circle? Explain.

114. Can a hexagon whose sides are each 4 in. long be drawn within a circle whose radius is 5 in.? Can it be inscribed in the circle? Explain.

115. If the side of a regular hexagon is 5 in., how long is the radius of the circumscribed circle? How long is its circumference?

116. The part of a circle between an arc and its chord is called a **Segment.**

How long would be the perimeter of each segment of Fig. 19, if the radius of the circle were $24\frac{1}{2}$ in.?


FIG. 20.

117. Bisect each arc of your copy of Fig. 19. Join the middle point of each arc with the extremities of its chord as in Fig. 20. How many sides has the regular polygon you have thus formed? How would its area be found if the length of one side and the apothem were known?

If in the same way a polygon of 24 sides were inscribed in the circle, and then another of double that number of sides, and so on, we should soon have a polygon whose sides were so small that the perimeter of the polygon could not be distinguished from the circumference of the circle, and the polygon and the circle would appear to be the same.

118. A circle may be considered as a polygon of an infinitely great number of sides, its circumference being the perimeter of the polygon. To what would the radius of the circle correspond?

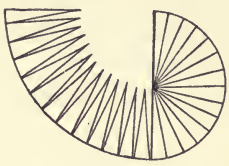


FIG. 21.

119. Cut out a small circle and fold it in halves. Fold it again, and continue folding until many small sectors are made by the folds. Cut along the folds and place the circle as in Fig. 21.

To what is the sum of the bases of these sectors equal? To what is their altitude equal?

120. If a circle is considered as a polygon of an infinitely great number of sides, the circumference is the sum of those sides and the radius of the circle is the apothem of the polygon. Can you see the reason for the following fact?

The area of a circle is equal to one half the product of its circumference and radius.

This may be expressed by the formula $A = C \times \frac{r}{2}$ or $A = \frac{Cr}{2}$, in which "A" stands for "area of circle," "C" for "circumference," and "r" for "radius."

121. Find the area of a circle whose radius is 7 ft.

Find the areas of circles of the following dimensions:

122. Radius 10 ft.

123. Circumference 110 ft.

124. Diameter 18 in.

125. Circumference 77 ft.

126. Radius 1 ft. 9 in.

127. Circumference 5 ft. 10 in.

128. Measure or estimate the diameter of the face of a clock or watch and find its area.

129. The minute hand of a clock in a tower is 3 ft. 6 in. long. What is the area of that part of the clock face over which the exact middle line of the hand passes in 15 min.?

130. What is the area of a sector which is $\frac{1}{6}$ of a circle, if the arc of the sector is 55 in.? Represent.

131. A cow is tied to a stake in a field by a rope 20 ft. long. What is the area of the surface over which she can move?

132. A statue whose base is 8 ft. square is placed in the center of a grass plot having a circumference of 1254 ft. How many square feet of the grass plot are around the statue?

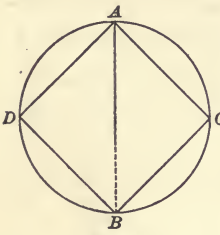
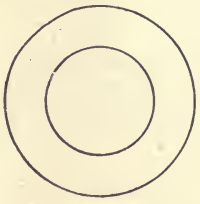


FIG. 22.

133. *AB*, a diameter, is 14 in. How long is one side of the inscribed square? What is its area? Find the area of each segment cut off by the square. Find the length of its perimeter.

134. Turn to Fig. 19, page 379. If the radius of the circle is 8 in., what is the area of the circle? Of the hexagon? Of each segment cut off by the inscribed hexagon?

135. A circle 10 in. in diameter is cut from a square 1 ft. in diameter. What is the area of the remaining surface?


FIG. 23.

136. The circles in Fig. 23 have the same center. The diameter of the smaller circle is 16 in. and the diameter of the larger circle 28 in. Find the area of each circle. Find the area of the circular ring which is left when the smaller circle is cut from the larger.

137. Circles which have a common center are called **Concentric Circles.**

Draw two concentric circles, one with a radius of 5 in., and the other with a radius of 7 in. Find the area of the circular ring which lies between their circumferences.

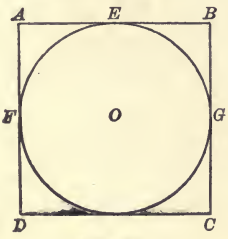


FIG. 24.

138. If the larger circle in Fig. 23 were 16 in. in diameter and the circular ring were 2 in. wide, what would be the diameter of the inner circle? Its area?

139. Draw a circle, and circumscribe a square about it as in Fig. 24. If the radius of the circle is 3 in., how long is one side of the square?

140. If a side of the circumscribed square is 8 ft., how long is the circumference of the circle? The perimeter of the irregular figure *FAE*? What is the area of *FAE*?

The area of a circle may be easily found by a formula derived from the formula $A = \frac{Cr}{2}$. $C = \frac{22}{7} \times 2r$, or $\frac{44}{7}r$. Substituting this value in the formula $A = \frac{Cr}{2}$, we have $A = \frac{44}{7}r \times \frac{r}{2}$, or $A = \frac{22}{7}r^2$. Thus, to find the area of a circle whose radius is 5 in., we take $\frac{22}{7}$ of r^2. $\frac{22}{7}$ of $25 = 78\frac{4}{7}$. *Ans.* $78\frac{4}{7}$ sq. in.

141. Find by the formula given in the note the area of a circle whose radius is 9 in. $1\frac{3}{4}$ in. 12 in. $3\frac{1}{2}$ in. $\frac{7}{8}$ in.

142. Each member of a geometry class numbering 24 pupils constructed a pasteboard cylinder and a pasteboard cone. Supposing the diameter of each base of those figures to be 7 cm., how many square inches of pasteboard were in the bases of the figures?

143. A grass plot in the form of a semicircle, whose straight edge is 15 ft., has within it a round bed of pansies 5 ft. in diameter. Represent and find the area of the grass plot not occupied by the pansies.

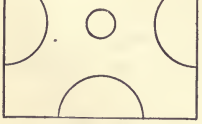


FIG. 25.

144. A garden 28 ft. square has flower beds arranged as in Fig. 25. Each semicircle is 12 ft. in diameter, and the small circle is 4 ft. in diameter. Find the area of the ground space not within the flower-beds.

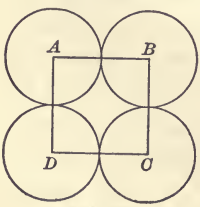


FIG. 26.

145. Turn to Ex. 61, page 337, and find the area of the irregular figure which is left when the sectors are subtracted from the square.

146. Four circles whose centers are *A*, *B*, *C* and *D*, and whose radii are 7 in., are placed as in Fig. 26. What is the area of the square *ABCD*?

147. Find the area of the surface which is included between the circles.

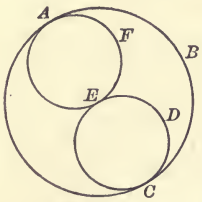


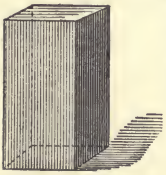
FIG. 27.

148. The radius of the large circle which is 14 in. equals the diameter of each of the small circles. Find the area of a small circle. Of the large circle. Of the irregular figure *ABCDEF*.

Observe that *ABCDEF* is one half of the space remaining when the small circles are subtracted from the large circle.

SOLIDS

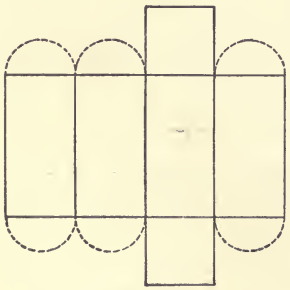
NOTE TO TEACHER. All the solids treated here are right solids, and the bases of the figures are regular polygons. Pupils should model these solids. The problems that follow require that models should be used as an objective basis for work until the pupils are able to visualize the forms accurately.


Quadrangular Prism. Bases Squares.

149. A **Prism** is a solid whose bases are polygons and whose sides are rectangles.

Mention some objects that are in the form of a prism.

150. Copy Fig. 28 on paper or cardboard, cut it out, and fasten its parts together so as to make a quadrangular prism.

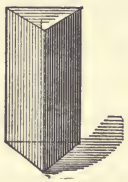

FIG. 28.

151. If the base of the prism were 8 in. square and the altitude of the prism 10 in., what would be the area of all the surfaces of the prism? How many inch cubes would equal it?

152. What are the cubic contents of a drawer which is 18 in. square and 4 in. deep?

153. Give the reason for the following **rule for finding** the cubic contents of a prism.

Multiply the area of the base by the altitude.



Triangular Prism. Bases Triangles.

The volumes of all prisms and cylinders are found in the same way.

154. Approximately $1\frac{1}{4}$ cu. ft. equal 1 bu. How many bushels of wheat can be stored in a bin 20 ft. long, 8 ft. wide, and 10 ft. deep?

155. Find the value of the apples that fill a bin 8 ft. long, 6 ft. wide, $4\frac{1}{2}$ ft. deep, at \$.75 a bu.

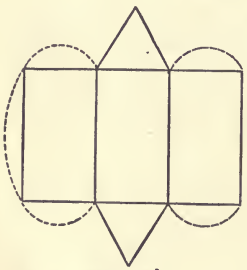


FIG. 29.

156. From the outline given in Fig. 29 construct a triangular prism.

157. If each base edge of the prism you constructed were 4 in. and each lateral edge were 8 in., how many inches would there be in all the edges of the prism? Find the area of its lateral or side surface. Find the area of its entire surface. Find its volume.

158. Find the area of the lateral surface of a triangular prism whose altitude is 9 in. and each side of whose base is 5 in. Find its entire surface. Find its volume.

159. Find the entire surface and the volume of a triangular prism each side of whose base is 6 in. and whose altitude is 4.5 in.



Pentagonal Prism. Bases Pentagons.

160. Given a pentagonal prism, the perimeter of whose base is 15 in. and whose altitude is 8 in. Find the area of a base. (See table of ratios, p. 376.) Find the entire surface. Find the volume. Find the sum of all its edges.

161. Construct a hexagonal prism, the perimeter of whose base is 18 in. and altitude 7 in. Find its entire surface and volume.

162. Given a hexagonal prism the perimeter of a face of which is 24 in. and altitude 10 in. Find the sum of all its edges. Its entire surface. Its volume.

163. Turn to Ex. 103, p. 377. Suppose the square *GHJK* to represent the base of a quadrangular prism of wood 10 in. high. Suppose the wood to be cut away until a hexagonal prism of the same height remains, whose base is represented by the hexagon *ABCDEF*. Find the number of cubic inches cut away.

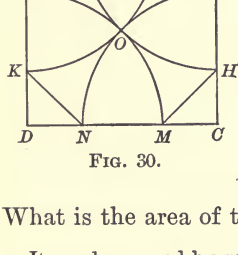


FIG. 30.

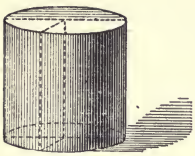
164. Reproduce the square *ABCD*. With *D* as a center, and *DO*, which is $\frac{1}{2}$ the diagonal, as a radius, draw the arc *LOM*. With *A*, *B*, and *C* as centers and with radii equal to *DO*, draw equal arcs. If the side of the square is 8 in., how long is *DO*? *DL*? *AL*? *DK*? *AE*?

165. What is the area of each of the triangles cut off from the square? What is the area of the octagon remaining?

It can be proved by geometry that if the corners of a square are cut off by the method given in Ex. 164, a regular octagon will be left. This method is used by carpenters in marking off the end of a square piece of lumber in order to change it to an octagonal form.

166. Find the area of the largest possible regular octagon that can be cut from a 16-in. square.

167. A piece of lumber 4 ft. long and 16 in. square was changed into an octagonal prism and used as a newel post. The newel post was as large as it could be made from the piece of lumber. How many cubic inches of wood were cut away? How many cubic inches were in the newel post?



Cylinder.
Bases Circles.

168. As the number of sides of a regular prism is increased, the base approaches more nearly to a circle, and the prism more nearly to a cylinder.

Draw a rectangle and construct two circles whose circumferences are equal to a side of the rectangle as in Fig. 31. Cut out and combine the figures so that they inclose a cylinder.

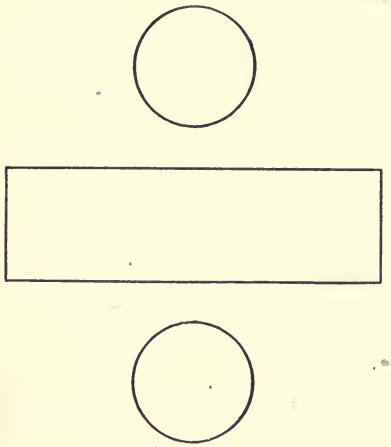


Fig. 31.

169. If the diameter of each circle were 14 in. and each of the shorter sides of the rectangle were 16 in., what would be the area of the entire surface of the cylinder? If the area of the base of a figure is 154 sq. in., how many cubic inches of sand would be required to cover it to the depth of 1 in.? 8 in? What would be the volume of this cylinder?

170. Find the entire surface of a cylinder 10 in. high, the diameter of whose base is 1 ft. 2 in. Find its volume.

171. How is the area of the entire surface of a cylinder found? How is the volume of a cylinder found?

172. How many cubic feet in a circular cistern 4 ft. in diameter and 7 ft. deep? How many gallons will it hold?

231 cu. in. = 1 gal.

173. How many square inches of tin are there in a dozen tin pails of cylindrical shape, the diameter of each being 8 in. and the height 10 in.?

174. A cylindrical tank 10 in. in diameter and 28 in. in height is full of water. How many gallons will remain in it when a pail of similar shape 5 in. in diameter is filled from it?

175. A cylinder 7 in. in diameter and 8 in. in height, outside measurement, was placed within another cylinder 14 in. in diameter and 8 in. in height, inside measurement. How many cubic inches of space were between the two cylinders?

176. A grindstone 28 in. in diameter was worn off until it was 21 in. in diameter. If the grindstone was 4 in. thick, how many cubic inches were worn off?



Triangular Pyramid. Base a Triangle.

177. A solid whose base is a polygon and whose sides are triangles meeting at a common point is a **Pyramid.**

Construct an equilateral triangle, and with each side as a base construct an isosceles triangle as in Fig. 32. Cut out the figure and bring the isosceles triangles together in such a way that their vertices meet in a common point and a triangular pyramid is formed.

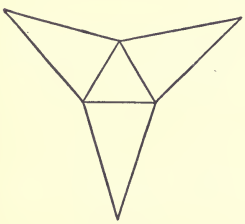


FIG. 32.

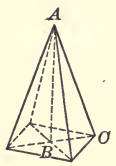
178. The point where all the faces of a pyramid meet is called the **Apex** of the pyramid.

A perpendicular from the apex to the base meets the base at its center. That perpendicular is the **Altitude** of the pyramid.

A line from the apex to the middle point of a side of the base is perpendicular to that side. It is called the **Slant Height** of the pyramid.

If a side of the equilateral triangle which you have constructed were 10 in., what would be the area of the base of the pyramid? If each of the equal sides of the isosceles triangles were 13 in., what would be the distance from the vertical

angle of each triangle to the middle point of each side of the base? What would be the area of the lateral surface of the pyramid? Of the entire surface?



Quadrangular pyramid. Base a square.

179. If a side of the base of the quadrangular pyramid whose apex is A, is 10 in., how long is the distance from center of the base to middle point of a side? If the altitude AB is 12 in., how long is the slant height? What is the area of the lateral surface of the pyramid?

180. If a side of a base of a quadrangular pyramid were 18 in., and the slant height were 12 in., how long would a lateral edge be? Find the sum of all the edges of the pyramid. Find its entire surface.

181. Given a side of the base of a quadrangular pyramid 40 in., the altitude 21 in., find the slant height. Find the area of all the surfaces of the pyramid.

182. Given a quadrangular pyramid whose base is 20 in. square, the slant height 26 in. Find the entire surface of the pyramid. Find the altitude of the pyramid.

183. Given a hexagonal pyramid, one side of the base being 16 in., and a lateral edge 17 in. Find the slant height of the pyramid. Find its lateral surface. Find the sum of all its edges.

184. It can be proved by geometry that the volume of a pyramid equals $\frac{1}{3}$ of the volume of a prism having the same base and altitude.

Find the volume of the pyramid described in Ex. 179. Ex. 181. Ex. 182.

185. Find the contents of a pyramid whose base is a triangle, each side of which is 8 ft. and whose altitude is 21 ft.

186. If a prism of wood whose base is a square 18 in. in dimensions and whose altitude is 10 in., be cut away until

a pyramid is left having the same base and altitude as the prism, how many cubic inches of wood must be cut away?

For data for the following problems see table of ratios, p. 376.

Given a side of a base 6 in. and the altitude 10 in., find the volume of:

187. A pentagonal pyramid.

188. An octagonal pyramid.

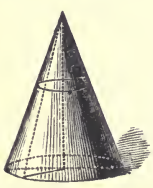
189. A hexagonal pyramid.

190. A pyramid whose base is a decagon.

191. The perimeter of the base of a hexagonal pyramid is 54 in., and a lateral edge is 15 in. Find a side of the base. Find the distance from the center of the base to the vertex of an angle of the base. Find the altitude of the pyramid. Find its volume.

192. Given a quadrangular prism and a quadrangular pyramid. The perimeter of the base of each solid is 5 ft. 6 in. The altitude of the prism equals the slant height of the pyramid, which is 1 ft. 8 in. The lateral surface of the prism equals how many times the lateral surface of the pyramid? Which has the greater altitude, the quadrangular prism or the quadrangular pyramid?

193. At 5¢ per square foot, what would be the cost of painting the sides of a steeple which is an octagonal pyramid, each side of the base being 8 feet and the slant height being 75 feet?



Cone. Base a circle.

194. The great pyramid of Gizeh was originally 480 feet high, with a square base 764 feet on each side. How many cubic feet of masonry were there in it?

195. As the number of sides of a pyramid is increased, the base of the pyramid

approaches more closely to a circle and the pyramid to a cone.

Draw and cut out a sector, and also a circle whose circumference equals the arc of the sector as in Fig. 33. With them construct a cone.

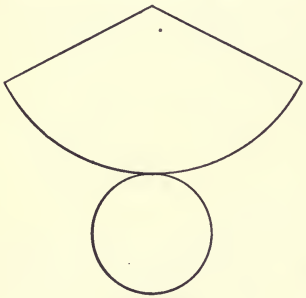


FIG. 33.

196. What would be the area of the convex or curved surface of the cone if the radius of the sector from which it were made were 4 in. and the arc 11 in.? What would be the circumference of the circle which forms the base of the cone? The diameter? The area?

197. Would it be possible to make a cone by using as a base a circle exactly equal to the circle from which a sector is cut to form the curved part of the cone? Explain.

198. What is the area of the curved surface of a cone if the circumference of the base is 4 ft. 8 in. and the slant height is 2 ft. 9 in.? What is the area of the entire surface?

199. Can a cone be constructed having the diameter of its base 10 in. and the slant height 4 in.? Explain.

200. How many square inches of tin are needed to make a funnel in the shape of a cone, the circumference of the base being 5 in. and the slant height $4\frac{1}{2}$ in.?

201. Four conical towers, each having a diameter of 4 ft. and a slant height of 12 ft., ornament a pavilion in a park. Find the cost of gilding them at 15¢ per square foot.

202. Find the entire surface of a cone the radius of whose base is $24\frac{1}{2}$ in. and the slant height of which is 4 ft. 2 in.

203. A line from the apex of a cone to the center of its base is perpendicular to the base. It is the altitude of the cone. What is the slant height of a cone whose altitude is 24 cm., if

the diameter of the base is 14 cm.? What is the area of the entire surface of the cone?

204. As a cone is a pyramid of an infinitely great number of sides, its volume is equal to $\frac{1}{3}$ that of a cylinder whose altitude and base are respectively equal to those of the cone.

Find the volume of a cone whose diameter is 5 in. and altitude 10 in.

Find volumes of cones having the following dimensions:

205. Radius 10 in., altitude 2 ft.

206. Diameter 15 in., altitude 11 in.

207. Circumference 5 ft. 6 in., altitude 1 ft. 10 in.

208. Radius 5 in., slant height 1 ft. 1 in.

209. What is the volume of the largest possible cone that could be cut from a prism 1 ft. long, whose base is 8 in. square?

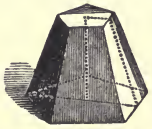
210. A cylinder whose diameter is 8 in. and altitude 10 in. is cut entirely across, parallel to its base at a distance of 3 in. from its base. What kind of solids are formed? What is the area of a base of each?

211. When a solid is cut entirely through in such a way that two plane surfaces are formed, the surfaces are called **Sections.**

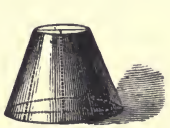
Represent a section made by cutting across a cylinder in such a way that the section is not parallel to the base.

212. Sections parallel to their bases were made of (*a*) a quadrangular prism, (*b*) a hexagonal pyramid, (*c*) a cone, (*d*) a triangular pyramid. What was the shape of each section?

213. Represent a section not parallel to the base of each of the above figures.



Frustum of Pyramid.



Frustum of Cone.

214. If a pyramid or a cone is cut by a plane parallel to its base, the part below the plane is called a **Frustum** of the pyramid or of the cone. See illustrations on page 391.

Construct a frustum of a cone or of a pyramid.

215. Each of the bases of the frustum of a triangular pyramid is what figure?

216. If each side of the lower base of a frustum of a triangular pyramid is 10 in., each side of the upper base is 8 in., and the slant height is 7 in., what is the area of the lateral surface?

If we find the area of one of the trapezoids that compose the lateral surface of the frustum of a pyramid and then multiply that area by the number of trapezoids we shall have the area of the lateral surface, but it is more convenient to find that area by multiplying the average length of the perimeters of the upper and lower bases by the slant height.

217. Find by each method the area of the lateral surface of the frustum of a triangular pyramid of the following dimensions, and then try to find why the results are the same: Edge of lower base 5 in., edge of upper base 3 in., slant height 8 in.

218. The area of the upper base of the frustum of a square pyramid is 100 sq. in., the area of the lower base 144 sq. in., and the slant height 10 in. Find the entire surface.

219. A quadrangular pyramid, each side of whose base is 16 in., is cut by a plane so that each side of the upper base of the frustum is 11 in. long. The slant height of the frustum is 10 in. What is the entire surface of the frustum?

220. What is the lateral surface of a frustum of a hexagonal pyramid, the perimeter of the lower base being 42 in., that of the upper base 24 in., and the slant height being 5 in.?

221. Find the lateral surface of the frustum of a hexagonal pyramid if a side of the lower base is 17 in., a side of the upper base 15 in., and the slant height is 1 ft.

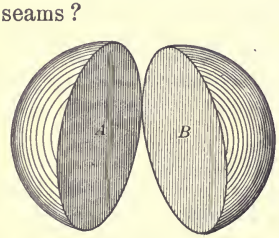
222. Find the convex or curved surface of a frustum of a cone, the upper base of which is 40 in. in circumference, the lower base 60 in., and the slant height 10 in.

Remember that the frustum of a cone is a frustum of a pyramid of an infinitely great number of sides.

Find missing measurements in frustums of cones:

	Circum. upper base	Circum. lower base	S. height	Convex surface
223.	8 in.	1 ft.	7 in.	?
224.	2 ft. 3 in.	3 ft. 2 in.	8 in.	?
225.	11 in.	15 in.	?	78 sq. in.
226.	1 ft. 4 in.	2 ft. 6 in.	?	1 sq. ft. 86 sq. in.

227. How many square feet of tin are used in constructing a tin pail in the shape of a frustum of a cone whose smaller base is 9 in. in diameter, upper base 1 ft., and whose slant height is 1 ft. 2 in., no allowance being made for overlapping at the seams?



Hemispheres

228. If a perfectly round ball 7 in. in diameter were cut into two hemispheres, *A* and *B*, what would be the area of each plane surface of the hemispheres?

229. It can be proved by geometry that the curved surface of a hemisphere is exactly twice as great as its plane surface. What, then, would be the area of the outside surface of the ball mentioned in Ex. 228?

230. Find the surface of a sphere whose diameter is 1 ft. 9 in. 1 ft. $5\frac{1}{2}$ in.

231. Find the surface of a sphere whose radius is 8 in.

232. A flagstaff is surmounted by a ball 1 ft. in diameter. Find the cost of gilding it at 30¢ a square foot.

233. What is the entire curved surface of a hemispherical dome whose height is 35 ft.?

234. If the earth were an exact sphere 8000 miles in diameter, what would be the area of its surface?

235. Solids that have the same shape are said to be **Similar Solids.**

Think of two boxes of the same shape, each dimension of the larger box being twice the corresponding dimension of the smaller. If the larger box is 8 in. by 4 in. by 2 in., what are the dimensions of the smaller? What is the area of the surfaces of each? What is the volume of each? What is the ratio of their areas? Of their volumes?

236. In the case of similar figures, every line of one figure has a corresponding or **Homologous Line** on the other figure, and every angle on one figure has a **Homologous Angle** on the other figure.

Take two similar right triangles of different dimensions and point out the homologous lines and angles.

SUGGESTION TO TEACHER. Similar solids should be handled and examined by the pupils. The magnitudes of their homologous angles, lines, surfaces, and volumes should be compared until the following principles are realized.

1. *On similar solids, homologous angles are equal.*

2. *On similar solids, any two homologous lines are to each other as any other two homologous lines.*

3. *On similar solids, homologous surfaces are to each other as the squares of their homologous lines.*

4. *The volumes of similar solids are to each other as the cubes of their homologous lines.*

237. A side of a base of a quadrangular pyramid is 6 in. The altitude of the pyramid is 4 in. What is the slant height? The sides of the base of a similar pyramid are each 12 in. What is the altitude? The slant height?

238. Find the area of one of the sides of the smaller pyramid. Of one of the sides of the larger. Find the ratio of their areas. Find the volume of the smaller pyramid. Of the larger. Find the ratio of their volumes.

239. Image two similar quadrangular pyramids whose homologous lines are in the ratio of 1 to 3. What is the ratio of the areas of their bases? If a lateral edge of the smaller pyramid is 8 in., what is the lateral edge of the larger? Assume dimensions for the figures, and find the areas of a triangular side of each solid, and also the ratio of those areas.

240. The slant height of the frustum of a hexagonal pyramid is 12 in. A side of the lower base is 8 in. A side of an upper base is 6 in. Find the perimeters of the bases of a similar frustum, the slant height of which is 3 in.

241. Find the lateral surface of each of those frustums. Find the ratio of the surfaces.

242. If a bucket 8 in. in diameter holds 3 gal., how many gallons can be poured into a bucket of similar shape whose diameter is 16 in.?

243. If it takes 110 sq. in. of tin to make a milk can 5 in. in diameter, how many square feet of tin will be required to make a similar can 20 in. in diameter?

244. If the volume of a cone whose altitude is 6 in. is 54 cu. in., how many cubic inches are there in a similar cone whose altitude is 10 in.?

245. A solid iron ball weighs 7 lb. What would be the weight of a similar ball whose diameter is twice as great as that of the first?

246. The cost of gilding a vase was $1.60. What would be the cost of gilding a larger vase of the same shape, the

larger vase being twice as high as the smaller? If the cubic contents of the larger vase were 328 cu. in., what were the cubic contents of the smaller?

247. Two similar cylinders are respectively 2 in. and 8 in. in diameter. If a section is made parallel to the base of each, what is the ratio of the area of the section of the greater cylinder to that of the less?

248. A pyramid whose base is 6 ft. square and whose altitude is 4 ft. is cut by a plane parallel to its base and 2 ft. above it. The pyramid above the cut equals what part of the original pyramid?

249. If a coal bin 4 ft. long holds 20 bu. of coal, how many bushels can be put into a bin of similar shape which is 8 ft. long?

250. If a city lot one side of which is 80 ft. is worth $12,000, what is the value of a lot of the same shape, the corresponding side of which is 40 ft.?

251. 16 sq. ft. of galvanized iron were used in making a water tank for a stove. How many square feet must be used to make a tank of the same shape, each edge of which is twice that of the first tank? If the first tank held 30 gal., how many gallons would the second tank hold?

252. A certain freight car contains 24,000 cu. ft. of space. How many cubic feet of space will be contained in a model of this car, which is $\frac{1}{30}$ as long as the original?

253. What is the ratio of a diagonal of a face of a liter to a diagonal of a face of a stere?

254. What is the ratio of the sum of the surfaces of a liter to the sum of the surfaces of a stere? What is the ratio of a liter to a stere?

255. If each of the sides of a polygon is trebled, the resulting polygon equals how many times the original polygon?

256. If each of the lines of a frustum of a hexagonal pyramid were made 5 times as long, the volume of the resulting solid would be how many times the original solid?

257. If a straw stack 5 ft. high contains 3 tons, how many tons are there in a stack of similar shape 10 ft. high?

258. How many balls of lead 2 in. in diameter will weigh as much as a ball of lead 8 in. in diameter? What is the ratio of the sum of all the surfaces of the 2-inch balls to the surface of the 8-inch ball?

ARCS AND ANGLES

259. Circumferences of circles are considered to be divided into 360 equal parts called degrees, marked °. How many degrees in a semicircumference? In a quadrant?

260. Over how many degrees does the minute hand of a clock pass in 15 min.? In 45 min.?

261. How many degrees are described by the hour hand of a clock in 4 hr.? In $2\frac{1}{2}$ hr.? In 5 hr. 30 min.?

262. If a circumference is divided by a chord in such a way that the greater arc is 4 times the less, how many degrees are there in each arc?

263. If a regular hexagon is inscribed in a circle, how many degrees are there in each arc cut off by a side of the hexagon?

264. How many degrees are there in each arc subtended by a side of a regular inscribed octagon? By a side of a regular inscribed decagon? Dodecagon? Heptagon? Nonagon?

265. An arc 20 ft. long equals how many degrees of a circumference 160 ft. long? Of a circumference 240 ft. long?

266. A horse trotted 1760 ft. on a circular race track 1 mi. in length. Over how many degrees of its circumference did he pass?

267. If a circumference is 24 in., how long is an arc of 60°? 90°? 30°? 150°? 120°? 45°? 75°? 108°?

268. Find the length of an arc of 110° of a circumference which is 20 ft. Of a circumference whose radius is 7 ft.

269. How long is an arc of 45° in a circle whose diameter is 6 ft. 5 in.? In a circle whose radius is 4 ft. 8 in.?

270. How long is an arc subtended by the side of a regular pentagon inscribed in a circle whose circumference is 40 ft.? In a circle whose diameter is 10 ft.?

271. In order to make calculations more exact, a degree is divided into 60 equal parts called minutes, marked ′, and a minute is divided into 60 equal parts called seconds, marked ″.

TABLE OF ANGULAR MEASURE

60 seconds (″) = 1 minute (′)

60 minutes = 1 degree (°)

Do not confound minutes and seconds that are measures of arcs and angles with minutes and seconds that are measures of time.

An arc which is one minute is what part of a circumference? An arc which is one second is what part of a circumference?

272. How many minutes in 25° 30′?

273. How many minutes in $4\frac{1}{2}$°? In 720″? In 5° 55′?

274. How many seconds in 35′ 25″? In 8° 10′ 20″? In 90° 15′ 25″?

275. Express 50° 15′ 30″ as seconds. As minutes. As degrees.

Express in each denomination of angular measure:

276. 7° 10′ 20″.

277. 8° 10″.

278. 4′ 30″.

279. 7° 20′ 40″.

280. 15° 15′ 15″.

281. 9′ 20″.

282. 10° 45″.

283. 8° 6′ 6″.

284. 4′ 50″.

285. 12° 12′ 12″.

286. Express in degrees, minutes, and seconds, 42784 sec. 31125 sec. 57241 sec.

287. Express in higher denominations, 97860 sec. 77825 sec.

Add:

288. $8° \ 5' \ 50''$ + $6° \ 11' \ 27''$

289. $6° \ 7' \ 24''$ + $20° \ 37' \ 48''$

290. $8° \ 29' \ 33''$ + $17° \ 31' \ 47''$

Find difference:

291. $8° \ 7' \ 25''$ − $3° \ 8' \ 16''$

292. $24° \ 16' \ 38''$ − $7° \ 19' \ 49''$

293. $85° \ 21' \ 36''$ − $17° \ 27' \ 54''$

Multiply:

294. $41° \ 17' \ 25'' \times 8$

295. $16° \ 17' \ 19'' \times 9$

296. $23° \ 28' \ 39'' \times 11$

Divide:

297. $15)18° \ 36' \ 45''$

298. $11)19° \ 36' \ 48''$

299. $24)13° \ 19' \ 28''$

300. $15)7° \ 8' \ 43''$

301. The sum of two arcs, one $49° \ 1' \ 28''$, the other $16° \ 38' \ 59''$, is how much less than the whole circumference?

302. How many degrees, minutes, and seconds in $\frac{5}{7}$ of a circumference?

FIG. 34.

303. Think of the position of the hands of a clock at 12 o'clock. Imagine two equal lines BO and AO in the same position (Fig. 34). Suppose AO to remain fixed, and that BO makes a complete revolution around the point O and returns to its former position, the point B describing a circle. At the different stages of its revolution BO makes different angles with AO, corresponding in the number of degrees to the arcs described. As the angle of the whole revolution is considered an angle of 360°, the angle formed by the two lines, when $\frac{1}{4}$ of the revolution has been made, equals 90°, or a right angle. When BO has made $\frac{1}{2}$ of a revolution, and the two lines are so placed that each is a continuation of the other (Fig. 35), the angle formed by them is an angle of 180°, or a straight angle.



FIG. 35.

How many right angles form a straight angle?

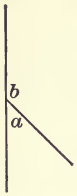


FIG. 36.

304. How many degrees are there in the sum of the angles a and b?

305. Reproduce Fig. 36 several times, changing the dividing line between the angles to make it lie in different directions. The sum of the angles is always how many degrees?

306. Two angles, whose sum is equal to 180°, are said to be **Supplements** of each other.

How many degrees are there in the supplement of an angle of 100°? Of 179½°? Of 3°? Of a right angle?

307. How many degrees are there in the supplement of an angle which is $\frac{5}{6}$ of a right angle?

308. How much greater or less than its supplement is an angle of 80°? 90°? 130°? 170°? 50°? 75°?

309. How many degrees are there in an angle whose supplement is twice the given angle?

310. How many degrees are there in an angle which is twice its supplement?

311. What is the ratio of an angle of 110° to its supplement? To a straight angle? To a right angle?

312. A fan, which when opened was semicircular in shape, was opened $\frac{2}{3}$ of its extent. What angle was formed by the outside edges of the sticks?

313. A branch of a tree made an angle of 45° with the trunk of the tree. What was the supplement of that angle?

314. How many degrees are there in the angle formed by the hands of a clock at 2 o'clock? 4 o'clock? 6 o'clock?

315. How many degrees are there in the supplement of an angle formed by the hands of a clock at 1.30 P.M.?

316. What kind of an angle is the supplement of an acute angle? Of a right angle? Of an obtuse angle? Explain.

317. When is an angle greater than its supplement? Less than its supplement?

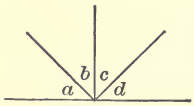


FIG. 37.

318. How many degrees are there in the sum of all the angles, a, b, c, d, formed at a given point and on the same side of a straight line? Explain.

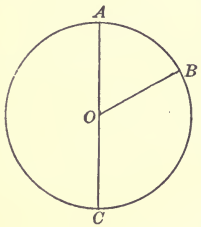


FIG. 38.

319. An angle formed by radii of a circle contains just as many degrees as the arc which is included between the ends of the radii.

O is the center of the circle of which the arc AB is 60°. How many degrees are there in the angle AOB? In BOC? In the straight angle AOC?

In naming an angle by three letters, the letter at the vertex is placed between the other two letters.

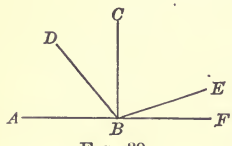


FIG. 39.

320. The angle ABC is a right angle. ABD is 10° more than DBC. CBE is 4 times EBF. How many degrees in DBC? In ABD? In EBF? In EBC? In DBE? In ABE?

321. Three angles, a, b, and c, are formed at the same point and on the same side of a straight line. $a = 3$ times b, and $c = 4$ times b. How many degrees are there in each?

Let x = number of degrees in angle b.

322. Four angles, a, b, c, and d, are formed on one side of a straight line at the same point. b has 10° more than a, c has 10° more than b, and d has 10° more than c. How many degrees are there in each?

323. How many degrees are there in the sum of an angle of 17° 3′ 15″ and an angle of 17° 4′ 21″?

324. How many degrees, minutes, and seconds are there in the supplement of an angle of 105° 15′ 20″?

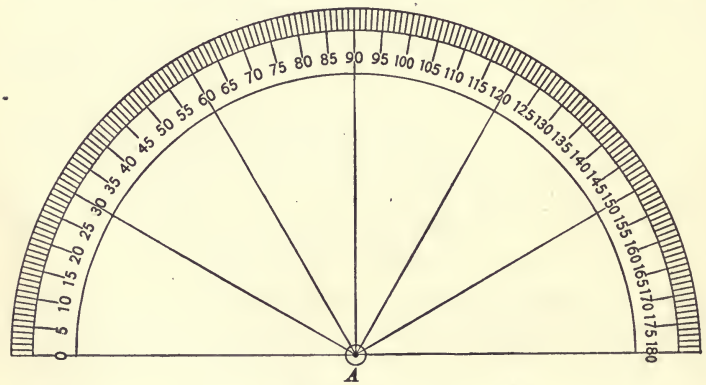
325. How many degrees, minutes, and seconds are in the supplement of an angle which is 4 times an angle of 7° 40′ 50″?

326. Give the measurement of an angle which is 5 times the supplement of an angle of 150° 10′ 24″.

327. Give the measurement of an angle whose supplement is 6 times an angle of 10° 20′ 30″.

328. The figure below represents a **Protractor,** a device for measuring or constructing angles. The point at the center marked *A* in this figure must always be placed at the vertex of the angle that is measured or constructed. The degrees corresponding to the angle are marked upon the arc.

Make a protractor. Lay off by means of it an angle of 20°.



329. Draw an angle and find by the protractor the number of its degrees.

330. Find by the protractor how many degrees there are in an angle of an equilateral triangle. Of a regular hexagon.

Suggestion to Teacher. Let pupils measure angles found in decorative designs, as in wall paper, carpet, parquetry, also the angles formed by branching stems, the angles of crystals, and of other natural objects.

331. How many degrees are there in each angle of a square? In all the angles of a square?

332. Draw a square. With each corner of the square as a center and with a radius less than $\frac{1}{2}$ the side of the square draw arcs inside the square ending in its sides. (See p. 337, Fig. 10.) Cut out the 4 sectors and place them so that their vertices are at the same point. What figure is formed?

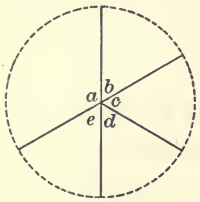


FIG. 40.

333. How many degrees are there in all the angles a, b, c, d, e formed about a common point in Fig. 40?

334. Angle 1 (Fig. 41) is 89° 10′, angle 2 is 48° 30′, angle 3 is 90° 5′. How many degrees in angle 4?

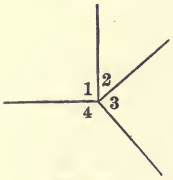


FIG. 41.

335. How many degrees are there in each of 8 equal angles whose vertices are at the same point?

336. There are 4 angles a, b, c, and d around a common point. $a = 3$ times b, $c =$ twice b, and $d =$ twice a. Find the number of degrees in each. Represent.

337. Construct around a common point an angle of 75°, an adjacent angle of 65°, and another adjacent angle of 80°. How many degrees are there in the remaining angle?

338. How long is the perimeter of a sector of 60° of a circle whose radius is 15 in.? What is the area of the sector?

339. If a sector of 45° were cut from a circle whose diameter is 20 in., how long would be the perimeter of the remaining figure? What would be its area?

340. Find the perimeter and the area of a sector of 40° of a circle whose radius is 18 in.

341. What is the area of the figure that remains when a sector of 36° is cut from a circle whose radius is 3 ft. 6 in.? How long is the perimeter of the figure?

342. Construct an angle of 50°, an angle of 60°, and an angle of 70°. Cut them out and place them side by side so that their vertices are at a common point. What kind of an angle do they form?

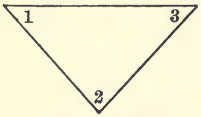


FIG. 42.

343. Draw and cut out a triangle, marking its angles 1, 2, 3, as in Fig. 42.

Cut off the corners 1 and 3 as in Fig. 43. Place them beside corner 2 as in Fig. 44.

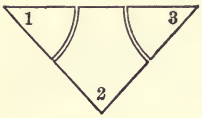


FIG. 43.

The angles will have their vertices at a common point and will cover all the surface around that point on the same side of a straight line. Hence they are equal to two right angles.

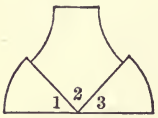


FIG. 44.

344. Repeat the experiment with triangles of different shapes until you see the truth of the following:

The sum of the angles of a triangle is equal to two right angles, or 180°.

By geometry this is proved to be true in all cases.

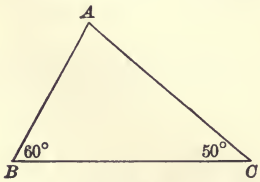


FIG. 45.

345. How many degrees are there in angle A of Fig. 45?

346. If A were 83° and B were 75°, how many degrees would angle C be?

347. If A were 91° and C 41°, how many degrees would angle B be?

348. If one angle of a triangle is a right angle, how many degrees are there in the sum of the other two angles? What kind of angles are they?

349. If one of the angles of a right triangle is 38°, how many degrees are there in the other acute angle?

350. At one extremity of a line construct an angle of 35°. Construct an angle of 75° at the other end of the line. Prolong the added lines until they meet. How many degrees are there in the angle formed by their meeting?

351. Fold an isosceles triangle so that the equal sides coincide. Can you see that the following statement is true?

In an isosceles triangle the angles opposite the equal sides are equal.

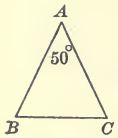


FIG. 46.

352. How many degrees are there in each angle of the isosceles triangle ABC? Explain.

353. How many degrees are there in each angle of an isosceles triangle in which a base angle is 80°? Explain.

354. Draw two equal lines forming a right angle and join their extremities. What kind of a triangle is formed? How many degrees are there in each angle?

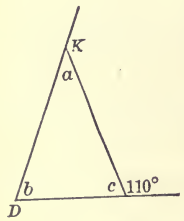


FIG. 47.

355. Find the measurement of each angle of an isosceles triangle in which a base angle is 3 times the vertical angle. Represent.

356. How many degrees are there in angle c? How many degrees in each angle of the isosceles triangle in Fig. 47?

357. An angle formed by a side of a polygon and a prolongation of an adjacent side is called an **Exterior Angle.**

Show two exterior angles in Fig. 47, and tell how many degrees in each.

358. Reproduce Fig. 47 and make an angle exterior to b. How many degrees are there in it? Can an isosceles triangle have two exterior angles at the base unequal? Explain.

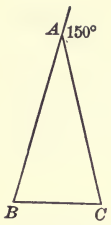


FIG. 48.

359. How many degrees are there in each angle of the isosceles triangle ABC (Fig. 48), whose base is BC?

360. The triangle ADC (Fig. 49) is isosceles. A, the vertical angle, is 40°. CB is perpendicular to AD. How many degrees are there in each angle of the triangles ABC and DBC?

361. The triangle ABC (Fig. 50) is isosceles. AE bisects the angle A. Angle DBC is 100°. Find each angle of the triangles AEB and AEC.

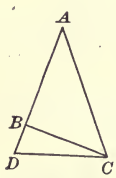


FIG. 49.

362. In a scalene triangle ABC, A is a right angle, and the angle B is 8 times the angle C. How many degrees are there in each angle of the triangle? Represent.

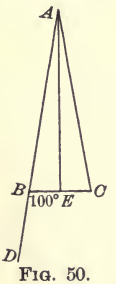


FIG. 50.

363. How many degrees apart are the equator and the north pole? How many degrees apart are two places which are on exactly opposite points of the equator?

Recall your knowledge of meridians and parallels of latitude by reference to your geography, if necessary.

364. How many degrees apart are two places, one of which is on the equator, and the other half way between the equator and the south pole?

365. How many degrees apart are two points, one of which is at the south pole, and the other 30° from the north pole?

366. Considering the circumference of the earth as 25,000 miles, how far apart are two places on the equator which are 60° apart? 180° apart? 120° apart?

In the following problems the equator and meridian circles are assumed to be circles 25,000 miles in circumference.

367. How long is an arc of the equator which is 50°? 70°?

368. How long is an arc of 50° of a circle of latitude which is 1800 miles in circumference?

369. An arc of 75° of a parallel of latitude 120 miles in circumference equals how many miles?

370. How many miles from the equator is a place 20° north of it? About how many miles from the equator are you?

371. If a ship sailed 2000 miles on the equator, through how many degrees of longitude would it pass?

372. If a ship sailed 200 miles on a circle of latitude, the circumference of which was 4000 miles, through how many degrees would it pass?

373. How many degrees from the equator is a place that is 500 miles north of it?

374. A village is 2° 10′ 30″ east of a certain city, and another village is 17° 49′ 30″ west of the city. How many degrees apart are they? If the circle of latitude upon which they are situated is 1200 miles in circumference, how many miles apart are the villages?

375. What is the difference in longitude between a place 20° 13′ 48″ east longitude, and a place 15° 15′ 55″ west longitude? Draw a diagram to illustrate the problem.

Find the difference of longitude between:

376. A, 17° 15′ 30″ east, and B, 16° 16′ 58″ west.

377. C, 40° 30′ 20″ east, and D, 50° 10′ 25″ west.

378. E, 17° 13′ 21″ west, and F, 19° 18′ 24″ west.

379. G, 41° 16′ 29″ east, and H, 31° 17′ 27″ east.

380. J, 61° 16′ 38″ east, and K, 15° 15′ 45″ west.

381. L, 6′ 17″ east, and M, 20′ 30″ west.

382. A certain lighthouse is 6° 15′ 20″ north of the equator, and another lighthouse on the same meridian is 11° 44′ 40″ south of the equator. How many miles apart are they?

383. The earth revolves on its axis once in 24 hr. Through how many degrees does a point on the earth's surface turn in one hour?

384. A point on the equator revolves at the rate of how many miles in one hour?

LONGITUDE AND TIME

As the earth's motion in revolving on its axis is from west to east, when it is sunrise at a certain place, points west of that place are still in darkness.

385. Suppose the sun to rise at 6 o'clock at a certain place, how much less than 6 o'clock is the time of day at a place 15° west of that place? What is the time 15° east of that place?

386. When it is noon at Denver, is it A.M. or P.M. at Boston? New York? San Francisco? Philadelphia? Portland, Oregon? Portland, Me.? Washington, D.C.? St. Louis?

387. Find the difference in longitude and the direction from Chicago of a place whose real time is 2 hr. later than Chicago time. 3 hr. earlier. $5\frac{1}{2}$ hr. earlier. 4 hr. 30 min. later?

388. When it is noon at Buffalo, what time is it at a place 15° east of Buffalo? 15° west? 60° east? 40° west?

389. When it is midnight at St. Paul, what time is it at a place 30° north of St. Paul? 30° south? 30° west? 30° east?

390. If a point moves 15° in 1 hr., or in 60 minutes of time, how far will it move in one minute?

391. If a point moves $\frac{1}{4}$ of a degree, or 15′ in one minute of time, how far will it move in one second of time?

15° of longitude cause a difference of 1 hr. of time.
15′ " " " 1 min. of time.
15″ " " " 1 sec. of time.

392. What is the difference in longitude of two places between which there is a difference of 2 hr. 10 min. 17 sec. of time?

hr.	min.	sec.
2	10	17
		15
32	34	15

Since a difference of 1 hr. of time is caused by a difference of 15° in longitude, 1 min. of time by 15′ of longitude, and 1 sec. of time by 15″ of longitude, we multiply the number of hours, minutes, and seconds by 15 to find the corresponding number of degrees, minutes, and seconds of longitude.

393. When it is 4 P.M. at Anda it is 6.30 P.M. at Roseville. What is the difference in longitude, and which place is further east?

Find difference in longitude and relative position of places whose simultaneous time reckonings are as follows:

394. A, 3 hr. 30 min. P.M. B, 5 hr. 10 min. 30 sec. P.M.

395. C, 11 hr. 30 min. A.M. D, 12 hr. 30 min. P.M.

396. E, 10 hr. 30 min. 30 sec. A.M. F, 2 hr. 30 min. 45 sec. P.M.

397. G, Noon H, 10 hr. 30 min. 25 sec. A.M.

398. J, 9 hr. 30 min. P.M. K, Midnight.

399. L, 8 hr. 40 min. 30 sec. P.M. M, 5 hr. 20 min. 10 sec. P. M.

400. N, 7 hr. 30 min. 15 sec. A.M. O, 1 hr. 10 min. 15 sec. P.M.

401. P, 9 hr. 20 min. 10 sec. A.M. Q, 3 hr. 5 min. 30 sec. P.M.

402. The longitude of the following places is reckoned from the meridian of Greenwich.

Portland, Me.,	70° 15′ 18″ W.	Chicago, Ill.,	87° 37′ 0″ W.
Boston, Mass.,	71° 3′ 50″ W.	New Orleans, La.,	90° 5′ 0″ W.
New York City,	74° 0′ 36″ W.	Omaha, Neb.,	95° 56′ 14″ W.
Washington, D.C.,	77° 2′ 48″ W.	Paris, France,	2° 20′ 0″ E.
Pittsburg, Pa.,	80° 2′ 0″ W.	Rome, Italy,	12° 28′ 0″ E.
Louisville, Ky.,	85° 30′ 0″ W.	Vienna, Austria,	16° 23′ 0″ E.
Nashville, Tenn.,	86° 49′ 0″ W.	St. Petersburg, Russia,	30° 18′ 0″ E.

The difference in time between Portland, Me., and a place west of it is 2 hr. What is the longitude of that place?

403. In what longitude is a place that has 2 hr. 10 min. later time than Louisville?

404. In what longitude is a place where it is half past ten in the morning at the same instant when it is noon at Pittsburg?

What is the longitude of a place which has:

405. 3 hr. 30 min. later time than Boston?

406. 2 hr. 20 min. earlier time than New York City?

407. 1 hr. 10 min. earlier time than Nashville?

408. 2 hr. 15 min. later time than New Orleans? Louisville?

409. 3 hr. 15 min. 30 sec. later time than New Orleans?

410. 1 hr. 20 min. later time than Paris?

411. 1 hr. 40 min. later time than Rome?

412. 3 hr. earlier time than Rome?

413. 4 hr. earlier time than St. Petersburg?

414. 3 hr. later time than Washington, D.C.?

415. 4 hr. 30 min. later time than Boston?

416. A difference of $75° 45' 15''$ between two places causes how much difference in time?

$$15\overline{)75° 45' 15''}$$
$$5\ \ 3\ \ 1$$

Since 15° of longitude cause a difference of 1 hr. in time, 15′ of longitude, 1 min. in time, and 15″ of longitude 1 sec. in time, we divide the number of degrees, minutes, and seconds of longitude by 15 to find the corresponding numbers of hours, minutes, and seconds of time.

417. A city is $30° 15' 45''$ west of a certain meridian. What is the difference in time between that city and all places on that meridian?

418. There are two meridians $24° 48' 30''$ apart. When it is 2 P.M. at places on the western meridian, what time is it at the places on the eastern meridian?

What is the difference in time between the following places:

419. Portland, Me., and Omaha?

420. Boston and Chicago?

421. Washington, D.C., and New Orleans?

422. Pittsburg and Omaha?

423. New York and Paris?

424. Boston and St. Petersburg?

425. Washington and Rome?

426. Nashville and Vienna?

427. Two persons, one in Paris and the other in Rome, agreed to read a certain poem at the same time. If the time selected by the one in Paris is 9 P.M., at what time by the clocks in Rome must the other person begin reading the poem?

A system of standard time has been adopted in the United States by which the difference in time between places differs by whole hours or not at all. The meridians 60°, 75°, 90°, 105° and 120° west from Greenwich are called time meridians. Places within $7\frac{1}{2}$° east or $7\frac{1}{2}$° west of the meridian of 75° have the time of that meridian, which is called Eastern Time. The time within $7\frac{1}{2}$° either side of the meridian of 90° is called Central Time. The time within $7\frac{1}{2}$° either side of the meridian of 105° is Mountain Time. The time within $7\frac{1}{2}$° either side of the meridian of 120° is Pacific Time.

428. How many hours by standard time are there between places which have Eastern Time and those which have Mountain Time? Pacific Time? Central Time?

429. If school begins at 9 o'clock, how long have the children in Denver been in school when the morning session begins on the Pacific coast? How long have the children in Boston been in school? The children in Chicago?

430. If the afternoon session begins at 1 P.M. and closes at 3 P.M., in what part of the country is the afternoon session just beginning when the children in Washington, D.C., are being dismissed?

431. A train entered a city at 10 P.M. Central Time. After a stop of 10 min. the train left the city at 9 hr. 10 min. Mountain Time. In what direction was the train running? What was the longitude of the city?

432. Is the present time of Boston slower or faster than the old local time?

MISCELLANEOUS EXERCISES

1. Simplify $\frac{16\frac{2}{3}}{100}$ $\frac{16\frac{2}{3}}{50}$ $\frac{16\frac{2}{3}}{33\frac{1}{3}}$ $\frac{16\frac{2}{3}}{83\frac{1}{3}}$ $\frac{16\frac{2}{3}}{66\frac{2}{3}}$.

2. Write a complex fraction in which each number in the numerator is prime and each number in the denominator is composite. Simplify it.

3. Make a fraction whose numerator is the only prime between 90 and 100, and whose denominator is the product of all the primes between 80 and 90. Express that fraction as per cent.

Which is greater, and how much:

4. $3\sqrt{441}$ or $\sqrt{3721}$?

5. $5\sqrt{676}$ or $13\sqrt{121}$?

6. What number squared equals 77,284?

7. Express 2 mi. 20 rd. 3 yd. as yards. As rods. As miles.

8. At 7¢ a foot, what is the cost of $2\frac{1}{2}$ mi. of telephone wire?

9. At 7¢ a foot, how much will it cost to fence a square lot containing 1521 sq. rd.?

10. If $x = 4$, how much is x^2? $\sqrt{x}$? $5\sqrt{x}$? $\sqrt{9x}$? $7\sqrt{25x}$? $x\sqrt{64}$? $\sqrt{21+x}$? $\sqrt{x+45}$? $\sqrt{85-x}$?

11. If $x = 9$, and $y = 16$, how much is $\sqrt{x}+\sqrt{y}$? $\sqrt{x+y}$?

Solve.

12. $8 : x = x : 2$.

13. $9 : x = x : 16$.

Write mean proportionals between the following numbers:

14. 2 and 32.

15. 2 and 18.

16. 2 and 50.

17. 8 and $4\frac{1}{2}$.

18. 3 and 27.

19. $2\frac{1}{2}$ and 10.

20. $12\frac{1}{2}$ and 50.

21. $8\frac{1}{3}$ and 75.

22. $6\frac{1}{4}$ and 100.

23. $\frac{2}{3}$ and 54.

24. Name all the demominations in the table of metric linear measure. Of metric square measure. Of metric cubic measure. Of metric measure of capacity. Of metric weight. Give the meaning of the prefixes in the metric tables.

25. What is the weight of a cubic centimeter of water? Of a liter of water?

26. A tank 3 meters long and 2 meters wide is filled with water to the depth of $1\frac{1}{2}$ meters. How many kiloliters of water are in it?

27. How many meters are there in the perimeter of a right triangle whose base is 27 centimeters and altitude 120 centimeters?

28. There are 2400 square decimeters in the surface of a cube. How many cubic centimeters does it contain?

29. Find the cost of digging a cellar 2 dekameters long, $1\frac{1}{2}$ dekameters wide, and 6 meters deep at 10¢ a cubic meter.

30. The measure of a meter was found by taking as nearly as possible $\frac{1}{10000000}$ of the distance from the equator to the pole. When a man has traveled one kilometer north from the equator, how far is he from each pole?

31. What are the dimensions of a cube that holds a milliliter? A kiloliter?

32. What is the unit of land measure in the metric system? What are its dimensions?

33. One side of a piece of land in the form of a right triangle is 2.7 kilometers, the side perpendicular to it is 3.6 kilometers. What is the value of the land at \$ 50 a hectare? What is the cost of fencing the land at 20¢ a meter?

34. Give the dimensions of the unit of wood measure in the metric system.

35. At \$1.50 per stere, what is the value of a pile of wood 8 m. long, 4 m. wide, and $2\frac{1}{2}$ m. high?

36. A circle is inscribed in a square whose side is $10\frac{1}{2}$ in. Find the area of the circle.

37. Can you write two fractions, such that the quotient of the greater divided by the less is $\frac{3}{8}$?

38. Add 3 to each term of a proper fraction and determine whether the resulting fraction is greater or less than the original fraction.

39. Add 3 to each term of an improper fraction and determine whether the resulting fraction is greater or less than the original fraction.

40. Name three perfect squares whose sum is 14. 21. 26. 29. 30. 35. 38. 42.

41. Name two perfect cubes whose sum is 9. 35. 65. 133.

42. The Kohinoor diamond weighs 103 carats. The Star of Brazil weighs 125 carats. The smaller equals what per cent of the larger? The larger equals what per cent of the smaller?

43. Virginia tobacco contains 7% nicotine. How much nicotine in a ton of it?

44. Some India rubber was bought for $12\frac{1}{2}¢$ a pound where it was grown, and was sold at 50¢ a pound in this country. What was the per cent of increase in price?

45. How many degrees in an arc which is 40% of a circumference? 60% of a circumference? 50% of a quadrant?

46. If an arc of 45° is 17 in. long, how long is the whole circumference?

47. Find the number of which 40 is $\frac{2}{3}$%.

48. 120 rd. are $8\frac{1}{3}$% of how many rods? $12\frac{1}{2}$%? $6\frac{1}{4}$%?

49. If the decoration of a building required 3000 cu. yd., 25 cu. ft of stone, and that was 7% of the whole amount used in constructing the building, how much was used?

50. Draw four isosceles right triangles whose equal sides are each 6 in. Arrange them so as to form a square. An oblong. A right triangle. A rhomboid. A trapezoid. An irregular pentagon. An irregular hexagon. Find the perimeter and area of each figure.

51. If the rainfall on a certain day were $\frac{5}{8}$ of an inch, how many gallons of water would fall on an acre of land?

52. A boat can sail 15 mi. an hour down the river, and 10 mi. an hour up the river. If it sails down the river for 2 hr. and then returns, how long a time will elapse between its departure and its arrival?

53. $1500 equal 25% more than A's money, and 25% less than B's. How much more has B than A?

54. A box 28 in. by 18 in. by 14 in. is filled with packages of coffee, each 7 in. by $3\frac{1}{2}$ in. by 3 in. Find the value of all at 10¢ per package.

55. If 5 boys do a piece of work in 8 hr., how long will it take a man who works twice as fast as a boy?

56. A dealer bought a number of stoves for $240, paying the same amount for each. If he had bought another dozen of stoves at the same price the cost of both invoices of stoves would have been $360. How much did each stove cost?

57. Mr. Dean earns $1.25 per day and his brother $1.75 per day. How many days more are required for Mr. Dean to earn $350 than for his brother to earn it?

58. A man bought a cow for $35, a horse for $3\frac{1}{5}$ times as much as the cow, and a wagon for $1 more than $\frac{1}{3}$ of the cost of the cow and the horse. How much was paid for both horse and wagon?

59. A man and his wife received $270 each year from money which they had at interest. The man received $30 more than 3 times as much as his wife. How much did each receive?

60. The sum of $24,000 was divided between A, B, and C so that A received $\frac{3}{4}$ as much as B, and C $4000 less than A and B together. How much did each receive?

61. A man has two fields, containing 10 A. and $12\frac{1}{2}$ A. respectively. Find the length, in rods, of the side of a square field equal in area to both fields.

62. Construct, if possible, triangles whose sides have the following lengths. Measure their angles with a protractor, and tell whether each triangle is right, acute angled, or obtuse angled.

(*a*) 2 in., 3 in., 4 in. (*b*) 2 in., 4 in., 5 in. (*c*) 3 in., 4 in., 5 in.

(*d*) 3 in., 4 in., 8 in. (*e*) 3 in., 4 in., 7 in. (*f*) 3 in., 4 in., 6 in.

63. Three lines being given, when is it impossible to construct a triangle with them?

64. Can a pyramid be constructed whose base is 8 in. square and whose slant height is 3 in.? Explain.

65. A toy table 10 in. high is an exact model of a study table 30 in. high. If a leg of the large table is $19\frac{1}{2}$ in. long, how long is a leg of the small table? If the area of the top of the small table is 120 sq. in., what is the area of the top of the large table? If a drawer of the small table contains 40 cu. in., how many cubic inches does the corresponding drawer of the large table contain?

66. A train running at the rate of 40 mi. an hour starts from Newburg, to go to Ironton, a distance of 245 mi. At the same time another train going 30 mi. an hour starts from Ironton to go to Newburg. In how many hours will they meet?

67. Atmospheric pressure is computed to be 15 lb. to the square inch. At that rate how many pounds of pressure are upon the top of a round table 15 in. in diameter?

68. Suppose Fig. 38, page 401, to represent the upper surface of a cheese 16 in. in diameter and 6 in. high, from which a part has been cut whose upper surface is represented by the sector AOB. How many cubic inches are in the part that is left?

69. A cylindrical tank $1\frac{3}{4}$ ft. in diameter and 6 ft. high is half full of water. Assuming that a cubic foot of water weighs 1000 oz., how many pounds of water are in the tank?

ANSWERS

Page 7. —**6.** Diff. = 396. **7.** 693. **8.** Sum = 1332.
Page 8. —**15.** Sum = 14.
Page 11. —**41.** 13,333 min. 222 hr.; 22 da. **44.** 232,277,043.
Page 12. —**50.** 123,952. **54.** $42,805,223. **56.** $80,602,555. **57.** 51,178,800 sq. mi.
Page 13. —**58.** $75,568,200. **59.** 47,200,000 mi.
Page 14. —**70.** $841,134,651. **71.** $150,700,575. **72.** 2,692,936,773 lb. **73.** 1,487,900,000.
Page 15. —**77.** $\frac{1}{21}$ of it = $476,190\frac{9}{21}$. **85.** Sum = 14,778.
Page 16. —**86.** 546. **87.** 54 yr. **89.** 16.
Page 17. —**98.** $1843.
Page 23. —**157.** 160 pt.; 80 qt.; 20 gal.
Page 24. —**161.** *a* 12; *b* 18; *c* 12; *d* 8; *e* $\frac{3}{28}$; *f* 1; *g* 8; *h* $1\frac{3}{5}$; *i* $\frac{1}{15}$; *j* 5; *k* $\frac{1}{2}$; *l* 1. **162.** *a* $2\frac{1}{2}$; *b* $1\frac{1}{2}$; *c* 8; *d* 14; *e* 7; *f* $21\frac{3}{5}$.
Page 27. —**198.** *a* 1424.234; *b* 727.65; *c* 2716.0815. **200.** *a* 287.703; *b* 2691.444; *c* 4283.617; *d* 28446.898; *e* 97333333.258.
Page 28. —**201.** *a* 12 in.; *b* 14 sq. yd.; *c* 8.3 sq. ft.
Page 29. —**210.** *a* 124.8585; *b* 24377.68125; *c* 325688.582535. **217.** 245.14 mi. **218.** 11.5 in.
Page 30. —**220.** 884.44 mi. **222.** *a* $137.96; *b* $163.61; *c* $215.61; *d* $1566. **223.** *a* 416.57; *b* 335.88; *c* 456.25; *d* 121538.027; *e* 681357.09.
Page 31. —**234.** *a* 155.15; *b* 270.535; *c* 311.15; *d* 706.175. **235.** 61.7 A. **236.** 158.2 sq. yd. **237.** *a* .96; *b* 299.92; *c* .2; *d* 5960.317; *e* 799.61; *f* 8297.593. **239.** 9.9; 101.7. **240.** 116.866; .364. **241.** 292.862; 479999.517. **242.** .629. **243.** 2.267. **244.** 9999.99. **245.** 9999999.9. **246.** 499.95.
Page 32. —**247.** 799999.992. **248.** .45. **249.** 9.25. **250.** $36.40. **251.** $48.75. **252.** $66.47. **253.** $88.46. **254.** $35.50. **255.** $268.25. **256.** $811.74. **257.** $.06. **258.** $96.04. **259.** $10.91. **261.** 1750.3. **262.** 653.5; 649.7; 649.75. **264.** 6.6 in.
Page 33. —**265.** 3 in. **266.** 3.1 in. **267.** 23.4 rd. **268.** 2 ft. **269.** 2.2 ft. **270.** 12 ft. **271.** 15 ft. **272.** 3.8 ft.
Page 34. —**273.** $3.80 total cost; $.47½ average cost.
Page 35 —**288.** 28 cm.
Page 36. —**291.** 28 ft. **293.** 54.6 cm. **294.** 54. cm. **297.** 4.5 cm.
Page 37. —**298.** 39 cm.; 24 cm. **299.** 27.2 cm.; 6.8 cm. **300.** 10.2 cm. **301.** 3.24 cm.; .54 cm. **302.** .54 cm.; .27 cm.; 1.08 cm.; 1.62 cm.; .54 cm. **305.** 40 cm.; 400 mm.
Page 38. —**310.** 100 sq. mm. **311.** 100 sq. cm.; 10,000 sq. mm. **313.** 100 sq. dm; 10,000 sq. cm.; 1,000,000 sq. mm. **315.** $218.09. **316.** $6300. **317.** $2.75. **318.** $1.25. **319.** $132.35. **320.** $387.75.
Page 39. —**321.** $110.93. **322.** $32.45. **324.** $297.16.

325. $238.75. **326.** $119.75. **327.** 23.75; 95; 142.5. **330.** *a* 2870.22. *b* 232.925; *c* 336.6; *d* 55.2; *e* 2000.7.

Page 40. — **332.** 7.7049. **336.** 61.9997. **337.** 217.802. **338.** 26.25 in. **339.** 24.75 in.; 14.625 in. **340.** 23.95.

Page 41. — **342.** 36.19; 25.85. **344.** 13 in. **346.** $10.44. **347.** $.45. **348.** .01792; .02816. **349.** $3.96. **350.** $210. **351.** $91.50.

Page 42. — **353.** $12.30. **354.** $6.34. **358.** 10; 100. **361.** 126.25. **362.** 3.2825. **363.** 787.5. **364.** 212.787. **365.** 3776.22.

Page 43. — **373.** 227.5; 42.25; 40.625. **374.** *a* 302.5; *b* 48.11; *c* 417.13; *d* 140.91; *e* 1.289. **375.** 16 sq. in. **376.** 116.25. **377.** 59.45. **380.** 2.257; 6.771; 18.056.

Page 44. — **384.** *a* .471; *b* 10.15; *c* 248.52; *d* 2.9282; *e* 21.9863; *f* 36.206; *g* 5.412; *h* .7381; *i* 2.39041; *j* .492. **386.** .045; .0245; .0014; .000168. **387.** *a* 2.25; *b* .0256; *c* 5.29; *d* .000081; *e* .0324; *f* 3.61; *g* 10.24; *h* .002601; *i* 7.84; *j* 16.6464; *k* .000625. **388.** 2.25 sq. dm. **389.** 1.56 sq. dm. **390.** 1.5625 sq. in. **391.** 26.25 sq. in. **392.** $3093.75; $3781.25.

Page 45. — **393.** 43.3125 sq. in. **401.** 87%.

Page 46. — **404.** 2.64. **405.** 1.68; 2.34; 3.78; 8.64; 8.1. **407.** 93. **408.** .7; 1.2; 3.75; .0915; .322; .186; .4245. **409.** $144; $1104.00. **410.** $62.51; $3062.99.

Page 47. — **420.** .369; .1476; .5904. **421.** *a* 4.91; *b* 6.47; *c* .72; *d* 3.6; *e* 49.1; *f* 2.65; *g* 106.1; *h* .221.

Page 48. — **422.** 2.5 in. **423.** .979; 3.841; 74.7; 523.59. **424.** $2476.95. **425.** $20.61. **427.** .004; .008; .123. **428.** .00216; .000252; .0151275; .003229. **430.** .2233+; 9.6777+; .0182+; .0677+; 25.4666+. **431.** 1.937+; 27.366+; 12.960. **433.** .42; 1.24; .493; .1969. **434.** .17; .051; .00357; .64.

Page 49. — **435.** .0144; .0216; .010066 .0231. **436.** 1.26; 1.98; .72. **437.** $.21. **438.** $.72. **439.** $83.33¼. **440.** $24.80; $132.20; $189.20; $2564.60; $13,569.40. **441.** 12.16; 11.68; 57.5. **442.** 144.06 A. **443.** $3.50. **444.** 1922.76; 2380.56; 4455.92; 6729.66. **445.** 97.375; 147.6. **446.** $.75; $.875; $1.25. **447.** $.33⅓. **448.** *a* $.01; *b* $.0075; *c* $.015. **449.** 100 qt.

Page 50. — **450.** $.01¾. **451.** $.02. **452.** 4.32. **453.** 18.8. **454.** 4.9. **455.** 12.5; 137; 479.1; 699; 53.5. **456.** 3690; 220; 910; 1520; 10.6; 99,000; 35,000. **457.** *a* 2610; *b* 12,100; *c* 355; *d* 250; *e* 1105; *f* 480; *g* 520; *h* 545; *i* 750; *j* 14850.

Page 51. — **458.** 80 lessons. **459.** 17 yd.; 31 yd. **460.** *a* 1.5; *b* 1.1; *c* 110; *d* 330; *e* 4.4; *f* .17; *g* 2.4; *h* 38; *i* 520; *j* 2600. **461.** *a* 7 ft.; *b* 2.5 ft.; *c* 1.1 ft.; *d* .25 cm.; *e* .8 in. **1.** 5438.053459. **2.** 11899.981. **3.** .21894. **4.** 248.231+. **5.** 6666. **6.** 363; 836; 1364; 1144; 803; 286. **7.** 12.

Page 52. — **8.** 61209.171911. **9.** 255889.0021. **10.** 2.1875 sq. ft.; 6 ft. **11.** 1.2 in.; 5.6 in. **12.** 1200 times. **13.** $255.09. **14.** $1422.566. **15.** $6000 wife; $3600 daughter; $2400 library. **16.** $.20. **18.** $183. **19.** 500 lb.; 200 lb.; 400 lb. **20.** $957. **21.** $9900 first year, $10,890 second year, $11,979 third year; $2979 more.

Page 53. — **24.** 12. **25.** 23; 19. **26.** 13. **27.** 19; 27; 18; **15.** **28.** 95%. **29.** 11. **30.** 7.893+. **31.** $33.96¾. **32.** 48.8 ft.

Page 54. — **33.** 34.55 ft. **34.** $3500. **36.** 89.25°. **37.** 20°. **38.** 66°; 114°. **39.** 28°. **40.** 19°; 21°; 20°. **41.** 9° below zero. **42.** 6° above zero. **43.** 10°.

Page 55. — **50.** 17.3 in. **51.** 8.9 in.

Page 57. — **76.** 70 in. **77.** 14 sq. in.

Page 58. — **80.** 11.875 m. **81.** 1.83 in.

Page 59. — **88.** $11.25. **89.** $40. **90.** 45 lb. **91.** $.025. **92.** $.025; $8.70. **93.** 8. **94.** $1432 cost; $232 lost. **95.** $5968. **96.** $46.25. **97.** $1.40. **98.** $2691. **99.** $1987.20.

Page 60. — **100.** $1143. **101.** $350.38. **102.** $27.42. **103.** $1102.50. **104.** $20. **105.** $14.85. **106.** 102 houses. **107.** 43 pupils. **108.** $17. **109.** 85.5 rd.

Page 61. — **110.** $129.50 spent; $170.50 left. **112.** 169; 2.25; 2.89. **115.** 605 sq. in. **116.** 12.4 in.; 9.61 sq. in. **117.** 12.5 in.; 37.5 sq. in. **118.** 30 in.; 337 5 sq. in.

Page 62. — **6.** 8; .49. **8.** $5.25.

Page 63. — **9.** 3000 sq. ft.

Page 64. — **28.** .00021.

Page 65. — **29.** .8. **30.** .135. **35.** 17. **36.** 4.5. **37.** 6.25. **38.** 4.6.

Page 66. — **39.** .0076. **40.** 1.38. **42.** 63. **43.** 77. **47.** 11; 41. **48.** 12.

Page 67. — **53.** 52; 112; 68; 128.

Page 69. — **66.** $51 = 3, 17$; $55 = 5, 11$; $57 = 3, 19$. **68.** $70 = 2, 5, 7$; $75 = 3, 5^2$; $80 = 2^4, 5$; $85 = 5, 17$; $90 = 2, 3^2, 5$; $72 = 2^3, 3^2$; $75 = 3, 5^2$; $78 = 2, 3, 13$; $81 = 3^4$; $84 = 2^2, 3, 7$; $87 = 3, 29$; $90 = 2, 3^2, 5$; $91 = 7, 13$; $92 = 2^2, 23$; $93 = 3, 31$; $94 = 2, 47$; $95 = 5, 19$; $96 = 2^5, 3$; $98 = 2, 7^2$; $99 = 3^2, 11$; $100 = 2^2, 5^2$. **69.** $a = 2^4, 3, 7$; $b = 2^7, 3^3$; $c = 5^2, 7^2$; $d = 2, 3^3, 41$; $e = 3^2, 71$; $f = 2^4, 5, 7$; $g = 3^3, 5, 23$; $h = 2^3, 3, 37$; $i = 2, 3, 5, 7^2$; $j = 2, 3^4, 5$; $k = 2^4, 3^4$; $l = 2^4, 3, 31$.

Page 71. — **83.** a 6; 10; 20; 210; b 30; 24; 30; 24; c 12; 24; 56; 24.

Page 73. — **101.** a 12600; b 240; c 504; d 48; e 134; f 90. **103.** a 2520; b 144; c 1008; d 336; e 120; f 140.

Page 78. — **155.** 4 lb. **156.** 7 ft., 14 pieces.

Page 79. — **161.** a 12; b 12; c 35; d 7; e 32; f 36; g 15; h 22; i 77; j 9; k 32; l 17; m 25; n 15; o 5; p 5; q 18; r 22; s 1; t 8; u 14; v 27; w 12; x 4.

Page 80. — **164.** 1; 900; 2500; 14,400; 225; 400. **166.** 9; 4; 16. **167.** 16 and 36; 9 and 49; 4 and 64; 1 and 81. **169.** 6.4; 12.1. **170.** 1332. **171.** 8; 27; 64; 125; 1. **174.** 35.84; 64. **175.** 3.84; 43.74; 20.58; 79.86; 7500; 103.68; 12,960. **176.** 8; 27; 8; 8; 27.

Page 81. — **177.** 8 cu. in.; 27 cu. in.; 64 cu. in.; 125 cu. in.; 216 cu. in.; 343 cu. in.; 512 cu. in.; 729 cu. in. **182.** 8. **183.** 8.

Page 82. — **187.** 600 sq. cm. **188.** $7.80. **193.** 1; 243; 3125. **194.** 64; 4096. **195.** 10,000,000. **196.** 12; 18; 50.

Page 83. — **197.** a 45; b 28; c 648; d 432; e 96; f 320; g 297; h 1375; i 637; j 4840; k 2400; l 14,175. **200.** .09; .21; .07. **201.** .12; .09; .18; .04. **202.** 6.16; 9.24; 5.04; 7.92; 6.93; 13.86; 18.48; 4.62. **204.** 5; 2; 6; 3. **205.** 3.6; .6; .084. **206.** .72; .48; .54. **207.** 9; 12. **208.** 15. **211.** 28 in.

Page 84. — **212.** $25. **217.** 82. **218.** .005; 6. **219.** 2; 6. **220.** 3; 2; 77. 1. .052624.

Page 85. — **3.** 9.7 ft. **4.** 19.1 ft. **5.** 104.375. **6.** 75.16. **10.** 2.761; 3.8; .013. **14.** 9 lb.; 15 lb.; 25 lb.; 50 lb.; 160 lb. **15.** $4\frac{1}{2}$ yr.

Page 86. — **16.** 1440. **17.** 5,17. **18.** $5\frac{1}{2}$. **19.** 72. **20.** 22; 16. **21.** 258.4. **22.** .315. **23.** $a\ \frac{1}{2}$; $b\ \frac{1}{10}$; $c\ \frac{1}{2}$; $d\ \frac{1}{2}$; $e\ \frac{1}{16}$; $f\ \frac{1}{4}$; $g\ 1\frac{1}{4}$; $h\ \frac{1}{18}$; $i\ \frac{1}{8}$.

Page 87. — **36.** $\frac{1}{4}$ wrong; $\frac{3}{4}$ right. **37.** $1600; $6400. **38.** $.12\frac{1}{2}$. **39.** 4 qt.; $.24. **40.** 870 cu. cm. **42.** 10 in.; 7 in.; 15 in.

Page 88. — **44.** 5 ft. **46.** 40.5 in. **48.** 18 in. **49.** 6 in.; 9 in. **50.** 16.2 in.

Page 89. — **55.** 19 ft. **56.** 7.2 ft. **58.** 25.875 in. **59.** 32 cm.; 48 cm.

Page 90. — **65.** 32 girls. **66.** $20; $400; $800. **67.** $1500; $2000; $3000. **68.** $1000; $700. **69.** 55 in. **70.** $.09; $.15; $.30; $.75; $1.20; $7.50. **71.** $5; $1; $.38.

Page 91. — **73.** *a* $1.20; *b* $.25; *c* $.02; *d* $1.80; *e* $1.12; *f* $.20. **74.** 45 yr. **75.** $264.44. **76.** $800 int.; $2000 am't. **80.** 80 sq. cm.; 25 sq. cm.; $33\frac{1}{3}$ sq. cm.

Page 92. — **84.** $83\frac{1}{3}\%$. **88.** 42 in.

Page 93. — **93.** 15; 25; 40; 55; 120. **94.** 7¢; 21¢; $17\frac{1}{2}$¢; 35¢; 63¢; 70¢. **95.** 11¢; 33¢; $27\frac{1}{2}$¢; 55¢; 99¢; $1.10. **97.** 83⅓. **102.** Base $\frac{1}{4}$; Perpen. $\frac{1}{8}$; Hypot. $\frac{5}{12}$. **103.** Base $\frac{1}{4}$; Perpen. $\frac{1}{8}$; Hypot. $\frac{5}{12}$.

Page 94. — **104.** 20 in. **105.** 19 odd; 23 even.

Page 96. — **12.** $\frac{1}{8}$; $\frac{1}{7}$.

Page 97. — **13.** $\frac{1}{4}$; $\frac{2}{4}$; $\frac{1}{2}$; $\frac{3}{4}$; 1. **14.** $\frac{1}{3\,2\,7}$. **15.** 1280. **16.** $\frac{1}{3\,1\,9}$. **17.** $\frac{16}{3\,0\,4}$; $\frac{32}{2\,8\,8}$; $\frac{80}{2\,4\,0}$; $\frac{120}{2\,1\,0}$. **18.** $\frac{1}{16}$. **19.** $\frac{1}{8}$. **20.** $\frac{1}{2\,0\,0\,0}$. **21.** $\frac{1}{1\,0\,9\,9}$.

Page 98. — **25.** $145\frac{6}{17}$. **26.** .79. **31.** 21 mi.; 28 mi.; 28 mi.; 14 mi. **33.** $\frac{2}{3}$; 3; $\frac{1}{2}$.

Page 99. — **36.** $\frac{3}{8}$; $\frac{1}{4}$. **37.** $\frac{14}{3\,3}$; $\frac{3}{4}$. **38.** $1\frac{2}{3}$; $\frac{3}{4}$. **39.** $1\frac{1}{2}$; $\frac{1}{3}$. **43.** $\frac{27}{64}$; $\frac{64}{2\,1\,6}$; $1\frac{9\,0\,2}{7\,2\,9}$; $\frac{2\,1\,6}{8\,0\,0\,0}$. **45.** $\frac{1}{2}$; $1\frac{1}{2}$; $1\frac{1}{2}$.

Page 101. — **61:** $45 per A.; $2600.

Page 102. — **72.** $3000; $2250; $3750. **1.** 70.31. **2.** 7.803. **3.** 28,000. **4.** 48 in. **5.** .9 cm. **6.** 1.69 cm.

Page 103. — **10.** $\frac{1}{8}$. **11.** 2, 3, 7, 13; 3, 3, 5, 11. **12.** $\frac{7}{1\,3}$; $\frac{7}{5}$; $\frac{7}{8}$. **13.** 8; $\frac{2\,4\,8}{1\,2\,5}$; $\frac{4\,4\,3}{1\,2\,5}$; $\frac{5\,1\,2}{1\,7\,2\,8}$. **14.** $\frac{1}{4}$; 1. **15.** 40 in. **16.** $\frac{2}{3}$. **17.** 3. **18.** The 3d. **19.** 4th; 6th; 8th. **21.** 400 rd.; 80 rd.

Page 104. — **24.** 77 mi. **25.** 1¢. **26.** $64. **27.** $4. **29.** $1.50. **31.** $.56; $.88. **32.** $.63; $.84.

Page 105. — **37.** $184. **38.** $\frac{25}{65}$. **41.** 20 sq. in. **43.** 36 sq. cm.; $175\frac{1}{2}$ sq. in. **44.** 18 sq. in.

Page 106. — **48.** 34 in. **49.** 8.5 feet. **50.** 48 in. or 4 ft.; 4 ft. 6 in.; 6 ft. **53.** 30 cm.; 80 cm.; 125 cm. **54.** 40 in.; 44 in. **55.** 28 in.

Page 107. — **56.** 2 ft.; 5 ft. **63.** 7.56 sq. in.

Page 109. — **12.** $\frac{15}{2}$; $\frac{10}{3}$; $\frac{25}{3}$; $\frac{50}{3}$; $\frac{50}{9}$; $\frac{70}{9}$; $\frac{41}{7}$; $\frac{81}{4}$; $\frac{60}{7}$; $\frac{107}{7}$; $\frac{87}{4}$; $\frac{62}{9}$; $\frac{29}{5}$; $\frac{10}{7}$; $\frac{47}{8}$; $\frac{82}{7}$. **14.** $\frac{73}{9}$; $\frac{88}{21}$; $\frac{136}{19}$; $\frac{460}{51}$; $\frac{151}{19}$; $\frac{83}{14}$; $\frac{63}{8}$; $\frac{94}{13}$; $\frac{106}{11}$; $\frac{148}{18}$; $\frac{40}{7}$; $\frac{23}{5}$; $\frac{47}{9}$; $\frac{49}{11}$.

Page 110. — **19.** $1\frac{1}{7}$; $5\frac{3}{8}$; $4\frac{1}{3}$; $2\frac{1}{4}$; $1\frac{1}{3}$; $1\frac{6}{13}$; $2\frac{1}{21}$; $7\frac{1}{25}$; $14\frac{1}{7}$. **21.** $1229\frac{1}{2}$; $1658\frac{3}{37}$; $31\frac{57}{67}$; 1; $3\frac{22}{225}$; $15\frac{6}{25}$.

Page 111. — **31.** 1 in.

Page 112. — **34.** *a* $\frac{1}{3}$; *b* $\frac{4}{15}$; *c* 35; *d* $\frac{8}{11}$; *e* $\frac{2}{5}$; *f* $\frac{3}{4}$; *g* $\frac{5}{12}$; *h* $\frac{1}{6}$; *i* $\frac{3}{10}$; *j* $\frac{1}{8}$; *k* $\frac{1}{6}$; *l* $4\frac{1}{2}$; *m* 3; *n* $2\frac{5}{8}$; *o* $5\frac{1}{8}$. **35.** 81 sq. cm. **36.** Sold 60 A.; 40 A. left. **37.** $\frac{1}{6}$; 6. **38.** $33,333\frac{1}{3}$. **39.** 8 cu. ft.; 27 cu. ft. **40.** 24 min.; 30 min.; **41.** $\frac{1}{21}$; $1\frac{1}{17}$. **42.** $\frac{1}{3}$; $1\frac{2}{3}$; $6\frac{2}{3}$.

Page 114. — **63.** $\frac{6}{7}$; $\frac{6}{7}$; $\frac{2}{3}$; $\frac{4}{5}$; $\frac{6}{7}$; $\frac{5}{11}$; $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{2}{11}$; $\frac{7}{22}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{5}{16}$; $\frac{1}{3}$; $\frac{7}{8}$; $\frac{6}{11}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{4}{7}$; $\frac{7}{8}$. **65.** 49; $\frac{3}{4}$. **66.** $\frac{21}{31}$; $\frac{11}{13}$; $\frac{12}{19}$; $\frac{11}{23}$; $\frac{13}{24}$; $\frac{5}{16}$; $\frac{1}{3}$; $\frac{1}{16}$; $\frac{7}{16}$; $\frac{6}{7}$; $\frac{2}{49}$; $\frac{2}{11}$; $\frac{1}{17}$; $\frac{2}{27}$; $\frac{3}{7}$.

Page 115. — **67.** 10 in.; 9 in.; 6 in. **68.** $.25. **69.** $.50.

Page 116. — **80.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{99}{100}$; $\frac{3}{4}$; $\frac{9}{125}$; $\frac{8}{125}$; $\frac{1}{20}$; $\frac{3}{8}$; $\frac{1}{8}$; $\frac{9}{16}$.

Page 117.—**91.** $\frac{1}{15}$. **98.** $\frac{11}{12}$; $\frac{13}{42}$; $\frac{19}{42}$; $\frac{11}{42}$; $\frac{13}{42}$. **99.** $1\frac{11}{12}$. **100.** $1\frac{1}{40}$.

Page 118.—**101.** $1\frac{17}{42}$. **102.** $\frac{11}{30}$. **103.** $\frac{13}{24}$; $\frac{19}{24}$; $\frac{11}{24}$; $\frac{7}{24}$; $\frac{13}{24}$; $\frac{17}{24}$. **105.** $\frac{1}{8}$; $1\frac{1}{12}$; $1\frac{5}{24}$; $1\frac{1}{12}$. **106.** *a* $\frac{1}{4}$; *b* $\frac{1}{10}$; *c* $\frac{1}{6}$; *d* $\frac{1}{24}$; *e* $\frac{1}{14}$; *f* $\frac{1}{14}$; *g* $\frac{1}{8}$; *h* $\frac{1}{8}$. **107.** *a* $1\frac{1}{12}$; *b* $1\frac{1}{12}$; *c* $\frac{3}{4}$; *d* $2\frac{1}{4}$. **108.** *a* $\frac{25}{28}$; *b* $1\frac{27}{28}$; *c* $\frac{53}{70}$; *d* $1\frac{23}{28}$. **109.** *a* $1\frac{1}{30}$; *b* $1\frac{17}{30}$; *c* $\frac{31}{60}$; *d* $2\frac{9}{60}$. **110.** *a* $2\frac{1}{5}$; *b* $1\frac{29}{30}$; *c* $1\frac{13}{30}$; *d* $2\frac{11}{120}$. **111.** *a* $1\frac{23}{24}$; *b* $\frac{11}{24}$; *c* $2\frac{9}{20}$; *d* $2\frac{37}{84}$. **112.** *a* $\frac{37}{42}$; *b* $2\frac{1}{33}$; *c* $\frac{31}{42}$; *d* $2\frac{11}{42}$. **113.** *a* $\frac{47}{84}$; *b* $1\frac{27}{60}$; *c* $1\frac{21}{40}$; *d* $\frac{1}{40}$. **114.** *a* $2\frac{1}{42}$; *b* $\frac{43}{90}$; *c* $1\frac{53}{90}$; *d* $1\frac{13}{40}$. **115.** *a* $\frac{7}{60}$; *b* $\frac{23}{60}$; *c* $\frac{23}{60}$; *d* $\frac{17}{200}$.

Page 119.—**116.** $\frac{1}{60}$; $\frac{1}{30}$; $\frac{2}{45}$; $\frac{1}{10}$; $\frac{2}{15}$. **117.** $\frac{7}{36}$; $\frac{7}{30}$; $\frac{7}{24}$; $\frac{1}{8}$; $\frac{7}{20}$. **119.** $1\frac{1}{8}$ mi. **122.** $\frac{5}{7}$. **123.** $1\frac{1}{4}$. **124.** $1\frac{17}{60}$; $\frac{13}{20}$. **125.** $2\frac{4}{35}$; $3\frac{7}{24}$. **126.** $2\frac{23}{36}$; $1\frac{1}{15}$. **127.** 100; 100,000; 1000. **129.** $\frac{819}{1000}$; $\frac{491}{1000}$. **130.** *a* .588; *b* .6895; *c* .146; *d* 1.295.

Page 120.—**132.** .647. **133.** .12; .055; .0177; .000393. **137.** 5.17625. **138.** .333. **139.** .6666+; .4285+; .8888+; .6; .56; .7857+; .7777+; .8333+; .68; .936; .8472+; .8645+; .1304+; .35; .3571+; .3809+; .225; .22. **140.** .568; .612$\frac{12}{43}$; .727$\frac{8}{11}$; .222$\frac{2}{9}$; .4; .04; .06; .625; .35.

Page 121.—**141.** .466$\frac{2}{3}$; .571$\frac{3}{7}$; .24; .222$\frac{2}{9}$; .363$\frac{7}{11}$; .5; .333$\frac{1}{3}$; 833$\frac{1}{3}$; .375. **142.** .666$\frac{2}{3}$; .5; .4; .333$\frac{1}{3}$; .285$\frac{5}{7}$; .25; .222$\frac{2}{9}$. **143.** .272$\frac{8}{11}$; .25; .230$\frac{10}{13}$; .214$\frac{2}{7}$; .2; .187$\frac{1}{2}$; .176$\frac{8}{17}$; .16$\frac{2}{3}$; .157$\frac{1}{7}$. **144.** .33$\frac{1}{3}$; .66$\frac{2}{3}$; .16$\frac{2}{3}$; .83$\frac{1}{3}$; .12$\frac{1}{2}$; .37$\frac{1}{2}$; .62$\frac{1}{2}$; .87$\frac{1}{2}$; 1.12$\frac{1}{2}$; .12$\frac{1}{2}$. **145.** 11$\frac{1}{9}$; 22$\frac{2}{9}$; 44$\frac{4}{9}$; 55$\frac{5}{9}$; 77$\frac{7}{9}$. **146.** 8$\frac{1}{3}$; 41$\frac{2}{3}$; 58$\frac{1}{3}$; 91$\frac{2}{3}$. **147.** 6$\frac{1}{4}$; 18$\frac{4}{7}$; 31$\frac{1}{4}$; 43$\frac{3}{4}$; 56$\frac{1}{4}$; 68$\frac{4}{7}$. **148.** 14$\frac{2}{9}$%; 28$\frac{4}{9}$%; 18$\frac{2}{11}$%; 45$\frac{5}{11}$%. **150.** *a* 20$\frac{5}{36}$; *b* 22$\frac{67}{105}$; *c* 16$\frac{1}{2}$; *d* 22$\frac{7}{18}$; *e* 17$\frac{9}{16}$. **151.** *a* 27$\frac{1}{12}$; *b* 41$\frac{2}{3}$; *c* 87$\frac{1}{4}$; *d* 162$\frac{1}{2}$.

Page 122.—**152.** 452$\frac{1}{12}$. **154.** 159$\frac{3}{4}$ A. **155.** 953 rd; $714.75. **156.** 127$\frac{3}{8}$ A. **157.** 3$\frac{1}{4}$ hr.; 2$\frac{3}{4}$ hr.; 3$\frac{1}{4}$ hr.; 2$\frac{3}{4}$ hr.; 4$\frac{3}{4}$ hr. **159.** *a* 1$\frac{1}{2}$; *b* 3$\frac{1}{4}$; *c* 13$\frac{1}{2}$; *d* 21$\frac{1}{2}$; *e* 4$\frac{5}{8}$; *f* 15$\frac{1}{2}$. **160.** $\frac{5}{8}$ yd. **161.** 7 in.

Page 123.—**163.** *a* 1$\frac{7}{8}$; *b* 3$\frac{8}{11}$; *c* 6$\frac{2}{3}$; *d* 6$\frac{5}{12}$; *e* 1$\frac{5}{8}$; *f* 3$\frac{1}{2}$; *g* 1$\frac{3}{4}$. **164.** 34$\frac{3}{4}$ ft.; 29$\frac{1}{4}$ ft. **166.** *a* 7$\frac{1}{2}$; *b* 5$\frac{4}{7}$; *c* 7$\frac{1}{4}$; *d* 5$\frac{1}{4}$; *e* 7$\frac{1}{2}$; *f* 5$\frac{2}{7}$; *g* 6$\frac{3}{4}$. **167.** *a* $\frac{3}{5}$; *b* 5$\frac{1}{4}$; *c* 2$\frac{1}{4}$; *d* 1$\frac{3}{5}$; *e* 1$\frac{1}{2}$; *f* 8$\frac{1}{11}$; *g* 2$\frac{1}{2}$; *h* 4$\frac{1}{2}$; *i* 6$\frac{2}{3}$; *j* 2$\frac{1}{2}$; *k* 5$\frac{1}{4}$; *l* 1$\frac{1}{4}$; *m* 3$\frac{1}{2}$; *n* 4$\frac{1}{10}$. **169.** *a* 2$\frac{1}{2}$; *b* 5$\frac{1}{10}$; *c* 6$\frac{1}{2}$; *d* 3$\frac{3}{14}$; *e* 5$\frac{5}{14}$; *f* 3$\frac{7}{12}$; *g* 3$\frac{1}{12}$. **Page 124.**—**172.** *a* 12$\frac{4}{7}$; *b* 2$\frac{11}{28}$; *c* 3$\frac{13}{14}$; *d* 3$\frac{5}{8}$; *e* 5$\frac{5}{8}$; *f* 1$\frac{7}{12}$; *g* 1$\frac{1}{2}$; *h* 6$\frac{1}{2}$; *i* 3$\frac{1}{2}$; *j* 2$\frac{11}{12}$; *k* 3$\frac{1}{4}$; *l* 1$\frac{47}{56}$; *m* 4$\frac{25}{56}$; *n* 1$\frac{13}{28}$. **175.** 1$\frac{1}{7}$; 1$\frac{1}{7}$; 2$\frac{2}{7}$; 6. **176.** *a* 3; *b* 1$\frac{1}{2}$; *c* 3$\frac{1}{4}$; *d* 9; *e* 5$\frac{1}{2}$; *f* 4$\frac{7}{9}$; *g* 5; *h* 4$\frac{1}{2}$. **177.** 643$\frac{1}{4}$ lb. **178.** 12$\frac{5}{8}$ lb.; 8$\frac{7}{8}$ lb.; 26$\frac{1}{4}$ lb.; 17$\frac{3}{8}$ lb.; 3$\frac{5}{8}$ lb.; 13$\frac{5}{8}$ lb.

Page 125.—**179.** 70$\frac{1}{3}$ mi. **180.** 57$\frac{11}{24}$ mi.; 34$\frac{5}{24}$ mi. **181.** 129$\frac{1}{2}$ A. **182.** 22,693$\frac{37}{40}$ A. **183.** 561$\frac{7}{8}$ lb. **184.** $\frac{1}{36}$. **185.** A $103,500; B $55,200; C $89,700. **186.** 89 A.; 93$\frac{5}{8}$ A.; 164$\frac{5}{8}$ A.; 169$\frac{1}{4}$ A.; 182$\frac{5}{8}$ A. **187.** 247$\frac{13}{32}$ rd.

Page 126.—**189.** $\frac{3}{10}$; $\frac{1}{9}$; $\frac{3}{8}$; $\frac{3}{11}$; $\frac{5}{12}$; $\frac{4}{9}$; $\frac{5}{14}$. **191.** *a* $\frac{7}{10}$; *b* $\frac{3}{10}$; *c* $\frac{7}{8}$; *d* $\frac{5}{12}$. **192.** *a* $\frac{5}{12}$; *b* $\frac{2}{7}$; *c* $\frac{2}{5}$; *d* $\frac{3}{4}$. **193.** *a* $\frac{1}{12}$; *b* $\frac{1}{7}$; *c* $\frac{2}{5}$; *d* $\frac{3}{8}$. **194.** *a* $\frac{3}{5}$; *b* $\frac{2}{5}$; *c* $\frac{1}{4}$; *d* $\frac{8}{15}$. **198.** $\frac{16}{49}$. **199.** $\frac{4}{9}$; $\frac{16}{25}$; $\frac{25}{49}$; $\frac{25}{81}$; $\frac{100}{121}$; $\frac{144}{169}$; $\frac{9}{49}$; $\frac{225}{256}$; $\frac{361}{400}$; $\frac{289}{324}$; $\frac{625}{900}$; $\frac{1681}{1849}$. **200.** $\frac{1}{4}$. **201.** $\frac{1}{3}$; 1 sq. ft. **202.** $\frac{8}{27}$.

Page 127.—**203.** $\frac{1}{8}$; $\frac{27}{64}$; $\frac{8}{343}$; $\frac{1}{216}$; $\frac{8}{1125}$; $\frac{27}{512}$; $\frac{125}{729}$; $\frac{343}{1000}$; $1\frac{61}{81}$; $\frac{343}{1728}$. **205.** $\frac{5}{12}$ sq. ft. or 60 sq. in. **206.** 32 in. perim.; $\frac{5}{12}$ sq. ft.; 60 sq. in. **207.** $\frac{9}{16}$ sq. ft.; 81 sq. in. **208.** $\frac{25}{36}$ sq. ft.; 100 sq. in. **209.** $\frac{1}{4}$. **210.** $\frac{7}{12}$. **211.** 105; $\frac{5}{27}$; $\frac{1}{10}$; $\frac{1}{2}$. **212.** 5$\frac{1}{2}$. **213.** *a* 9; *b* 20; *c* 15; *d* 56; *e* 6. **214.** *a* 70; *b* 14$\frac{2}{3}$; *c* 18; *d* 10$\frac{1}{2}$; *e* 10$\frac{2}{3}$. **215.** *a* 15; *b* 42; *c* 6$\frac{2}{7}$; *d* 6; *e* 23$\frac{1}{4}$. **216.** *a* 28; *b* 28; *c* 4$\frac{2}{3}$; *d* 9$\frac{1}{3}$; *e* 15.

Page 128.—**224.** *a* 231; *b* 348$\frac{1}{2}$; *c* 929$\frac{1}{3}$; *d* 700. **225.** *a* 512; *b* 1073$\frac{1}{4}$; *c* 676; *d* 948. **226.** *a* 495; *b* 1188$\frac{1}{2}$; *c* 408; *d* 1519. **227.** *a* 341; *b* 2239$\frac{1}{2}$; *c* 912; *d* 3904. **228.** *a* 1763; *b* 2616; *c* 1123$\frac{1}{2}$; *d* 1365.

Page 129.—**229.** *a* 2772; *b* 1998; *c* 1001; *d* 4794. **231.** $4.50; $5.00; $6.45; $7.44; $8.48; $9.23; $11.33. **232.** 120 sq. rd.; 100 sq. rd.;

144 sq. rd.; 150 sq. rd.; 15 sq. rd.; 128 sq. rd. **233.** $\frac{3}{4}$. **234.** 440 sq. rd. 900 sq. rd.; 1168 sq. rd.; 696 sq. rd. **236.** *a* 122; *b* 2220; *c* 477; *d* $2692\frac{4}{9}$. **237.** *a* 1990; *b* 1090; *c* 658; *d* 2952. **238.** *a* 219; *b* $1794\frac{1}{2}$; *c* 1298; *d* 492.

Page 130.—**240.** \$.75; $1.31\frac{1}{4}$; \$1.50; $1.87\frac{1}{2}$; \$2.25. **241.** $13\frac{1}{4}$ ft. **242.** 19 ft.; 38 ft.; $28\frac{1}{2}$ ft.; $23\frac{3}{4}$ ft. **243.** $2.66\frac{2}{3}$; \$3; \$4; $4.66\frac{2}{3}$; \$5; \$6. **245.** *a* $8\frac{1}{2}$; *b* $20\frac{5}{8}$; *c* $27\frac{1}{2}$; *d* $69\frac{3}{4}$; *e* $208\frac{1}{2}$; *f* $506\frac{1}{2}$. **246.** $6\frac{1}{2}$; $1\frac{19}{32}$; $11\frac{1}{3}$; $4\frac{22}{49}$; $39\frac{1}{16}$; $2\frac{7}{9}$; $2\frac{46}{49}$; $7\frac{9}{16}$. **247.** *a* $3\frac{3}{8}$; *b* $2\frac{19}{27}$; *c* $3\frac{1}{27}$; *d* $42\frac{7}{8}$; *e* $91\frac{1}{8}$; *f* $166\frac{3}{8}$; *g* $49\frac{5}{27}$; *h* $11\frac{25}{64}$.

Page 131.—**251.** $272\frac{1}{4}$ sq. ft. **252.** $71\frac{29}{40}$ sq. in. **253.** $99\frac{9}{98}$ sq. in. **255.** 22 in.; 66 in.; 44 in. **256.** 132 in.; 22 in.; 66 in.; 110 in. **257.** $14\frac{3}{8}$ in. **258.** 8. **259.** $5\frac{1}{2}$ ft. **260.** 8 in.; $25\frac{1}{7}$ in.

Page 132.—**262.** 4 in. diam.; $12\frac{1}{7}$ in. circ. **264.** 6 in.; $18\frac{8}{7}$ in. **265.** $9\frac{3}{7}$ in. **266.** $2\frac{5}{14}$ in. **267.** $7\frac{1}{14}$ in. **268.** $31\frac{3}{7}$ in. circ.; 20 in.; $17\frac{3}{7}$ in. **269.** 22 in.; 55 in.; 165 in. **270.** $5\frac{1}{2}$ in.; 11 in.; 11 in.; $16\frac{1}{2}$ in. **271.** 22 cm.

Page 133.—**274.** *a* $7196\frac{1}{4}$; *b* $7498\frac{4}{7}$; *c* $4218\frac{15}{28}$; *d* $7682\frac{5}{16}$. **275.** *a* $12{,}752\frac{4}{7}$; *b* $630\frac{1}{2}$; *c* $23{,}000\frac{2}{21}$; *d* $18{,}192\frac{1}{42}$. **276.** $4192\frac{9}{16}$; $1064\frac{25}{64}$; $616\frac{25}{36}$; $492\frac{4}{121}$; $1352\frac{49}{81}$. **277.** $1838\frac{17}{7}$; $4192\frac{25}{121}$; $3601\frac{17}{7}$; $2828\frac{225}{441}$. **278.** $448\frac{4}{5}$ mi. **279.** $25\frac{3}{10}$. **280.** $174\frac{3}{7}$. **281.** $162\frac{2}{15}$. **282.** $2.35\frac{3}{4}$. **283.** .01. **284.** $1\frac{377}{1000}$.

Page 134.—**285.** 1.377. **288.** *ab* .0008; *ac* .0002; *ad* .000028; *ae* .0000008; *af* .012; *bc* .0001; *bd* .000014; *bc* .0000004; *bf* .006; *cd* .0000035; *cf* .0015.

Page 135.—**301.** 3; 3. **303.** 4. **304.** *a* $\frac{7}{12}$; *b* $1\frac{1}{3}$; *c* $1\frac{1}{2}$; *d* $1\frac{11}{12}$; **305.** *a* $\frac{2}{3}$; *b* $\frac{4}{5}$; *c* $\frac{4}{5}$; *d* $\frac{2}{3}$. **306.** *a* $\frac{7}{8}$; *b* $\frac{5}{6}$; *c* $\frac{9}{10}$; *d* $\frac{3}{4}$. **307.** *a* $\frac{5}{8}$; *b* $2\frac{1}{2}$; *c* $\frac{5}{9}$; *d* $5\frac{3}{5}$. **308.** *a* $\frac{7}{8}$; *b* $\frac{3}{4}$; *c* $\frac{4}{5}$; *d* $1\frac{1}{3}$. **309.** *a* 6; *b* $2\frac{1}{4}$; *c* $3\frac{3}{8}$; *d* $\frac{4}{7}$. **310.** *a* $1\frac{1}{4}$; *b* $1\frac{1}{4}$; *c* $\frac{45}{77}$; *d* $\frac{2}{3}$. **311.** $3\frac{3}{8}$; $1\frac{3}{4}$; $2\frac{1}{2}$. **312.** *a* $\frac{5}{9}$; *b* $\frac{5}{6}$; *c* $\frac{4}{5}$; *d* $\frac{5}{6}$. **313.** *a* $\frac{3}{5}$; *b* $5\frac{7}{10}$; *c* $\frac{3}{4}$; *d* $\frac{2}{3}$. **314.** *a* $\frac{5}{6}$; *b* $2\frac{2}{3}$; *c* $1\frac{2}{7}$; *d* $1\frac{1}{2}$.

Page 136.—**323.** 56; 27; 44; 36; 30; 34; $28\frac{4}{7}$. **324.** $61\frac{1}{2}$; $54\frac{2}{3}$; $51\frac{1}{4}$; $47\frac{7}{8}$; $46\frac{1}{4}$; $44\frac{5}{12}$; $43\frac{1}{16}$. **325.** *a* $1\frac{2}{3}$; *b* $\frac{2}{5}$; *c* 5; *d* $\frac{1}{3}$. **326.** *a* $\frac{7}{10}$; *b* 3; *c* $\frac{8}{9}$; *d* $\frac{2}{3}$. **327.** *a* $1\frac{1}{15}$; *b* $2\frac{1}{4}$; *c* 2; *d* 28. **329.** $\frac{1}{8}$.

Page 137.—**330.** $3\frac{1}{3}$ A. **331.** $\frac{1}{6}$ ft. **332.** 3 in.; $1\frac{1}{2}$ in. **333.** $3\frac{1}{2}$ in. **334.** $18\frac{4}{7}$ in.; 6 in.; 3 in. **335.** 30 yd.; $22\frac{1}{2}$ yd.; 12 yd.; 9 yd.; 6 yd.; $4\frac{1}{2}$ yd.; 2 yd.; $1\frac{1}{2}$ yd.; $1\frac{1}{8}$ yd.; $\frac{9}{10}$ yd.; $\frac{6}{7}$ yd. **336.** 5. **337.** 50. **338.** 3 hr.; 36 min.; 24 min.; 12 min.; 6 min.; 3 min. **340.** $5\frac{1}{2}$; $1\frac{6}{7}$; $7\frac{6}{15}$; $6\frac{1}{8}$; $2\frac{1}{7}$.

Page 138.—**343.** *a* $1\frac{1}{7}$; *b* $\frac{3}{10}$; *c* $\frac{1}{3}$; *d* $\frac{3}{10}$; *e* $\frac{2}{7}$; *f* 2. **344.** *a* $\frac{1}{4}$; *b* $\frac{3}{20}$; *c* $\frac{4}{21}$; *d* $\frac{5}{11}$; *e* $\frac{1}{3}$; *f* $\frac{1}{3}$; *g* $\frac{1}{3}$; *h* $\frac{1}{8}$. **347.** $\frac{5}{8}$.

Page 139.—**351.** $\frac{a}{b}=5$; $\frac{a}{c}=12.5$; $\frac{a}{d}=250$; $\frac{b}{a}=.2$; $\frac{b}{c}=2.5$; $\frac{b}{d}=50$; $\frac{c}{a}=.08$; $\frac{c}{b}=.4$; $\frac{c}{d}=20$; $\frac{d}{a}=.004$; $\frac{d}{b}=.02$; $\frac{d}{c}=.05$. **352.** 1.2. **353.** 2.5; 1.16$\frac{2}{3}$. **354.** \$1.50; \$5.55; $10.12\frac{1}{2}$. **355.** 2400 sq. ft. **356.** 48. **357.** 61 yd. **358.** $15\frac{5}{36}$ hr. **359.** 56 bbl. **360.** 4.63 + da.

Page 141.—**386.** \$15,000. **387.** \$12,000; \$28,000. **388.** \$14,000.

Page 142.—**389.** \$10,000. **390.** 72; 6. **391.** \$.08; \$.76. **392.** $\$1.42\frac{1}{2}$. **393.** \$49. **394.** \$1.92. **1.** .08; 1.2; .0004; .003; .06. **2.** .004; .00012; .02; .00002. **3.** 3.6; .003; .09; 480. **4.** 8720; $8470\frac{6}{7}$; $8235\frac{5}{7}$. **5.** 343; 1053. **6.** 17 hr. **7.** 6. **8.** 3; 3. **9.** 16 yr. **10.** $\frac{198}{450}$; $\frac{44}{405}$. **11.** $\frac{1}{324}$; $\frac{1}{378}$.

Page 143.—**12.** $7\frac{1}{3}$. **13.** $26\frac{2}{7}$. **14.** $834\frac{1}{3}$. **15.** 264. **16.** 219. **17.** 42. **18.** $\frac{3}{11}$; 3; $1\frac{1}{7}$; $\frac{8}{49}$; $10\frac{1}{44}$; $5\frac{1}{4}$; $57\frac{3}{4}$; 6; $3\frac{1}{7}$. **19.** $10\frac{11}{14}$; $\frac{8}{7}$; $5\frac{11}{14}$;

$11\frac{5}{11}$; $1\frac{17}{154}$; $6\frac{1}{14}$. **20.** $10\frac{3}{4}$; $5\frac{3}{14}$; $\frac{2}{7}$; $9\frac{6}{11}$; 5; $4\frac{6}{11}$. **21.** $\frac{1}{2}$; $\frac{10}{11}$; $\frac{5}{16}$; 5; 20; $1\frac{1}{2}$; 4; $\frac{81}{100}$; 16. **22.** \$.53. **23.** 28; 56; 49; 70. **24.** 28; 36; 20; 44. **25.** \$1500; \$7500. **26.** \$.32. **27.** \$.48; \$2.88. **28.** 6. **29.** $\frac{25}{144}$¢. **30.** $\frac{1}{50}$¢.

Page 144. — **31.** $2\frac{1}{2}$¢. **32.** $\frac{1}{100}$¢. **33.** $\frac{1}{100}$¢. **34.** $\frac{1}{3}$¢. **35.** \$1.87$\frac{1}{2}$. **36.** \$.62$\frac{1}{2}$. **37.** $5\frac{1}{2}$ in. **39.** $13\frac{3}{7}$ in. **40.** $12\frac{1}{2}$ in.

Page 145. — **42.** $16\frac{1}{2}$ in.; $2\frac{3}{4}$ in. **44.** 8 in.; 16 in.; 32 in. **45.** $3\frac{7}{12}$ in.; $3\frac{37}{77}$ in.; $5\frac{7}{33}$ in. **46.** $8\frac{8}{15}$ in. **47.** $20\frac{4}{15}$ in. **50.** $17\frac{1}{2}$ ft.; $22\frac{1}{2}$ ft.

Page 146. — **53.** 880; 660. **54.** 50 in.; 75 in. **55.** 4500 sq. ft.; 2250 sq. ft. **56.** 19,872. **57.** $7\frac{21}{40}$ yr. **58.** 103 lb. **59.** The latter is $\frac{1}{4}$ greater. **60.** 6 sq. ft. **61.** $6\frac{1}{25}$ sq. in. **63.** $\frac{5}{8}$. **64.** \$900.

Page 147. — **65.** \$5.50. **66.** \$60.27. **67.** \$4600. **69.** \$2100. **70.** 600. **71.** 500 books. **72.** 13 ft. **73.** 52 in. **74.** 77 in.

Page 148. — **75.** 7 in. **76.** 12 in. **79.** 10 in. **80.** 50 in. **81.** 15 in.; 35 in.; 25 in.; 55 in.

Page 149. — **83.** 54 sq. ft. **84.** 54 sq. ft. **85.** 58 sq. ft. **86.** 56 sq. ft. **87.** 9 cu. ft. **88.** $\frac{5}{27}$; 8 cu. ft. **89.** \$2; \$2492. **90.** 6 wk. **91.** \$.02$\frac{1}{2}$; \$.25; \$3.00. **92.** \$15.25.

Page 151. — **8.** \$69.93. **10.** \$40.

Page 152. — **20.** 58 pints. **21.** 88 gills. **24.** 17 pt.; 11 pt. **27.** $\frac{1}{20}$; $\frac{3}{28}$; $\frac{3}{56}$.

Page 153. — **33.** $5\frac{1}{2}$ qt.; $1\frac{3}{8}$ gal. **35.** 7 qt. 1 pt. 2 gi. **36.** 4 gal. 1 qt. 1 pt. **37.** 2 gal. 1 qt. 1 pt. 2 gi. **40.** $\frac{7}{8}$ qt.; $\frac{7}{32}$ gal. **41.** $1\frac{3}{4}$ pt. **42.** $5\frac{1}{2}$ pt.; $2\frac{3}{4}$ qt.; $1\frac{1}{8}$ gal. **43.** 22 pt. is 1 pt. greater. **45.** \$.25; \$.87$\frac{1}{2}$; \$1.18$\frac{3}{4}$; \$.43$\frac{3}{4}$; \$.03$\frac{1}{8}$; \$.21$\frac{7}{8}$.

Page 154. — **48.** 5 gal. 3 qt. 1 gi. **49.** 13 gal. 2 qt. 1 pt. **50.** 18 gal. 2 qt. 1 pt. 1 gi. **51.** 14 gal. 2 qt. 1 pt. 1 gi. **52.** 17 gal. 2 qt. 1 pt. **53.** 27 gal. 2 qt. 1 pt. **54.** 59 gal. 2 qt.

Page 155. — **58.** 6 gal. 1 qt. 1 pt. 2 gi. **59.** 8 gal. 2 qt. 2 gi. **60.** 6 gal. 2 qt. 1 gi. **61.** 5 gal. 3 qt. 1 gi. **62.** 5 gal. 3 qt. **64.** 35 gal. 2 qt. 1 pt. **65.** 94 gal. 1 pt. 3 gi. **66.** 23 gal. 1 pt. 1 gi.; 38 gal. 2 qt. 3 gi.; 54 gal. 1 gi. **67.** 698 gal. 1 qt. **68.** \$167.58; \$32.58. **69.** 5.

Page 156. — **70.** 16. **71.** 32. **73.** 4 gal. 1 qt. 3 gi. **74.** 2 gal. 1 qt. 2 gi. **75.** 2 gal. 1 qt. 1 pt. $3\frac{1}{4}$ gi.; 1 gal. 3 qt. 1 pt. $3\frac{1}{2}$ gi.; 1 gal. 2 qt. 1 pt. $\frac{1}{2}$ gi.; 1 gal. 1 pt. $3\frac{1}{2}$ gi. **76.** 160.

Page 157. — **80.** 7 pt.; $3\frac{1}{2}$ qt.; $\frac{7}{16}$ pk. **81.** 43 pt.; $21\frac{1}{2}$ qt.; $2\frac{11}{16}$ pk.; $\frac{43}{64}$ bu. **83.** \$.80; \$.08$\frac{3}{4}$; \$.13$\frac{1}{2}$. **84.** \$.12$\frac{1}{2}$; \$.87$\frac{1}{2}$; \$1.37$\frac{1}{2}$. **85.** \$1.00; \$1.25; \$4. **86.** $3\frac{1}{2}$ pk.; $1\frac{1}{2}$ qt. $1\frac{3}{4}$ pt.; 24 qt. **87.** $\frac{3}{4}$ bu.; $\frac{1}{11}$ pk.; $\frac{3}{14}$ qt.; $\frac{5}{1\,\frac{3}{4}}$ bu. **88.** 1 qt.; 3.5 pk.; 110 pt. **89.** 1.5 pk.; 12 qt. 24 pt. **90.** .5 pk.; .125 bu. **91.** 8 pt. **92.** \$.80. **93.** .28 pk.; 4.08 qt. **94.** 30 bu. 3 pk. 5 qt. 1 pt. **95.** 104 bu. 1 pk. 1 qt. 1 pt. **96.** 17 bu. 1 qt. **97.** 32 bu.

Page 158. — **98.** 4 bu. 1 pk. 4 qt. **99.** 22 bu. 2 pk. 2 qt. 1 pt. **100.** 7 bu. 1 pk. 1 qt. 1 pt. **101.** 4 bu. 1 pk. 4 qt. **102.** 23 bu. 2 pk. 1 pt. **103.** 62 bu. 3 pk. 4 qt. 1 pt. **104.** 44 bu. 2 pk. 1 qt. **105.** 71 bu. 3 pk. 4 qt. **106.** 27 bu. 1 pk. 6 qt.; 41 bu. 5 qt.; 54 bu. 3 pk. 4 qt. **107.** 14 bu. 1 pk. 6 qt. **108.** 406 bu. 3 pk. 4 qt. **109.** \$1.60. **110.** 1 bu. 1 pk. 1 qt. 1 pt. **111.** 1 bu. 1 pk. 6 qt. $\frac{1}{2}$ pt. **112.** 4 bu. 3 pk. 3 qt. $1\frac{1}{2}$ pt.; 3 bu. 7 qt. $1\frac{3}{4}$ pt.; 2 bu. 1 pk. 5 qt. $1\frac{3}{4}$ pt. **113.** 5. **114.** 616 sq. cm.

Page 159. — **119.** 537 cl. **121.** 7000 l.; 70 Hl.; 700,000 cl.; 7,000,000 ml. **122.** 250 l.; 25,000 cl.; .25 Kl.; 2500 dl.

Page 160. — **125.** 5252.007 l. **126.** 12,591.825 l. **127.** 7632.244 l. **128.** 797.756 l. **129.** 626.375 l.; 1002.2 l.; 1503.3 l. **131.** 59.817 l.; 39.878 l.; 19.939 l. **133.** 9.275 l.; 12.509 l. **134.** 17.51 l.; 175.1 dl.; 1751 cl. **135.** $9.375. **136.** $.72½. **137.** $33.75. **138.** $3.75. **139.** 1500 Dl.

Page 161. — **142.** 88 oz.; 5½ lb.; $\frac{11}{100}$ cwt.; $\frac{11}{1000}$ T. **143.** $3\frac{11}{1000}$ T.; $70\frac{19}{100}$ cwt.; 7025¾ lb.; 112,412 oz. **144.** $12\frac{8}{11}$ cwt.; 55⅝ lb. **145.** $\frac{1}{15}$ lb.; $\frac{1}{12}$ cwt.; $\frac{3}{11}$ cwt. **146.** 14 cwt.; 17 lb.; 2 oz.; 1500 lb. **147.** 12.5 cwt.; 1250 lb.; 20,000 oz. **148.** .15 cwt. **149.** $.50; $.65; $1.00; $.15; $1.42½. **150.** 5000 lb.; 60 lb. **151.** 1500 lb. **152.** 9 cwt. **154.** 19 T. 6 cwt. 12 lb. 3 oz. **155.** 39 T. 10 cwt. 77 lb. 13 oz. **156.** 24 T. 1226 lb. 4 oz. **157.** 37 T. 297 lb. 2 oz.

Page 162. — **158.** 6 T. 15 cwt. 88 lb. 13 oz. **159.** 3 T. 13 cwt. 2 lb. 8 oz. **160.** 4 T. 25 lb. 12 oz. **161.** 13 T. 1800 lb. 4 oz. **162.** 22 T. 1900 lb. **163.** 1 lb. 12 oz. **165.** 277 lb. 12 oz. **166.** 44 T. 3 cwt. 25 lb. **167.** 189 T. 9 cwt. 11 lb. 4 oz. **168.** 46 T. 1 cwt. 76 lb. 8 oz.; 76 T. 16 cwt. 27 lb. 8 oz.; 122 T. 18 cwt. 4 lb.; 138 T. 5 cwt. 29 lb. 8 oz. **169.** 2 T. 8 cwt. 80 lb. 2 oz. **170.** 3 T. 6 cwt. 90 lb. 2 oz. **171.** 3 T. 8 cwt. 74 lb. 12⅜ oz.; 2 T. 11 cwt. 56 lb. 1⅜ oz.; 1 T. 14 cwt. 37 lb. 6½ oz. **172.** 3 T. 12 cwt. $72\frac{8}{11}$ lb. **173.** 10,080 lb.; 280 lb.

Page 163. — **174.** $123. **180.** 75,000 g.; 7,500,000 cg.; 75,000,000 mg.; 7500 Dg.; 750,000 dg.; 750 Hg. **181.** 1.86 g.; 1860 mg.; .186 Dg.; 18.6 dg.; .0186 Hg.; .00186 Kg. **182.** 56,075 g. **183.** 85 g. **184.** 1209.6 Dg.; 120960 dg.; 1,209,600 cg.; 12,096,000 mg. **185.** .5 Kg.

Page 164. — **186.** $1.32. **187.** $1.345. **188.** $11.375. **191.** 7800 sec.; 130 min.; 2½ hr. **192.** 219 da.; 7½ hr.; 45 min. **193.** 52½ da.; 1251¾ hr.; 75,085¾ min.; 4,505,142¾ sec. **194.** $\frac{1}{24}$ hr.; $\frac{1}{10}$ da.; 1¾ min. **195.** 43¾ da.; 8 hr.; 45 min.; 40 sec.; 17 yr. **196.** 1.8 min.; .03 hr. **197.** 3 min.; 39 min.; 57 min. **198.** 69 min. **199.** $18; $27; $63; $49.50. **200.** 1½ yr.; $27; $36; $22.50.

Page 165. — **201.** 696 hr. **202.** 1848; 1892; 1904; 2000; 2108. **203.** $25; $24. **204.** $36. **205.** 4320; 103,680; 37,843,200. **206.** 3,029,529,600. **207.** 4 yr. **208.** 5 hr. 48 min. 46 sec.; 23 hr. 15 min. 4 sec.; 44 min. 56 sec. **209.** 44 min. 56 sec.

Page 166. — **210.** 5 hr. 16 min. 40 sec. **211.** 2 hr. 53 min. 20 sec., an average of 26 sec. a day. **212.** 14 hr. 30 min. **214.** 8760 hr. **218.** 58 da. **219.** 59 da. **220.** 38 da. **221.** 47 da.

Page 167. — **224.** Aug. 3, 1876. **225.** Dec. 13, 1492; Feb. 23, 1898. **226.** Aug. 16. **229.** April 1; Jan. 1. **232.** 79 da. 8 hr. 1 min. 10 sec. **233.** 62 da. 4 hr. 31 min. 30 sec. **234.** 101 da. 8 hr. 1 min. 45 sec. **235.** 95 da. 3 hr. 46 min. 45 sec. **236.** $9.23⅛.

Page 168. — **237.** $11.54. **238.** 21 da. 22 hr. 30 sec. **239.** 13 da. 14 hr. 10 min. 30 sec. **245.** 78 yr. 11 mo. 2 da.; 84 yr. 8 mo. 20 da.; 75 yr. 27 da.; 72 yr. 5 mo. 20 da.; 83 yr. 2 mo.; 58 yr. 4 mo. 2 da. **246.** Whittier, 4 yr. 6 mo. 22 da.; Longfellow, 3 yr. 9 mo. 2 da.; Lowell, 15 yr. 8 mo. 27 da.; Tennyson, 6 yr. 2 mo. 11 da.; Dickens, 8 yr. 8 mo. 12 da.

Page 169. — **247.** Emerson, 11 yr. 10 mo. 18 da.; Whittier, 22 yr. 2 mo. 29 da.; Longfellow, 11 yr. 9 mo. 15 da.; Lowell, 21 yr. 2 mo. 3 da.; Tennyson, 22 yr. 3 mo. 27 da. **248.** 54 da. 8 hr. 21 min. 10 sec. **249.** 104 da. 16 hr. 16 min. 30 sec. **250.** 27 da. 3 hr. 42 min. 30 sec.; 32 da. 14 hr. 3 min.; 43 da. 10 hr. 44 min. **251.** 22 hr. 30 min.; 90 hr.;

900 hr. **252.** 5 da. 10 hr. 40 sec. **253.** 2 da. 6 hr. 12 min. 15 sec. **254.** 3 da. 2 hr. 32 min. 6 sec.; 2 da. 14 hr. 6 min. 45 sec.; 1 da. 22 hr. 35 min. $3\frac{3}{4}$ sec. **256.** 91 in.; 223 in. **257.** 90 in.; $7\frac{1}{4}$ ft.; $2\frac{1}{4}$ yd. **258.** $2\frac{1}{16}$ mi.; 660 rd.; 3630 yd.; 10,890 ft. **259.** $\frac{1}{4}$; $\frac{3}{55}$; $\frac{1}{5280}$. **260.** $\frac{1}{4}$ yd.; $\frac{1}{11}$ rd.; $\frac{1}{5280}$ mi. **261.** $1\frac{7}{12}$ yd.; $\frac{1}{66}$ rd.; $\frac{15}{211200}$ mi. **262.** 100 rd.; 4 yd.; $1\frac{1}{8}$ ft.; $7\frac{1}{2}$ in.; 6 ft.

Page 170.—263. $\frac{7}{11}$ ft.; $\frac{3}{11}$ yd.; $\frac{1}{4}$ rd.; $\frac{1}{18}$ mi.; $\frac{1}{12}$ rd.; $\frac{1}{4}$ ft.; $1\frac{1}{8}$ yd. **264.** 280 rd.; $19\frac{1}{2}$ yd.; $16\frac{1}{2}$ ft.; $10\frac{1}{2}$ in.; 5 ft.; $3\frac{1}{18}$ yd. **265.** .6 ft.; .2 yd. **266.** 38.5 yd.; 7 rd.; .021875 mi. **267.** 16 rd.; 48 rd.; 112 rd. **268.** 8 yd. $1\frac{1}{2}$ in. **269.** 10 yd. 2 ft. $7\frac{1}{2}$ in. **270.** 15 mi. 4 yd. 1 ft. 4 in. **271.** 18 mi. 3 yd. 1 ft. 6 in. **272.** 12 mi. 251 rd. $\frac{1}{2}$ yd. 1 ft. 4 in. = 12 mi. 251 rd. 2 ft. 10 in. **273.** 17 mi. 221 rd. 1 yd. 2 ft. 8 in. **274.** 31 mi. 30 rd. 5 yd. 1 ft. 2 in. **275.** 24 mi. 2 rd. 1 yd. 2 ft. 11 in. **276.** 26 mi. 2 rd. 4 yd. 1 ft. **277.** 28 yd. 2 ft. **278.** 25 ft. 11 in.

Page 171.—279. 1 ft. 4 in. **280.** $16\frac{1}{2}$ mi. **281.** 14 mi. 260 rd. **282.** 10 yd. 6 in. **283.** 20 mi. 8 rd. 5 yd. **284.** 14 mi. 160 rd. 1 yd. 2 ft. **285.** 73 mi. 165 rd. 4 yd. 2 ft. 11 in. **286.** 5 mi. 161 rd. 4 yd. 6 in.; 8 mi. 82 rd. 3 yd. 1 ft. 6 in. **287.** 17 yd. 2 ft. 9 in. **288.** 15 yd. **289.** 12 yd. 1 ft. 4 in. **290.** 1 mi. 168 rd. 1 yd. 10 in.; 1 mi. 5 rd. 2 yd. 2 ft. $\frac{1}{4}$ in.; 244 rd. 1 ft. 11 in. **291.** 3 mi. 258 rd. 1 yd. 6 in.; 3 mi. 55 rd. 2 ft. 11 in.; 2 mi. 230 rd. 2 ft. 6 in. **292.** 3 mi. 240 rd. 2 yd. 10 in.; 2 mi. 160 rd. 1 yd. 1 ft. $6\frac{3}{4}$ in.; 1 mi. 280 rd. 1 yd. 5 in. **293.** 960.

Page 172.—294. Don 7 in. longer; Don 23 lb. 14 oz. heavier; Don 7 mo. 11 da. older. **295.** 50 yd. **296.** 1.8 ft. **300.** 780 sq. in.; 596 sq. in.; 368 sq. in. **301.** 42 sq. ft.; 187 sq. ft.; 167 sq. ft.

Page 173.—302. 257 sq. yd.; 307.25 sq. yd.; 745 sq. yd. **303.** 153,680 sq. rd.; $960\frac{1}{2}$ A.; $1\frac{64}{1280}$ sq. mi. **304.** 14,040 sq. in.; $97\frac{1}{2}$ sq. ft.; $10\frac{1}{2}$ sq. yd. **305.** $75\frac{1}{2}$ sq. yd.; $2\frac{60}{121}$ sq. rd.; $679\frac{1}{2}$ sq. ft.; $\frac{155}{6561}$ A. **306.** 220 A.; 10 sq. rd. **307.** 22 sq. yd.; $2\frac{1}{7}$ sq. ft.; 30 sq. in.; $\frac{1}{10}$ sq. ft. **308.** 400 A.; 60 sq. rd.; 14.52 sq. yd.; 1.575 sq. ft.; 100.8 sq. in.; .03 sq. ft.; .02 sq. yd. **309.** 2.4 sq. ft. **310.** 45 sq. yd. 7 sq. ft. 2 sq. in. **311.** 73 sq. yd. 6 sq. in. **312.** 13 sq. yd. 5 sq. ft. 10 sq. in. **313.** 14 sq. yd. 7 sq. ft. 25 sq. in. **314.** 146 sq. yd. 5 sq. ft. 120 sq. in. **315.** 115 sq. yd. 5 sq. ft. 96 sq. in. **316.** 6 A. 121 sq. rd. 1 sq. yd. 1 sq. ft. 24 sq. in.; 9 A. 1 sq. rd. 11 sq. yd. 5 sq. ft. 44 sq. in.; 11 A. 41 sq. rd. 22 sq. yd. 64 sq. in.

Page 174.—317. 32 A.; 44.8 A.; 96 A. **318.** 90 sq. in. **319.** 16 sq. yd. 2 sq. ft. $49\frac{1}{2}$ sq. in. **320.** 5 sq. yd. 1 sq. ft. $29\frac{1}{2}$ sq. in. **321.** 5 A. 20 sq. rd. 5 sq. yd. 1 sq. ft. 18 sq. in.; 2 A. 90 sq. rd. 2 sq. yd. 5 sq. ft. 9 sq. in.; 3 A. 66 sq. rd. 23 sq. yd. 5 sq. ft. 36 sq. in. **322.** $\frac{9}{16}$ sq. ft. 81 sq. in. **323.** $3\frac{1}{2}$ ft.; $\frac{25}{36}$; 100 sq. in. **324.** 1 sq. mi. 10 sq. rd. **325.** 50 A. 40 sq. rd. **326.** 482 A. **327.** $105.60. **328.** $5\frac{7}{48}$ sq. ft.

Page 175.—329. 352 sq. ft. **330.** *a* $24.89; *b* $64. **331.** *a* $38.67; *b* $25.73. **332.** *a* $30.07; *b* $29.07. **333.** *a* $17.73; *b* $28.27. **334.** *a* $13.60; *b* $16.36. **335.** Neither; 2 in. **338.** 36 sq. in.; 49 sq. in.; 81 sq. in.; 100 sq. in. **339.** 12 in.

Page 176.—341. 8832 cu. in.; 47,280 cu. in. **342.** 1512 cu. in.; 1080 cu. in.

Page 177.—353. 36 ft. **354.** $67.50. **355.** 12 cu. ft. **356.** *a* 16 cu. ft.; *b* 15 cu. ft.; *c* 32 cu. ft.; *d* 80. cu ft. **357.** 700 cu. ft. **358.** 81 cu. yd.; 20 cu. yd.

Page 178.—359. $294. **360.** $356.50. **361.** 45 sq. in. **362.** 6; 27. **363.** 4; 12; 9; 2. **364.** 144; 12; 1728; $\frac{1}{1728}$. **365.** $\frac{1}{231}$.

366. They are equal; Latter contains 144 sq. in. more. **368.** 18 cu. ft. **369.** 150 cu. ft.; 259,200 cu. in.; 1122$\frac{6}{7}$ gal. **370.** 9 cu. yd. 19 cu. ft. 101 cu. in. **371.** 14 cu. yd. 2 cu. ft. 1700 cu. in.

Page 179. — **372.** 16 cu. yd. 3 cu. ft. 2 cu. in. **373.** 13 cu. yd. 9 cu. ft. 72 cu. in. **374.** 54 cu. yd. 25 cu. ft. 575 cu. in. **375.** 15 cu. yd. 19 cu. ft. 127 cu. in. **376.** 40 cu. yd. 10 cu. ft. 83 cu. in. **377.** 11 cu. yd. 10 cu. ft. 275 cu. in. **378.** 60 cu. yd. 17 cu. ft. 24 cu. in. **379.** 61 cu. yd. 15 cu. ft. 500 cu. in. **380.** 5 cu. yd. 14 cu. ft. 272 cu. in.; 11 cu. yd. 1 cu. ft. 544 cu. in. **381.** 1 cu. yd. 9 cu. ft. 705$\frac{3}{4}$ cu. in. **382.** 1 cu. yd. 7 cu. ft. 436$\frac{1}{2}$ cu. in. **383.** 3 cu. yd. 14 cu. ft. 60 cu. in.; 2 cu. yd. 3 cu. ft. 36 cu. in. **385.** 128 cu. ft. **387.** 8 cords.

Page 180. — **388.** $12.50; $45. **390.** 4200 cm.; 420 dm.; 4.2 Dm.; .42 Hm. **391.** .375 Km.; 37.5 Dm.; 3750 dm.; 375,000 mm. **392.** 4.287 Km.; 42,870 dm.; 42.87 Hm. **393.** 132 cm.; 27.5 cm. **394.** 13.44 m.; 1.344 Dm. **395.** 393.7 in.; 3.93 in. **396.** 39,370 in.; 15748 in.; 4724.4 in.; 27.559 in.

Page 181. — **397.** $9. **398.** 126 Km. **399.** $16.50. **400.** 6.4 cm. **401.** 1.5 dm. **402.** 16 cm. **403.** 11 cm. **405.** 25 mi.; 45 mi.; 2 mi.; 4.2 mi. **406.** 5.805 mi. **407.** 28.665 mi.; 3.989 mi.; 374.3 mi.; 4490.5 mi. **408.** 120 Km.; 376 Km.; 135.2 Km. **409.** 100 sq. mm. **410.** 100 sq. cm.; 10,000 sq. mm. **411.** 100 sq. dm.; 10,000 sq. cm.; 1,000,000 sq. mm.

Page 182. — **412.** 100 sq. m.; 10,000 sq. dm.; 1,000,000 sq. cm.; 100,000,000 sq. mm. **418.** 3,000,250 sq. m.; 30002.5 sq. Dm.; 3.00025 sq. Km. **419.** 7,203,000 sq. mm.; 72,030 sq. cm.; 720.3 sq. dm. **420.** 1,020,300 sq. mm.; 10,203 sq. cm.; 102.3 sq. dm.; 1.0203 sq. m.; .010203 sq. Dm.; .00010203 sq. Hm.; .0000010203 sq. Km. **421.** 36 sq. cm.; 25 sq. cm.; 1 sq. cm. **422.** 36 cm.; 28 cm.; 3.2 m. **423.** 2.45 sq. dm.; 8.4 dm. **424.** 100 sq. m.; 40 m.

Page 183. — **425.** $7.50. **427.** 400 m. **429.** 100 sq. dm.; 10,000 sq. dm.; 1,000,000 sq. dm. **430.** $10,497.50; $65,274. **431.** 7 A.; 4 A.; 6 A.; 3 A. **432.** 1200 a.; 3000 a.; 700 a.; 350 a.; 500 a.; 240 a.; 14,000 a.; 20,000 a.; 10 a.

Page 184. — **438.** 600 sq. dm. **439.** $\frac{1}{2}$ m.; 140 dm. **440.** 1000. **441.** $150. **442.** $60; $94,520; $166,940. **444.** 210 s.; $315. **445.** $330.75. **446.** 5 cd.; 8 cd.; 10.5 cd.; 3.2 cd.; 1.59 cd.

Page 185. — **448.** 20 s.; 28 s.; 38 s.; 14 s.; 25 s.; 48$\frac{1}{2}$ s.; 16$\frac{1}{4}$ s.; 44$\frac{1}{2}$ s.; 96$\frac{1}{4}$ s. **452.** 7 l.; 120 l.; 35$\frac{5}{11}$ l.; 15$\frac{5}{11}$ l. **453.** $14.03. **454.** $29.26 **457.** 1000 g. **458.** .0352 oz. **1.** 25,000. **2.** 7.

Page 186. — **3.** $\frac{7}{32}$; $\frac{23}{36}$. **4.** 456. **5.** $\frac{17}{11 \cdot 14}$. **6.** $\frac{1}{75}$. **7.** 26,460 = $2^2, 3^3, 5, 7^2$; 60,060 = 2^2, 3, 5, 7, 11, 13. **8.** $1\frac{23}{36}$. **9.** $\frac{30}{6}$; $\frac{21}{6}$. **10.** $3\frac{2}{3}$; $2\frac{1}{3}$; **2.** **11.** 3; 3. **12.** $42\frac{3}{4}$. **13.** 4; 4. **14.** 4. **15.** $15\frac{1}{3}$. **16.** $\frac{25}{49}$; $\frac{9}{25}$; $\frac{4}{81}$; $\frac{121}{625}$. **17.** $\frac{2}{5}$. **18.** $\frac{5}{9}$; $\frac{9}{10}$; $\frac{8}{9}$; $\frac{5}{8}$; $\frac{8}{10}$. **20.** 1760 cu. yd. **21.** $360.

Page 187. — **22.** 9; $31.50. **23.** $8. **24.** $6.04. **25.** $6.44. **26.** $3.30. **27.** $3. **28.** $6.22. **29.** $21.60. **30.** $3.20. **32.** $22.90. **33.** $36.50. **34.** $25.375. **35.** $31.625. **36.** 11 yd. 1 ft.; 3$\frac{20}{33}$ yd.; 1$\frac{53}{66}$ yd. **37.** 14 A. **38.** $125. **39.** 480 **40.** 576 cu. ft.; 4$\frac{1}{2}$ cords.

Page 188. — **41.** 1000 g.; 3000 g.; 15 g.; 1000 g.; 1 g.; 1,000,000 g.; 4,000,000 g. **42.** 1 Kg.; 3 Kg.; .015 Kg.; 1 Kg.; .001 Kg.; 1000 Kg.; 4000 Kg. **44.** 165 lb.; 19.36 lb.; .33 lb.; .385 lb.; .00869 lb. **45.** 72.6 lb.;

.605 lb.; .0020328 lb.; 21.78 lb.; .352 lb. **46.** $.24. **47.** $.90. **48.** Nothing. **49.** $16. **50.** $28. **51.** 600 sq. dm. **52.** $3\frac{3}{8}$ sq. in. **53.** $15.

Page 189. — **54.** 16 days. **55.** 13 cwt. 74 lb. 7 oz. **56.** 32. **57.** 22 yd. w. **58.** 21 yd. 6 in. **59.** 8600. **60.** 1 yd. 2 ft. 5 in.; 1 yd. 3 in.; 2 ft. $\frac{3}{8}$ in. **61.** 2 ft. 5 in. **62.** 4 T. 9 cwt. 25 lb. **63.** $32\frac{1}{2}$ mi. **64.** 11 A. M. **65.** 1560 sq. rd.

Page 190. — **66.** 8.62\frac{1}{2}$. **67.** $28.08. **68.** 58 yr. 10 mo. 26 da. **69.** June 2. **70.** 14 wk. 1 da. **71.** $6.77. **72.** 81. **73.** 11 wk. 5 da. **74.** $62.86 **75.** $14.583. **76.** 13 sq. yd.

Page 191. — **77.** $2\frac{4}{9}$ sq. yd. **78.** 9 ft. **80.** 8 ft. **81.** 216 sq. cm. **82.** 120 sq. yd. **84.** 96 sq. in. or cm. **85.** 96 sq. in. or cm. **87.** 12 in.; 8 in.

Page 192. — **90.** 11 in.; 7 in. **91.** 83 sq. in. **92.** 40 in.; 64 sq. in. **93.** 52. **95.** 56 sq. in. **96.** 4 in.; 320 sq. in.

Page 193. — **97.** 270 sq. ft.; 108 sq. ft.; 162 sq. ft. **98.** 540 sq. in. **99.** 1296 sq. in. **101.** 193 sq. in.

Page 194. — **107.** 24 sq. ft. **108.** 800 sq. rd. **109.** 5.83\frac{1}{3}$. **110.** 5; 6 yd.; $26.25. **111.** 8; 80 yd. **112.** $47.25. **113.** 8; $88.

Page 195. — **114.** 14.96\frac{1}{2}$. **115.** $77.96. **119.** 24 sq. in. **120.** 1. **121.** 488; 384; 96; 8; 512. **122.** 4175 gal. 2 qt. 1 pt. **123.** $3\frac{1}{4}$. **124.** 552.

Page 196. — **125.** $.02$\frac{1}{12}$; $17.50. **126.** $16. **127.** $.948. **130.** $920. **131.** $333.38.

Page 197. — **132.** $870. **133.** $135. **134.** $255.36; $257.64; $262.20; $253.08. **136.** $528.75. **137.** $672; $472. **138.** $270. **139.** $800; $742.

Page 198. — **142.** 160 sq. rd. **143.** 73.33\frac{1}{3}$. **144.** 13 wk. **145.** $4.95.

Page 200. — **13.** $1\frac{1}{3}$. **14.** $\frac{1}{3}$. **15.** 2. **16.** $\frac{3}{7}$.

Page 201. — **23.** 4. **24.** $\frac{5}{14}$. **25.** $\frac{15}{38}$. **26.** $\frac{4}{11}$. **27.** $\frac{4}{15}$.

Page 202. — **35.** $.06; $1.08. **36.** $7\frac{1}{2}¢$; $1.30. **37.** $.80 gain.

Page 203. — **52.** $4. **53.** $.50 gain.

Page 204. — **63.** 132 sq. in. **64.** 144 cu. in. **65.** $.13; $.52; $.65; $.32$\frac{1}{2}$; $.48$\frac{3}{4}$; $.78; $.81$\frac{1}{4}$; $1.17. **66.** $336; $280. **67.** 39 in.

Page 205. — **74.** $.25. **75.** 5. **77.** $5.25.

Page 206. — **85.** 8. **86.** $.50. **87.** $.25. **89.** $1.00.

Page 207. — **92.** $\frac{1}{12}$. **93.** $\frac{1}{32}$. **94.** $\frac{1}{48}$. **95.** $\frac{1}{6}$. **96.** $\frac{3}{8}$. **97.** $\frac{1}{48}$. **98.** $2\frac{1}{2}$. **99.** 3.

Page 208. — **114.** 240 sq. ft. **115.** $\frac{7}{8}$.

Page 209. — **121.** 1800; 2400; 24,000. **124.** 210; 140; 320; $77\frac{1}{2}$. **125.** 90; 270; $143\frac{1}{3}$; $93\frac{1}{3}$; 50.

Page 210. — **1.** 31. **2.** 4.41; 10.24; 1.5625; 1.0404; 4.012009. **3.** .000008; .512; 1.728; 1.092727; 1.157625. **4.** .3125. **5.** *a* .025; *b* .15; *c* .01875. **6.** 251. **7.** 4. **8.** 83. **9.** $\frac{28}{225}$. **10.** 2; 3; 5; 10; 12; 4. **11.** 8\frac{1}{8}$. **12.** $\frac{2}{3}$; $2\frac{2}{3}$. **14.** 33. **15.** 10.67. **16.** 96. **17.** 2; 3. **18.** 144; $341\frac{1}{4}$. **19.** $1\frac{1}{2}$; 2; $3\frac{1}{2}$; $2\frac{1}{2}$; 3; $4\frac{1}{2}$; 6.

Page 211. — **20.** 1500. **21.** 2 mo. **22.** $4\frac{1}{8}$ cm.; $1\frac{1}{2}$ cm. **23.** $1.00; $4.00; 1.87\frac{1}{2}$; $.06$\frac{1}{4}$; $.31$\frac{1}{4}$; $.01$\frac{9}{16}$; $.04$\frac{1}{16}$. **24.** $.50. **25.** 25%. **26.** 7000%; 6%; $\frac{1}{160}$%; 9%; 70%. **27.** $\frac{3}{40}$; $\frac{7}{8}$; $\frac{9}{16}$. **30.** 3.28. **31.** 111.1. **32.** .46$\frac{2}{3}$; .83$\frac{1}{3}$; .79$\frac{1}{6}$; .94$\frac{4}{9}$; .06$\frac{1}{4}$. **33.** $11\frac{1}{9}$%; $22\frac{2}{9}$%; $33\frac{1}{3}$%; $44\frac{4}{9}$%; $55\frac{5}{9}$%; $66\frac{2}{3}$%; $77\frac{7}{9}$%; $88\frac{8}{9}$%. **34.** 21,120 ft.

Page 212. — **35.** $151\frac{5}{11}$ rd. **36.** $2.12. **37.** $1\frac{13}{21}$. **38.** $.13. **39.** $.50. **40.** $721.45. **41.** $48\frac{3}{8}$ A.. **42.** $718. **43.** 8 qt. **44.** 2 pk. $6\frac{2}{3}$ qt. **45.** 13. **46.** 571 axes; $1\frac{1}{2}$ lb. left. **47.** $7\frac{133}{176}$ mi. **48.** 8.

Page 213. — **49.** 19 bu. 2 qt. **50.** $6. **51.** 6; 15; 22. **52.** $93.86. **53.** $152\frac{1}{10}$. **54.** 270. **55.** $33\frac{1}{4}$ ft.

Page 214. — **61.** 50 ft.; **31.** **62.** 7 A. **63.** 168½ yd. **64.** $3200. **65.** $243 lost. **66.** $62.50 lost. **67.** $270.41⅔. **68.** $794.91.

Page 215. — **69.** $1495.15. **71.** $883¾. **72.** $680. **73.** $38.50.

Page 216. — **6.** $5.28; $10.56.

Page 217. — **15.** $1500. **17.** $500. **18.** $25. **20.** $1.20; $.10.

Page 218. — **23.** *a* 44.64; *b* 24; *c* 23.76; *d* 54; *e* 16; *f* 50.4; *g* 36; *h* 126; *i* 37.44; *j* 38.88; *k* 80; *l* 120; *m* 122.4; *n* 128. **24.** 136.35; 147.46; 169.68; 180.79. **27.** 600. **28.** son $38,400; daughter $96,000; library $19,200. **32.** 16⅔%; 12½%; 33⅓%; 50%; 66⅔%; 100%.

Page 219. — **42.** 62½%. **43.** 16⅔%. **44.** 80%.

Page 220. — **46.** 50%. **47.** 41⅔%. **48.** 25%; 75%. **49.** 8%. **50.** 6¼%. **51.** 5%. **52.** 25%. **53.** 29 17/27%; 33⅓%; 47 7/24%; 59 7/27%; 29⅛%; 91⅔%.

Page 221. — **55.** 20%; 60%; 30%; 50%. **56.** $25; $6.25; $31.25; $43.75; $34.375. **57.** 11⅓%; 11⅛%; 5⅝%; 2⅔%; 17½%; 2⅝%. **58.** 12½%. **61.** 50%; 80%; 100%. **62.** 6⅔%; 16⅔%; 11½%; 33⅓%; 25%; 40%. **63.** 50%. **67.** 378 mi. **68.** 40; 42.

Page 222. — **74.** 200; 40. **75.** $200; $48; $160. **76.** $915. $305; $183. **77.** $50,000: $88,800; $44,440; $64,260. **78.** 100 mi. **79.** 2000 bu. **80.** 2500 bu.

Page 223. — **81.** 1200 A. **82.** $12,336.67; $10,486.17. **83.** 40 yr. **84.** 2 yr. **85.** 2½ T. **86.** 16⅔ sq. ft. **87.** 1 1/8 ft. **88.** $120. **89.** $28.75. **90.** $1100; $1518. **91.** $2000. **92.** $20. **93.** $1700. **94.** $2; 10%.

Page 224. — **96.** 68¢ per lb. **97.** *a* 4¢; *b* 52¢; *c* $1.08; *d* $1.48; *e* $2.20; *f* $3.80. **99.** $40. **100.** $10. **101.** $3. **102.** 300 bu. **103.** 200 lb. **104.** 300 A. **105.** 400 yd.

Page 225. — **106.** 20 ft. **107.** 1000 lb. **108.** 56 plum; 42 pear; 238 peach. **109.** $56. **110.** 23%. **111.** 18%. **112.** 19%. **113.** Oak 2 1/12; maple 3 3/4; beech 2 59/128; walnut 2 7/64. **114.** 240 cu. in.; 16⅔%. **115.** 455 da. **116.** 7.7 oz. **117.** 500 rd.

Page 226. — **118.** 180 mi. **119.** $19.04. **120.** 48⅝%. **121.** 93¾%. **124.** $60. **125.** $1.32. **126.** $1.20; $.12. **127.** $120. **128.** 10%. **129.** 10%. **131.** $12.50 lost. **132.** $1.00.

Page 227. — **133.** 100%. **134.** 20 yr.; 16⅔ yr.; 12½ yr.; 11⅓ yr.; 8⅓ yr. **135.** 33⅓%; 66⅔%; $.63; $1.26. **136.** 25%; 33⅓%; 41⅔%. **137.** 12½%; 62½%; 25%. **138.** $2133⅓; $16. **139.** $26,666⅔. **140.** $8640. **141.** $1600; $1520. **142.** $270. **143.** $7142⅞.

Page 228. — **144.** $5708.80. **145.** $8000; $6000. **146.** $5333⅓. **147.** $5000. **148.** $2222⅔. **149.** 42. **150.** 11,111⅛ bu. **151.** $2500. **152.** $15,000; $12,500; $9375; $18,750; $25,000. **154.** $1200.

Page 229. — **155.** $6300. **156.** $2100. **157.** $7200. **158.** $8100. **159.** $10,000. **160.** $3823; $12,823. **161.** $12605.30. **162.** $12,187.67. **163.** $3.36; $3.20; $3.24; $3.60.

Page 230. — **164.** $19.20; $16; $16.80; $24. **165.** $28. **166.** $19.94. **167.** $.90. **168.** $.81. **169.** $.70. **170.** $1. **171.** $3.28. **172.** $1.75.

Page 231. — **175.** $57.50; $517.50. **176.** $456.25; $1368.75. **177.** $277.75. **178.** $168. **179.** $120; $7880. **180.** 10%. **181.** $20; 10%. **182.** $1800.

Page 232. — **183.** $1800. **184.** $225. **185.** $200; $5250.25. **186.** $3.90. **189.** $1.81; $34.37.

Page 233. — **191.** $1423.75; $142.38; $1281.37. **192.** $52.50. **193.** $3.50. **194.** $13.13. **195.** $15.53. **196.** $71.25.

Page 234. — **198.** $72.50. **199.** $337.50; $362.50. **200.** $1320. **201.** $405. **202.** $200. **203.** $86.63. **204.** $228.48. **205.** $267.75. **206.** $342. **207.** $246.96. **208.** $342.

Page 235. — **210.** $31.92; $25.08; $28.08. **211.** $27.07; $13.90. **212.** $3180.60. **213.** $243.20. **214.** $483.00. **215.** $346.10.

Page 236. — **216.** $485.60. **217.** $503.10. **218.** $1444.80. **219.** $348.30. **220.** $54. **221.** $1197. **222.** The offers are the same.

Page 237. — **225.** $48. **226.** $140. **227.** $49. **228.** $160. **229.** $150. **230.** $50. **231.** $950. **232.** $150. **233.** $77. **234.** $2. **235.** $28. **236.** $33. **237.** $67.50. **238.** $100. **240.** $.071. **241.** $.126. **242.** $.187. **243.** $.256. **244.** $.317. **245.** $.462. **246.** $.533. **247.** $.555$\frac{1}{9}$. **248.** $.445$\frac{5}{9}$. **249.** $.526$\frac{1}{9}$. **250.** $.416$\frac{1}{9}$. **251.** $.456$\frac{1}{2}$.

Page 238. — **253.** $.308; $.462; $1.078; $3.85. **255.** $25.20. **256.** $58.55. **257.** $61.81. **258.** $56.29. **259.** $52.17. **260.** $20.83. **261.** $29. **262.** $48.46. **263.** $40.95. **264.** $84.23. **265.** $15.98. **266.** $40.33. **267.** $104.14. **268.** $111.14. **269.** $22.65. **270.** $83.94. **271.** $115.62. **272.** $191.56. **273.** $230.19.

Page 239. — **276.** At 6% $28.70; $258; $1.38; $43.37; $65. At 3% $14.[illegible]; $1.29; $.69; $21.69; $32.50. **278.** $2.63. **279.** $9.62. **280.** $14.86. **281.** $70.43. **282.** $8.28. **283.** $24.13. **284.** $24.75. **285.** $37.21. **286.** $4.16. **288.** $234.60.

Page 240. — **289.** $1296.90. **290.** $882.47. **291.** $868. **292.** $149.04. **293.** $846.44. **294.** $167.43. **295.** $949.35. **296.** $1008.89. **297.** $251.28. **298.** $1052.65. **300.** 1 yr. 6 mo. 6 da. **301.** $21; $48. **302.** $762.60. **303.** $474.83. **304.** $725.90. **305.** $838.60. **306.** $801.85. **307.** $900.20. **308.** $756.12.

Page 241. — **312.** $9.11. **313.** $3.63. **314.** $10.20. **315.** $20.42. **316.** $38.50. **317.** $12.50. **318.** $9.80. **319.** $20. **320.** $6.05. **321.** $16.86. **322.** $43.58.

Page 242. — **324.** $28. **325.** $27.33. **326.** $21.03. **327.** $27.56. **328.** $15. **329.** $15.40. **330.** $140.63. **331.** $62.07. **332.** $163.20. **333.** $150.70. **334.** $61.27.

Page 243. — **340.** $29.29. **341.** $9.34. **342.** $24.03. **343.** $8.29.

Page 244. — **352.** $432.08.

Page 245. — **354.** $4.20. **355.** $7.75; $6.46; $9.04. **356.** $40.94; $20.47. **357.** $7.75; $5.17. **358.** $1.65. **359.** Amt. $505.25.

Page 246. — **361.** $306.71. **362.** $330.83. **363.** $337.09; 342.40; $344.91. **367.** $191.30.

Page 247. — **373.** $437.75.

Page 248. — **374.** $344.85.

Page 249. — **378.** $276.55. **379.** $533.18. **380.** $83.99. **381.** $326. **383.** $40.63.

Page 250. — **384.** $25.90. **385.** $27.95. **388.** $5.83.

Page 251. — **390.** $24; $276. **391.** $380. **392.** $345.33; $395.33; **393.** $3.75; $246.25. **394.** $3.38; $446.62. **395.** $12; $888. **396.** $11.25; $738.75. **397.** $4.50; $895.50. **398.** $12.64; $637.36. **399.** $788.

Page 252. — **400.** $596. **401.** $496.56. **402.** $860.14. **403.** $655.16. **404.** $739.04. **406.** $37.70. **407.** $888.20. **408.** 841.-31. **409.** $737.67. **410.** $1490. **411.** $424; $390.08. **412.** $1380; $1605; $225.

Page 253.—**413.** $1476.60. **414.** $1485. **415.** $746.25. **416.** $646.75. **417.** $1697.17. **418.** $1196. **419.** $996.67. **420.** $247.50. **421.** 1st sale profit $24.67; 2d sale $24.63; 3d sale $24.

Page 254.—**422.** $50. **423.** $3418.75. **424.** $3474.70. **426.** $12; $1200.

Page 255.—**427.** $16.88; $20. **428.** $118.13. **429.** $993.75. **430.** $16.60. **431.** $92. **432.** $5.39. **433.** $29.60. **434.** $30.25. **435.** $53. **436.** $92.

Page 256.—**437.** $960. **438.** $442.50. **439.** $5.50. **440.** $51. **441.** $39. **442.** $80. **443.** $32.50. **444.** $750.

Page 257.—**445.** $1670; $3340. **446.** $2702 more. **447.** $4761.60 less. **448.** $4061.20 more. **449.** $11,070 less. **450.** $41,087.50 more. **451.** $83,000 more. **452.** $96; $800. **453.** $12,364.63.

Page 258.—**454.** $24,181.25. **455.** $12.50. **456.** $4331.25. **457.** $30.75 less. **458.** $192.40. **459.** $140.21; $771.84.

Page 259.—**460.** $617.50 more; $682.50 less. **461.** $180.25. **462.** $120. **463.** $100. **464.** $101.50. **465.** $99.50.

Page 260.—**466.** $209.13. **468.** $79.20. **469.** $87.12. **470.** $25.84. **471.** $29.03. **474.** $20,403.

Page 261.—**475.** 624. **476.** $45,000; $.01¼. **477.** $106.19. **478.** $30.48. **479.** $101.69. **480.** $1.50. **481.** $6.25. **482.** $.50; $.06; $.02.

Page 262.—**483.** $32.24. **484.** $27.50. **485.** $34.38. **486.** $1500 **487.** $67.50.

Page 263.—**488.** $1500. **489.** $325. **490.** $112.50; $112.50. **491.** $712.50. **492.** $4600. **493.** $1080. **494.** $3979.92. **495.** $55.40.

Page 264.—**496.** $1120. **497.** $120. **498.** $12,500. **499.** $86,400. **500.** $96,000; $8000. **501.** $75,000; lose $1000.

Page 265.—**1.** 2, 3^3, 5^3; 2^4, 3^2, 5, 11. **2.** 5. **3.** 720. **4.** .0061. **5.** 87. **6.** 28. **7.** 3¾ bu., $216.69. **9.** 24. **10.** 51⅓. **11.** $\frac{13}{100}$; $\frac{9}{50}$; $\frac{9}{40}$; $\frac{79}{50}$; $\frac{35}{4}$; $\frac{1}{8}$; $\frac{1}{500}$; $\frac{1}{120}$. **12.** 1. **13.** 63.36; 96; 2560. **14.** 216; 486; 1512. **15.** $\frac{25}{1}$; .09; 1.44; 6¼; .0036; $\frac{1}{3}$.

Page 266.—**19.** 144 in.; $45\frac{9}{11}$ in.; $22\frac{10}{11}$ in.; $81\frac{9}{11}$ in.; $81\frac{9}{11}$ in., $93\frac{9}{11}$ in.; $69\frac{9}{11}$ in. **20.** 25%. **21.** 12. **22.** 25%. **23.** 50%. **24.** 20%; 200%; $14\frac{2}{7}$%.

Page 267.—**28.** $1428. **29.** $146.54. **31.** 325. **32.** $11\frac{23}{35}$ qt. **33.** $138\frac{18}{35}$ mi. **34.** $13\frac{19}{20}$ mi.; $41\frac{17}{20}$ mi. **35.** 12½ ¢ less; no more nor less. **36.** $3\frac{1}{3}$.

Page 268.—**37.** 1⅔ ¢. **38.** $325. **39.** 3 bu. **40.** 240 mi. **41.** 900; 2400. **42.** 30; 90. **43.** $327. **44.** $144. **45.** $240. **46.** $13\frac{7}{11}$%. **47.** 75 ¢. **48.** $62.50. **49.** 26%.

Page 269.—**50.** $69\frac{1}{21}$%. **54.** 120 ft. **55.** $42. **58.** $23\frac{1}{4}$ in.

Page 270.—**59.** 21 rd. 2 ft. 4 in. **60.** 64 in. **61.** 54 ft. **62.** 8 ft. 2 in. **64.** 6.8 in. **65.** 29.6 in. **66.** 25.5 in.

Page 271.—**70.** 50%. **71.** 66 ¢. **72.** 10 ¢. **73.** $31\frac{1}{4}$ ¢. **74.** 12 ¢. **75.** 13 ¢. **76.** $21\frac{25}{42}$%. **77.** 315. **78.** 40%. **79.** 4%. **80.** $3\frac{1}{3}$%.

Page 272.—**82.** $4720. **83.** $90. **84.** 18%. **85.** 20%. **86.** 5 ¢. **87.** $87.50. **88.** $20.48. **89.** 132 lb. **91.** Lose 25%; gain 50%; g. 100%; l. 50%; g. 25%. **92.** 50%. **93.** $212. **94.** $500; $20\frac{5}{8}$%.

Page 273.—**95.** 20%. **96.** 100%; 50%. **97.** $17.40. **98.** 18. **99.** 1768. **100.** 10%. **101.** $79. **102.** $39.50. **103.** $1800.

Page 274.—**104.** $25.11. **105.** ⅔; 40%; $3.00; $4.50. **106.** 25%;

$12.50. **107.** $10,000. **108.** $510. **109.** $636.87. **110.** $9.40. **111.** $11.75. **112.** $11.75. **113.** *a* $11.07; *b* $10.82; *c* $11.44; *d* $10.50; *e* $10.88.

Page 275. — **114.** 40%; 20%. **115.** $.25; $.50; $.75. **116.** 30%. **117.** $6. **118.** $54.37. **119.** $20,736. **120.** $4650.

Page 276. — **121.** $60. **122.** *a* $\frac{2}{3}$; *b* $\frac{3}{7}$; *c* $1\frac{4}{11}$; *d* $\frac{9}{10}$. **123.** $34\frac{2}{3}$ in. **124.** $15.71. **125.** 66 ft. **126.** .183203125. **127.** .17. **128.** 4200 sq. cm. **129.** 154 ft. **130.** 8400 cu. ft. **131.** 28 sq. in. **132.** $19\frac{11}{16}$ A.

Page 277. — **133.** 30%; 20%. **135.** 400 sq. in. **136.** 25%; $16\frac{2}{3}$%; $12\frac{1}{2}$%; 10%. **137.** $33\frac{1}{3}$%; $46\frac{2}{3}$%; 60%. **138.** 50%. **139.** 140% gained. **140.** $1500; $1500. **141.** $11,484. **142.** 7 chickens; 6 doz. eggs; $2.08.

Page 278. — **143.** 275%. **144.** $5.50; $7.42; $9.08. **145.** 9.22\frac{1}{2}$; 27.67\frac{1}{2}$. **146.** L. & H. = $9.12; E. & C. = $27.35; St. P. & R. = $7.46; C. & St. P. = $22.37. **147.** $187. **148.** $101.35.

Page 279. — **149.** $83.25. **151.** $16\frac{2}{3}$%. **152.** $2332.25. **153.** $2. **154.** $14\frac{8}{17}$%. **155.** $\frac{4}{5}$%. **156.** $577.50. **157.** 10; 20. **158.** 27 cu. in.

Page 280. — **3.** $3.75.

Page 281. — **4.** $300; 80. **5.** $3907.50; $15,630. **7.** $3120. **8.** 9; 25. **9.** $7875. **10.** $15,300. **11.** $4900. **12.** $1960. **13.** 13; 17. **14.** $425.

Page 282. — **15.** $16,836; $1098. **16.** $252,500. **17.** $750 gain. **18.** $3750. **19.** $4200. **20.** $5000. **21.** $5; $4005; No.

Page 283. — **22.** $65. **23.** $60. **24.** $27.50. **25.** Neither. **27.** $6170. **28.** 24. **29.** 90; $360.

Page 284. — **30.** *a* $8410; *b* $5048.75; *c* $1693.38. **31.** $2057.50. **32.** $7098.75. **33.** $90. **34.** $37.50; $7.50. **35.** $2160; $80. **36.** 75; $225. **37.** 96; $384. **38.** *a* $400; *b* $210; *c* $80; *d* $800.

Page 285. — **39.** $2.50; 25%. **40.** $1.25; $.75; $.25. **41.** $3.25. **42.** *a* 2%, Albert $.10, Edgar $.06, Fred $.02, John $3.02; *b* 39%, Albert $1.95, Edgar $1.17, Fred $.39, John $3.39. **43.** Albert $.50, Edgar $.30, Fred $.10, John $.10.

Page 286. — **44.** 60%. Albert, $3.00; Edgar, $1.80; Fred, $.60; John, $.60. **45.** $300. **46.** 4%; 6%. **47.** 7%; $140; $3500; $490.

Page 287. — **48.** $5070. **49.** Mr. Smart $90; Mr. Howe $180; Mr. Blake $4500; Mr. Lee $630. **50.** At 12%, Mr. Smart $120; Mr. Howe $240; Mr. Blake $6000; Mr. Lee $840. At 5%, Mr. Smart $50; Mr. Howe $100; Mr. Blake $2500; Mr. Lee $350. At 7$\frac{1}{2}$%, Mr. Smart $75; Mr. Howe $150; Mr. Blake $3750; Mr. Lee $525. **51.** $1600. **52.** $40; $200; $140. **53.** $5020. **54.** $100; $500; $350. **55.** $4450. **56.** Mr. Blake $7000; Mr. Smart $600; Mr. Howe $700; Mr. Lee $1200.

Page 288. — **57.** $280; $40. **58.** $800. **59.** $8775; $810. **60.** 10%, $370. **62.** $7350. **63.** $44,100. **64.** $43,312.50; $7245.

Page 289. — **65.** 80; $400. **66.** 80; $480. **67.** $744. **68.** $11,625. **69.** $2496. **70.** $490. **71.** $490. **73.** 4%. **74.** 5%. **75.** $2\frac{22}{41}$%. **76.** 8%. **77.** $5\frac{155}{167}$%.

Page 290. — **78.** Shares $2\frac{2}{3}$% more. **79.** $1500; $75; $3\frac{7}{11}$% on bonds; $5\frac{5}{9}$% on stocks. **80.** $1147.50. **81.** $536.75. **82.** $13.50. **83.** $8400. **84.** $17.50. **85.** $450. **1.** 997. **2.** 75%. **3.** $21,000. **4.** $8400. **5.** 14 yr.

Page 291. — **6.** 2856 bu. **7.** $26.50. **8.** 3.5; 3.8. **9.** 21$\frac{1}{2}$ mi.;

$1\frac{7}{10}$ mi.; $1\frac{8}{10}$ mi. **10.** 22.92 mi. **11.** $11\frac{1}{2}\%$. **12.** 220 mm. **13.** 13. **14.** 600 sq. dm. **15.** 272 s. **16.** \$14. **17.** A gram; 5500 g.; 5.5 Kg. **18.** $5\frac{3}{5}$ Kg. **19.** 169.2 Kg.

Page 292. — **20.** 27 Kg. **21.** $\frac{1}{4}$. **22.** 30 lots; \$58,935. **23.** 45,360. **24.** \$3000. **25.** \$28.13. **26.** 40%. **27.** \$98. **28.** 25%. **29.** \$1334.38. **30.** \$5950.88. **31.** \$774.28. **32.** 50%; \$900; \$1228.33.

Page 293. — **33.** $37\frac{1}{2}\%$. **34.** 60%. **35.** \$1.65; $3\frac{1}{8}\%$. **36.** \$216; 25%. **37.** \$85.61. **39.** \$28.50; \$921.50. **40.** \$1875; \$5625. **41.** \$1059.90. **42.** \$1036.35. **43.** \$8970. **44.** \$7028.13.

Page 294. — **45.** \$3.75. **46.** \$9773.12$\frac{1}{2}$. **47.** \$54 gain. **48.** \$70 loss. **49.** \$26.25 loss. **50.** 125 shares; \$1000. **51.** \$1120. **52.** 30. **53.** 30¢.

Page 295. — **57.** $20\frac{8}{10}$ in. **58.** $23\frac{1}{4}$ in. **59.** 4 ft. **61.** 44 in. **62.** 8. **63.** 30. **64.** \$.06. **65.** \$33.73.

Page 296. — **1.** 70 da.; 600 min.; 1000¢. **2.** 16 ft.; 4 yr.; 3 lb.; 6 pk.; 12 gal. **9.** 13; 45; 38; 29; 8; 4; $3\frac{1}{3}$; $\frac{3}{10}$. **13.** 4 wk.; 5 wk.; 3 wk.; 7 wk.

Page 297. — **14.** 9 gal. **15.** 45 ft.; 51 ft. **17.** 66; 66. **18.** 8¢; 24¢. **21.** 25; 6.

Page 298. — **26.** 18; 10; 80; 2; 2; 36; 9. **27.** 18; 81; 25; 10; 3; 1. **28.** 35; 2; 28; 21; 49; 1. **29.** 45; 69; 169; 169. **30.** 12. **31.** 35. **32.** 14. **33.** 4. **34.** 105. **35.** 50. **36.** 9. **37.** 70 in.; 20 ft.; 50 yd. **38.** 10 in; 40 in. **39.** 40 in.; 12 ft. **40.** 48 in.; 6.6 in.; 15 in.

Page 299. — **41.** 56 in.; 28 ft. **42.** 70 in.; 27 in. **44.** 6 *b*. **45.** 11 *c*. **46.** 15 *x*. **47.** 5 *x*. **48.** 28 *y*. **49.** 43 *x*. **50.** 14 *x*. **51.** 49 *x*.

Page 300. — **53.** 5 in., 10 in., 15 in.; 9 in.; 18 in., 27 in. **54.** 9 in.; 18 in.; 18 in. **55.** 8 in., 24 in., 24 in.; 15 in., 45 in., 45 in.; 21 in., 63 in., 63 in. **56.** 5 in., 5 in., 15 in., 20 in.; 17 in., 17 in., 51 in., 68 in. **57.** 6 in., 6 in., 12 in., 18 in.; 1 ft. 4 in., 1 ft. 4 in., 2 ft. 8 in., 4 ft. **58.** 8 in., 16 in., 24 in., 32 in.; $7\frac{1}{2}$ in., 15 in., $22\frac{1}{2}$ in., 30 in. **59.** \$4; \$12; \$20. **60.** 15; 60. **61.** 3; 18.

Page 301. — **62.** 12 yr.; 36 yr.; 48 yr. **63.** 3 and 15. **65.** 9 ft.; 18 ft.; 27 ft. **66.** *AB* 12 ft., *BC* 24 ft., *CD* 12 ft., *DE* 36 ft., *EA* 12 ft.; *AB* $12\frac{1}{2}$ in., *BC* 25 in., *CD* $12\frac{1}{2}$ in., *DE* $37\frac{1}{2}$ in., *EA* $12\frac{1}{2}$ in. **68.** 5; 15; 60. **69.** \$1; \$3; \$6. **70.** 2; 4; 16.

Page 302. — **73.** 129; $118\frac{1}{2}$. **74.** 132. **75.** 99. **76.** 39; 55; 60; $35\frac{3}{8}$. **78.** 5 and 25; 3 and 27; 2 and 28. **79.** 8 and 16; 6 and 18; 4 and 20. **81.** \$3000; \$15,000. **82.** 7. **83.** \$3; \$27.

Page 303. — **84.** 6; 24; 30. **85.** 21. **86.** 9; 27; 36. **87.** 1; 2; 3. **88.** 10; 40; 50. **89.** 12, 48, 60; 15, 60, 75; 60, 240, 300. **90.** *AC* 18 in.; *BC* 54 in.; 144 in. **91.** *AC* 14 in.; *AB* 56 in. **93.** 5. **94.** 8. **95.** 7.

Page 304. — **97.** 4 qt.; 12 qt. **98.** \$32; \$160. **99.** 11 in.; 44 in.; 44 in. **100.** 40 in. **101.** *AB* 8; *BC* 16; cir. 96 in.; diam. $30\frac{6}{11}$. **102.** 2 ft.

Page 305. — **103.** 29 in. **105.** 7; 13; 16. **106.** 12; 24. **107.** 10; 70. **108.** 20 ft.; 60 ft. **109.** 9 ft. **110.** 10 ft. **111.** \$34.

Page 308. — **135.** 30. **136.** 29. **137.** 11. **138.** $7\frac{1}{2}$. **139.** 6. **140.** 10. **141.** 12. **142.** 5. **145.** 4. **146.** 2. **147.** 8. **148.** 3. **149.** 4. **150.** 12.

Page 309. — **151.** 58. **152.** $3\frac{1}{2}$. **153.** 1. **154.** $2\frac{2}{17}$. **155.** 3. **156.** 38. **166.** 3 and 14. **167.** 8 ft.; 23 ft. **168.** 7 ft.; 11 ft.; 11 ft. **169.** 8 yr.; 15 yr.

Page 310. — **171.** Chain $15; watch $35. **173.** Monday $.70; Tuesday $1. **174.** 40 A.; 60 A. **175.** 32 and 40. **176.** $21\frac{1}{3}$ and $68\frac{2}{3}$. **177.** *B* 20 mi.; *A* 30 mi. **178.** 90 ft.; 1800 sq. ft. **179.** 6.

Page 311. — **181.** *BC* 9 in.; *AB* 1 ft.; *AC* 1 ft. 3 in. **182.** *BC* 15 in.; *AC* 23 in.; *AB* 35 in. **183.** *YZ* 20 in.; *XY* 31 in.; *XZ* 37 in. **184.** *EF* 15 in.; *DE* 22 in.; *DF* 28 in. **185.** 7. **186.** 9 and 18. **187.** 5 yr.; 7 yr.; 9 yr.; 11 yr. **188.** 12 yr.

Page 312. — **189.** 13; 19. **190.** Mr. A $21.50; Mr. B $8.50; Mr. C $15.50; Mr. D $23.50. **191.** $333\frac{1}{3}$; $3333\frac{1}{3}$; $58333\frac{1}{3}$. **192.** 9; 27. **194.** $800; $900; $600. **195.** $9\frac{3}{4}$; $16\frac{2}{3}$; $13\frac{7}{8}$. **196.** $30,000; $50,000. **197.** $105; 18 wk.

Page 313. — **198.** 30 ft.; 90 ft.

Page 314. — **206.** 70. **207.** 216. **208.** 30. **209.** 15. **210.** 17. **211.** $11\frac{7}{11}$. **212.** 14. **213.** 20. **214.** $14\frac{16}{19}$. **215.** $21\frac{6}{7}$. **216.** $26\frac{16}{29}$. **217.** 50. **218.** 35. **219.** 28. **220.** 77. **221.** 70. **222.** 16. **223.** 12 in.; 8 in.; 10 in. **224.** 20 in.; 16 in. **225.** 30. **226.** 44 ft.; 121 ft.; 121 ft.

Page 315. — **227.** 231 ft.; 396 ft.; 91,476 sq. ft. **229.** 15 in.; 17 in; 8 in. **230.** 21; 24. **231.** 18; 24. **232.** 42; 48. **234.** 35¢; 40¢. **235.** 1 hr. 48 min. **236.** $25\frac{7}{17}$ first; $31\frac{13}{17}$ second; $50\frac{1}{17}$ third. **237.** 63. **238.** 21; 27; 36. **239.** 1st yr. $2400; 2d yr. $2700; 3d yr. $3150.

Page 316. — **240.** Upper base 12 in.; lower 24 in.; sides 8 in., 10 in. **241.** $3\frac{21}{22}$ ft.; $3\frac{13}{44}$ ft. **1.** $4.0425. **2.** $1147.50; $127.50; $76.50. **3.** $44. **4.** 9. **5.** $50. **6.** 25%; $33\frac{1}{3}$%. **7.** $x = 4$. **8.** $x = 3$. **9.** 37.

Page 317. — **10.** 92. **11.** 11 yr. **12.** 144. **13.** 10 dm.; 25 dm. **15.** $x = 200$. **16.** $x = 400$; $x = 8000$; $x = 408$. **17.** $x = 4000$; $x = 2000$; $x = 1200$. **18.** $x = 600$; $x = 300$; $x = 60{,}000$. **19.** $3000. **20.** $1400; $1000; $3500; $700; $875. **22.** $.151x$; $.342x$; $.474x$.

Page 318. — **25.** $1600. **26.** $1333.33\frac{1}{3}$. **27.** $1950. **28.** $700. **29.** $779.22. **30.** $1562.50. **31.** 1 yr. 1 mo. 18 da. **32.** 2 yr. 4 mo. 15 da. **33.** 4 yr. 3 mo. 6 da. **34.** 4 yr. **35.** 8 mo. **36.** 5 yr. 7 mo. 27 da. **38.** 4%. **39.** 5%.

Page 319. — **40.** 8%. **41.** 3%. **42.** $2\frac{1}{2}$%. **43.** $400. **44.** $84. **45.** $129.16. **46.** $75. **47.** $144. **48.** $75. **49.** 4%. **50.** $300. **51.** 1 yr. 8 mo. **52.** $16,666\frac{2}{3}$; $28,571\frac{3}{7}$. **53.** Neither.

Page 320. — **55.** $200. **56.** $4000; $720. **57.** $180; $19.80. **58.** $300; $7.50. **59.** $125; $18.75. **62.** $3.92. **63.** $6.075. **64.** $12.25. **65.** $1.13. **66.** $8.41. **67.** 2×9.

Page 321. — **68.** 3×36. **69.** 120 sq. in. **70.** 60 sq. in. **71.** $180. **72.** $1191.016. **73.** $191.016; $11.016. **74.** 464.10. **76.** $10,854; $8854; $5354. **77.** $324.65.

Page 322. — **2.** 6859; $\frac{1}{8}$; .027; $15\frac{5}{8}$; 1.728; .000000000000000343. **3.** 2401; .00032; $91\frac{1}{8}$; .216; 1.4641; $\frac{1}{8}$. **4.** 3; 4; $\frac{81}{1024}$. **5.** 96. **7.** 400; 900; 1600; 6400; 4900. **9.** 8000; 64,000; 216,000; 512,000; 125,000.

Page 323 — **17.** 19 sq. cm. **18.** 39 sq. in. **19.** 96 sq. in. **20.** 125 sq. in.

Page 325. — **26.** 24 ft. **27.** 25 in.

Page 326. — **30.** 2025. **31.** 1024.

Page 327. — **36.** 35. **37.** 25. **38.** 22. **39** 17. **40.** 41. **41.** 66. **42.** 47. **43.** 42. **44.** 96. **45.** 83. **46.** 51 **47.** 93. **48.** 72. **49.** 53. **50.** 45. **51.** 33. **52.** 81. **53.** 34. **54.** 71. **56.** 89; 49; 59. **57.** 225; 400. **60.** 112.

Page 328. — **61.** 495. **62.** 613. **63.** 886. **64.** 881. **65.** 969. **66.** 499. **67.** 739. **68.** 237. **69.** 153. **70.** 728. **71.** 715. **72.** 901. **73.** 821. **74.** 266. **75.** 346. **76.** 41 in.; 31 cm.; 27 ft.; 32 m. **77.** 132 in.; 376 ft.; 296 cm.; 252 dm. **78.** $192. **79.** $103.20; $88.80. **80.** $\frac{2}{3}$; $\frac{9}{11}$. **81.** $\frac{11}{14}$; $\frac{7}{22}$; $\frac{8}{13}$; $\frac{1}{3}$. **82.** $1\frac{1}{3}$. **83.** $1\frac{2}{3}$; $2\frac{1}{5}$; $1\frac{1}{4}$; $1\frac{3}{7}$; $2\frac{3}{5}$. **85.** .025. **86.** .07; .09; .05; .006. **87.** .023; .011; 5.4. **88.** 7.1; 7.8; 9.71; 6.19. **90.** 11.13+.

Page 329. — **91.** 5.19; 4.89; 9.21; 1.16; 1.64; 1.95; 2.02. **92.** 6.7 in.; 8.6 in.; 9.7 in. **93.** .121; .63. **97.** 25. **98.** 17. **99.** 41. **100.** 25. **101.** 61. **102.** 51. **103.** 200 mi. **104.** 26 in. **105.** 13 ft.

Page 330. — **106.** 45 ft. **107.** A rhombus, 20 in. **108.** 10 in. **109.** 5 ft. **110.** 10 ft. **111.** 20 ft. **112.** 169 ft. **113.** 168 ft. **114.** 16 cm. **115.** 120 ft.

Page 331. — **116.** $\frac{12}{41}$; $\frac{6}{2\frac{3}{8}}$; $\frac{2}{7}$. **118.** 24. **119.** 23.06+. **120.** 13.8+. **121.** 14. **122.** 21. **123.** 17.8+. **124.** 39 ft. **125.** 12.7+in. **126.** 14.1+ ft. **127.** 11.3+ ft. **128.** 10.6+ in.

Page 332. — **129.** 156 ft. **132.** 35 ft.; 26.9+ ft. **133.** 31.3+ ft. **134.** 24.7+ ft. **135.** 36.3+ ft. **136.** 11.1+ ft.

Page 333. — **1.** $450.45. **2.** 7849.999568. **3.** 20,677. **5.** 48%. **6.** 4. **7.** 24%. **8.** 16; 36; 64; 81; 100. **9.** $4172\frac{2}{3}$; **10.** $.18.

Page 334. — **11.** $120. **12.** $147. **13.** $78.14 more. **14.** $21,500. **15.** 20%. **16.** 8%. **17.** $53.86; $1023.34. **18.** $2400. **19.** 166.32 T.; 49.68 T. **20.** 26.91 T. **21.** 40.81 T.

Page 335. — **22.** $10\frac{5}{12}\not{\text{o}}$. **23.** 80,535. **24.** $2665.13. **25.** $3300; $3201. **26.** $360.15. **27.** 19 gal. 3 qt. 1 pt. **28.** 20%. **29.** $540. **30.** 388.2. **31.** $\frac{1}{5}$. **32.** $1\frac{1}{2}$. **33.** $1\frac{1}{3}$. **34.** 4. **35.** 4. **37.** 8.

Page 336. — **41.** $11\frac{2}{3}$. **42.** 7. **43.** $2\frac{1}{3}$. **44.** 7. **45.** 10 in.; 10 in.; 14 in. **46.** 9 in.; 12 in.; 15 in.; 18 in. **47.** $720. **48.** 2 yr. 3 mo. **49.** $3\frac{5}{8\frac{1}{3}}$%. **50.** $520. **51.** $560. **52.** 18. **53.** $316.80. **54.** $10\frac{5}{8}$ ft. **55.** $6.38.

Page 337. — **56.** 206.5+ in. **57.** 40 in.; 120 sq. in. **58.** 33.6+ in. **59.** 1080 sq. in.; 138 in. **60.** $16\frac{1}{2}$ in. **61.** $11\frac{19}{21}$ in.; $34\frac{2}{7}$ in. **62.** 17 cm.; 68 cm.

Page 338. — **64.** $112\frac{7}{12}$ ft.; $144\frac{3}{4}$ ft. **65.** $144\frac{3}{4}$ ft.; $434\frac{1}{4}$ ft. **66.** 128 ft.; 160 ft.; 272 ft.; 520 ft. **67.** 144 ft.; 180 ft.; 444 ft.; 870 ft. **68.** $216. **69.** $27.22. **70.** $12.83.

Page 339. — **71.** 1 sq. ft. **72.** 8 in. **73.** 125 cu. in. **74.** 3 in. **75.** 52; 246; $\frac{6}{11}$; $\frac{7}{9}$; 6.7. **76.** 5; 12; 9. **78.** 70 in. **79.** 48. **80.** 3 in.

Page 341. — **16.** $13\frac{1}{3}$. **17.** 6. **18.** 5. **19.** 12. **20.** 18. **21.** 6.

Page 342. — **22.** 5. **23.** 9. **24.** 9. **25.** 7. **26.** 20. **27.** 60. **28.** 15. **29.** $6\frac{2}{3}\frac{2}{3}$. **30.** $33\frac{1}{3}$. **31.** 50. **32.** 150. **33.** $133\frac{1}{3}$. **34.** 11. **35.** $\frac{3}{5}$. **36.** $5\frac{1}{3}$. **37.** $\frac{1}{20}$; 2. **44.** 6. **45.** 11. **46.** 4. **47.** 14. **48.** 14. **49.** 9.

Page 343. — **52.** $14. **53.** $.30. **54.** $.05$\frac{2}{3}$. **55.** $.26$\frac{1}{3}$. **56.** $75. **57.** 43\frac{1}{3}$. **58.** $44. **59.** $51. **60.** 30.66\frac{2}{3}$. **61.** $25.44. **62.** $.455. **63.** 4.16\frac{2}{3}$. **65.** $46. **66.** 61\frac{1}{3}$. **67.** 116.66\frac{2}{3}$.

Page 344. — **68.** $57. **70.** 10 mi. **71.** 11.66\frac{2}{3}$. **72.** $.41$\frac{2}{3}$. **73.** $.31$\frac{1}{4}$. **74.** 1.87\frac{1}{2}$. **75.** 1.66\frac{2}{3}$. **76.** $12.50. **77.** $187.50. **78.** 3507. **79.** 6 in.

Page 345. — **82.** 25 in.; 21 in.; 72 in.; 75 in.; $\frac{1}{3}$. **83.** 7 in.; 20 in.;

28 in. ; 28 in. **84.** 8 in. ; 8 in. ; 16 in. ; 24 in. **85.** 90 in. **86.** HF 13 in. ; FG 5 in. ; ABCD 240 sq. in. ; EFGH 60 sq. in. ; ratios 4 and $\frac{1}{4}$.

Page 346. — **87.** $2\frac{3}{4}$ in. ; $4\frac{13}{16}$ sq. in. **88.** 15 in. ; 12 in. **91.** 1 ft. **92.** 45 da.

Page 347. — **93.** 60 ; 120 ; 240 ; 150 ; 300. **94.** 2 ft. ; 4 ft. ; $3\frac{1}{2}$ ft. **95.** 15 da. ; $13\frac{1}{3}$ da. ; 20 da. **96.** $140. **97.** 24 hr. ; 15 hr. ; 12 hr. **98.** $1200 ; $2000. **99.** $1.40 ; $1.80 ; $4.60 ; $2. **100.** $4200 ; $1400 ; $490 ; $52.50.

Page 348. — **101.** 18 oz. ; 9 oz. ; 24 oz. **102.** $96 ; $84 ; $48. **103.** $110.75 ; $70.88. **104.** $1120 ; $1440 ; $1600 ; $2000. **105.** $297.88. **106.** $2125. **107.** 9 yr. **108.** 50¢ ; $\frac{3}{5}$; $\frac{2}{5}$; 60¢ ; 40¢. **109.** $.08 ; $.12.

Page 349. — **111.** 10 : 2 ; 30 : 6 ; 45 : 9 ; 70 : 14. **112.** 12¢ ; 2¢. **113.** $11 ; $33 ; $55. **114.** $15 ; $18 ; $21. **115.** $24 ; $16 ; $8. **116.** $30 ; $21. **117.** A $11.40 ; B $18.24 ; C $25.08. **118.** $12.25 ; $36.75 ; $21. **119.** $52.50 ; $70.

Page 350. — **120.** A $60 ; B $36 ; Helper $40. **121.** 2250 lb. ; 450 lb. ; 300 lb. **122.** $.75 ; $.50. **123.** 30 ; 40 ; 80. **124.** 15 ; 21 ; 39. **125.** $900 ; $1800. **126.** $857.14 ; $2142.86. **127.** $1100 ; $2400 ; **128.** $3000 ; $2250.

Page 351. — **129.** $36 ; $40. **1.** 4. **2.** 161 ; 156 ; 92. **3.** $1\frac{3}{7}$. **4.** 13. **5.** 56. **6.** 252. **7.** 2. **8.** 15. **9.** 216. **10.** 64 cu. in. **11.** $66\frac{2}{3}$ A. **12.** $14{,}088\frac{15}{16}$ T. **13.** $250. **14.** $626,175. **15.** $22.50.

Page 352. — **16.** $\frac{3}{20}$; $\frac{1}{40}$. **17.** $\frac{88}{99}$. **18.** $\frac{55}{11}$. **19.** .3700 ; .0016. **20.** 9000. **21.** 3. **22.** 33 bu. **23.** $127. **25.** 360. **26.** $2 \times 3 \times 3 \times 19$. **27.** 122.625. **28.** $17.50. **30.** 60%.

Page 353. — **31.** $277.20. **32.** $31\frac{1}{4}$ bu. **33.** $36.90. **34.** 20. **35.** $66\frac{2}{3}$%. **36.** $66\frac{2}{3}$%. **37.** 1020. **38.** 38¢. **39.** 33. **40.** 2 lb. 4 oz. **41.** 34%.

Page 354. — **42.** $3 ; 50%. **43.** $3. **44.** $.24 for fire-crackers, $.14 for candy, $.42 for lemons. **45.** $33\frac{1}{3}$%. **46.** $33\frac{1}{3}$%. **47.** $4. **48.** $13.50 ; $441 ; $220.50. **49.** $16\frac{2}{3}$ lb. **50.** 24 A. ; 60 A. **51.** $54.25. **52.** $1.25 ; $.45.

Page 355. — **53.** $600. **54.** $29.76. **55.** $20. **56.** $33\frac{1}{3}$% ; 10% ; 30% ; 15%. **57.** 50%. **58.** 20%. **59.** Gained $150 ; $12\frac{1}{2}$%. **60.** $1944 ; $216 ; $240. **61.** $350. **62.** $125. **63.** $2000. **64.** $5333.33\frac{1}{3}$.

Page 356. — **65.** $266. **66.** $7000. **67.** $217.13. **68.** $28,000. **69.** $3502.40 ; $4342.80 ; $2000 ; $1688.40. **70.** 50% ; $892.75. **71.** Mr. Low 55¢ ; Mr. Van 35¢ ; Mr. Dow 30¢ ; Mr. May 40¢. **72.** $7360.90. **73.** $430 ; $28\frac{2}{3}$%.

Page 357. — **74.** $468.72. **75.** 100%. **76.** $200. **77.** Buyer was loser ; $73.25. **78.** 7. **79.** 10. **80.** 10. **81.** 7. **82.** 8. **83.** $12 ; $38. **84.** 500 ; 1500 ; 2000. **85.** 6 in. ; 13 in. ; 13 in. **86.** 48.

Page 358. — **87.** 20 cm. ; 35 cm. ; 350 sq. cm. **88.** 70 in. **89.** 3 ; 36. **90.** 10 ; 30 ; 80. **91.** 15 ft. ; 45 ft. ; 60 ft. **92.** 80 in. **93.** 70¢ ; $16\frac{2}{3}$%. **95.** $21\frac{3}{5}$%.

Page 359. — **96.** $1 ; 25%. **97.** $4. **98.** $16. **99.** $.20. **100.** $.64 ; $1. **101.** $.50 ; $1. **102.** $.96 ; $1.60.

Page 360. — **105.** $1.84. **106.** $90. **107.** $20. **108.** $1. **109.** $2. **110.** $1.20. **111.** 12¢ ; $12\frac{1}{2}$%. **112.** 42¢ ; 100%.

Page 361. — **113.** 54¢ ; 200%. **114.** 85 ft. **115.** 24 ft. **116.** 76 ft. **117.** 336 ft. **118.** $6\frac{1}{2}$ ft. **119.** 1331 cu. in. **120.** 512 cu. in. **121.** 72. **122.** 35. **123.** 9. **124.** 1. **125.** 9. **126.** 36. **127.** 21. **128.** $4\frac{8}{7}$. **129.** $1\frac{1}{3}$. **130.** 18.

Page 362. — **131.** $55. **132.** $8.75. **133.** 279 mi. **134.** Alt. 2 ft. 9 in.; Base, 3 ft. 8 in.; Hyp. 4 ft. 7 in. **135.** 3.195 in. **136.** 14 bbl. **137.** $6180; $180. **138.** $21. **139.** 5. **140.** 50 da. **141.** 32. **142.** $144.

Page 363. — **143.** $1407. **144.** 7%. **145.** $150. **146.** $20; $30; $40. **147.** $1600; $6400. **148.** $600; $1200. **149.** $233.33⅓; $466.66⅔; $1400. **150.** $13.61; $15.55. **151.** A & X $94.50; C & Y $53; Third; $58.50.

Page 364. — **152.** $266.03. **153.** $221. **154.** $232.24. **155.** $170.81. **156.** $99.48. **157.** 1 ft.; 60%. **158.** 49 ft.; 28%. **159.** 20 bu. **160.** $10.60; $12.

Page 365. — **162.** 3; 6; 10. **163.** 15; 21. **164.** 12,150; $33.41; $2.19; $16.88; $52.48.

Page 369. — **13.** 28 sq. in. **14.** 56¼ A. **15.** 11 ft. **17.** 24 sq. in.

Page 370. — **21.** 432 sq. in. **22.** 200. **23.** 6. **24.** 7. **25.** 4. **26.** .4. **27.** 5. **29.** 336 sq. in. **30.** 56 sq. in.

Page 371. — **33.** $320.

Page 372. — **39.** 56 sq. in. **40.** 91 sq. in. **41.** 60 sq. in. **46.** 18 sq. in.; 450 sq. in.; 3⅛ sq. ft. **48.** 45 sq. in. **49.** 187 sq. in. **50.** 1 sq. ft. 81 sq. in.

Page 373. — **51.** 50 sq. in.; 25 sq. in.; 25 sq. in. **52.** 288 sq. cm.; 144 sq. cm. **54.** 21 sq. in. **56.** 5 in.

Page 374. — **57.** 2400 sq. rd.; 15 A. **58.** 35. **59.** 27. **60.** 300. **61.** 1200. **62.** 97½. **63.** 6. **64.** 10. **65.** 40. **66.** 30. **67.** 30. **68.** 2025 sq. in. **69.** 294 sq. in. **70.** 6 in. **71.** 40 in. **72.** 3 in. **73.** 72 sq. in.; 24 in. **74.** 12 in.

Page 375. — **75.** 36 in.; 540 sq. in. **76.** 15 in.; 120 sq. in. **77.** 12 in.; 15 in.; 48 in. **78.** Per. 90 in. **79.** 80 in.; 58 in. **80.** 8.6+ in.; 43+ sq. in. **81.** 6.9+ in.; 27.6+ sq. in. **82.** 173.2+ sq. in. **83.** 370.8+ sq. in.

Page 376. — **88.** Pen. 108.8+sq. in. **90.** 435.2+sq. in. **91.** 8.16+in.; 244.8+ sq. in. **92.** 489.6+ sq. in.

Page 377. — **93.** 333.2+ sq. in. **94.** 508.2+ sq. in. **95.** 940.8+ sq. in. **96.** 1499.4+ sq. in. **97.** 706.58+ sq. in. **98.** 662.4+ sq. ft. **99.** $9600. **100.** 43¢. **101.** 93.4+ sq. in. **102.** 4.8+ sq. in. **103.** 20.4+ sq. in. **104.** 64 sq. in.; 28 in.; 11.5+ sq. in.

Page 378. — **105.** 30 in. **106.** 3,748,500 sq. in. **107.** 580.8+ sq. in. **108.** 104184.5+ sq. in. **109.** 28 sq. in. **110.** 96.3 A.

Page 379. — **115.** 5 in.; 31⅜ in. **116.** 50⅛ in.

Page 380. — **121.** 154 sq. in. **122.** 314⅔ sq. ft. **123.** 962½ sq. ft. **124.** 254⅘ sq. in. **125.** 471⅝ sq. ft. **126.** 9⅝ sq. ft. **127.** $2\frac{23}{168}$ sq. ft. **129.** 9⅝ sq. ft. **130.** $10\frac{5}{192}$ sq. ft. **131.** 1257⅐ sq. ft.

Page 381. — **132.** 125,022½ sq. ft. **133.** 9.8+in.; 98 sq. in.; 14 sq. in.; 20.8+ in. **134.** 201.1+ sq. in.; 165.6+ sq. in.; 5.9+ sq. in. **135.** 65¾ sq. in. **136.** Area of ring 414⅔ sq. in. **137.** 75⅜ sq. in. **138.** 113⅐ sq. in.

Page 382. — **140.** 25½ ft.; 14⅔ ft.; 3⅜ sq. ft. **141.** 254⅘ sq. in.; 9⅝ sq. in.; 452⅔ sq. in.; 38½ sq. in.; $2\frac{13}{32}$ sq. in. **142.** 2772 sq. in. **143.** 68¾ sq. ft. **144.** 545½ sq. ft. **145.** $65\frac{5}{6}$ sq. in. **146.** 196 sq. in.

Page 383. — **147.** 42 sq. in. **148.** 154 sq. in.; 616 sq. in.; 154 sq. in. **151.** 448 sq. in.; 640. **152.** 1296 cu. in.

Page 384. — **154.** 1280 bu. **155.** $129.60. **157.** 48 in.; 96 sq. in.; 109.6+ sq. in.; 54.4+ cu. in. **158.** 135 sq. in.; 156.5+ sq. in.; 96.75+ cu. in. **159.** 111.6+ sq. in.; 68.85+ cu. in. **160.** 15.3+ sq. in.; 150.6+ sq. in.; 122.4+ cu. in.; 70 in.

Page 385.—161. 171+ sq. in.; 157.5+ cu. in. **162.** 84 in.; 140.4+ sq. in.; 102+ cu. in. **163.** 232− cu. in. **164.** 5.6+ in.; 5.6+ in.; 2.4− in.; 2.4− in.; 2.4− in. **165.** 2.88− sq. in.; 52.48+ sq. in. **166.** 211.8+ sq. in. **167.** 2120.6− cu. in.; 10167.3+ cu. in.

Page 386.—169. 1012 sq. in.; 1232 cu. in.; 2464 cu. in. **170.** 748 sq. in.; 1540 cu. in. **172.** 88 cu. ft.; 658$\frac{4}{7}$ gal. **173.** 3620$\frac{4}{7}$ sq. in.

Page 387.—174. 8$\frac{1}{8}$ gal. **175.** 924 cu. in. **176.** 1078 cu. in. **178.** 43+ sq. in.; 12 in.; 180 sq. in.; 223+ sq. in.

Page 388.—179. 5 in.; 13 in,; 260 sq. in. **180.** 15 in.; 192 in.; 756 sq. in.; **181.** 29 in.; 27$\frac{2}{3}$ sq. ft. **182.** 10 sq. ft.; 2 ft. **183.** 15 in.; 720 sq. in.; 198 in. **184.** 400 cu. in.; 11,200 cu. in.; 3200 cu. in. **185.** 193.2+ cu. ft. **186.** 2160 cu. in.

Page 389.—187. 204.+ cu. in. **188.** 576.+ cu. in. **189.** 306+ cu. in. **190.** 918+ cu. in. **191.** 9 in.; 9 in.; 12 in.; 831.6 cu. in. **192.** 2. **193.** $120. **194.** 93,391,360 cu. ft.

Page 390.—196. 22 sq. in.; 11 in.; 3$\frac{1}{2}$ in.; 9$\frac{5}{8}$ sq. in. **198.** 6$\frac{5}{12}$ sq. ft. 8$\frac{59}{396}$ sq. ft. **200.** 11$\frac{1}{4}$ sq. in. **201.** $45.26. **202.** 39 sq. ft.; 120$\frac{1}{2}$ sq. in. **203.** 25 cm.; 704 sq. cm.

Page 391.—204. 65$\frac{10}{21}$ cu. in. **205.** 2514$\frac{2}{7}$ cu. in. **206.** 648$\frac{3}{4}$ cu. in. **207.** 1$\frac{271}{576}$ cu. ft. **208.** 314$\frac{2}{7}$ cu. in. **209.** 201$\frac{1}{7}$ cu. in. **210.** 50$\frac{2}{7}$ sq. in.

Page 392.—216. 189 sq. in. **217.** 96 sq. in. **218.** 684 sq. in. **219.** 917 sq. in. **220.** 165 sq. in. **221.** 8 sq. ft.

Page 393.—222. 500 sq. in. **223.** 70 sq. in. **224.** 260 sq. in. **225.** 6 in. **226.** 10 in. **227.** 34$\frac{37}{672}$ sq. ft. **228.** 38$\frac{1}{2}$ sq. in. **229.** 154 sq. in. **230.** 9$\frac{5}{8}$ sq. ft.; 6 sq. ft. 98$\frac{1}{2}$ sq. in. **231.** 804$\frac{4}{7}$ sq. in. **232.** 94 ℘.

Page 394.—233. 7700 sq. ft. **234.** 201,142,857$\frac{1}{7}$ sq. mi. **237.** 5 in.; 8 in.; 10 in.

Page 395.—238. 15 sq. in.; 60 sq. in.; $\frac{1}{4}$; 48 cu. in.; 384 cu. in.; $\frac{1}{8}$. **239.** $\frac{1}{9}$; 24 in. **240.** 12 in.; 9 in. **241.** 504 sq. in.; 31$\frac{1}{2}$ sq. in.; 16. **242.** 24 gal. **243.** 12$\frac{3}{8}$ sq. ft. **244.** 250 cu. in. **245.** 56 lb. **246.** $6.40; 41 cu. in.

Page 396.—247. 16. **248.** $\frac{1}{8}$. **249.** 160 bu. **250.** $3000. **251.** 64 sq. ft.; 240 gal. **252.** $\frac{3}{8}$ cu. ft. **253.** $\frac{1}{10}$. **254.** $\frac{1}{100}$; $\frac{1}{1000}$. **255.** 9.

Page 397.—256. 125. **257.** 24 T. **258.** 64; 4. **260.** 90°; 270°. **261.** 120°; 75°; 165°. **262.** 72°; 288°. **263.** 60°. **264.** 45°; 36°; 30°; 51$\frac{3}{7}$°; 40°. **265.** 45°; 30°. **266.** 120°. **267.** 4 in.; 6 in.; 2 in.; 10 in.; 8 in.; 3 in.; 5 in.; 7$\frac{1}{2}$ in.

Page 398.—268. 6$\frac{1}{3}$ ft.; 13$\frac{1}{3}$ ft. **269.** 2 ft. 6$\frac{1}{4}$ in.; 3 ft. 8 in. **270.** 8 ft.; 6$\frac{2}{3}$ ft. **271.** $\frac{1}{21600}$; $\frac{1}{1296000}$. **272.** 1530′. **273.** 270′; 12′; 355′. **274.** 2125″; 29,420″; 324,925″. **275.** 180,930″; 3015$\frac{1}{2}$′; 50$\frac{31}{120}$°. **276.** 7$\frac{31}{180}$°; 430$\frac{1}{3}$′; 25,820″. **277.** 8$\frac{1}{360}$°; 480$\frac{1}{6}$′; 28,810″. **278.** $\frac{3}{40}$°; 4$\frac{1}{2}$′; 270″. **279.** 7$\frac{31}{300}$°; 440$\frac{2}{3}$′; 26,440″. **280.** 15$\frac{61}{240}$°; 915$\frac{1}{4}$′; 54,915″. **281.** $\frac{7}{45}$°; 9$\frac{1}{3}$′; 560″. **282.** 10$\frac{1}{80}$°; 600$\frac{3}{4}$′; 36,045″. **283.** 8$\frac{61}{300}$°; 486$\frac{1}{10}$′; 29,166″. **284.** $\frac{29}{36}$°; 45′; 290″. **285.** 12$\frac{61}{300}$°; 732$\frac{1}{5}$′; 43,932″. **286.** 11° 53′ 4″; 8° 38′ 45″; 15° 54′ 1″.

Page 399.—287. 27° 11′; 21° 37′ 5″. **288.** 14° 17′ 17″. **289.** 26° 45′ 12″. **290.** 26° 1′ 20″. **291.** 4° 59′ 9″. **292.** 16° 56′ 49″. **293.** 67° 53′ 42″. **294.** 330° 19′ 20″. **295.** 146° 35′ 51″. **296.** 258° 15′ 9″. **297.** 1° 14′ 27″. **298.** 1° 46′ 58$\frac{10}{11}$″. **299.** 33′ 18$\frac{2}{3}$″. **300.** 28′ 34$\frac{13}{15}$″. **301.** 294° 19′ 33″. **302.** 257° 8′ 34$\frac{2}{7}$″.

Page 400.—306. 80°; $\frac{1}{2}$°; 177°; 90°. **307.** 105°. **308.** 20° less; 0; 80° greater; 160° greater; 80° less; 30° less. **309.** 60°. **310.** 120°.

311. $\frac{11}{2}$; $\frac{11}{18}$; $\frac{11}{9}$. **312.** 120°. **313.** 135°. **314.** 60°; 120°; 180°. **315.** 45°.

Page 401. — **319.** 60°; 120°; 180°. **320.** 40°; 50°; 18°; 72°; 112°; 162°. **321.** $22\frac{1}{2}$°; $67\frac{1}{2}$°; 90°. **322.** 30°; 40°; 50°; 60°. **323.** 34° 7′ 36″. **324.** 74° 44′ 40″.

Page 402. — **325.** 149° 16′ 40″. **326.** 149° 8′. **327.** 117° 57′. **331.** 90°; 360°.

Page 403. — **333.** 360°. **334.** 132° 15′. **335.** 45°. **336.** *a* 90°, *b* 30°, *c* 60°, *d*, 180°. **337.** 140°. **338.** $45\frac{5}{7}$ in.; $117\frac{3}{7}$ sq. in. **339.** 75 in.; 275 sq. in. **340.** $48\frac{4}{7}$ in.; $113\frac{1}{7}$ sq. in. **341.** 34.65 sq. ft.; 26.8 ft.

Page 404. — **345.** 70°. **346.** 22°. **347.** 48°. **348.** 90°.

Page 405. — **349.** 52°. **350.** 70°. **354.** 45°, 45°, 90°. **355.** $25\frac{5}{7}$°; $77\frac{1}{7}$°; $77\frac{1}{7}$°. **356.** 70°; 70°; 40°.

Page 406. — **359.** A 30°; B 75°; C 75°. **360.** ACB 50°; BDC 70°; BCD 20°. **361.** ABE 80°; BAE 10°; AEB 90°. **362.** A 90°; B 80°; C 10°. **363.** 90°; 180°. **364.** 45°. **365.** 150°. **366.** $4166\frac{2}{3}$ mi.; 12,500 mi.; $8333\frac{1}{3}$ mi. **367.** $3472\frac{2}{3}$ mi.; $4861\frac{1}{9}$ mi. **368.** 250 mi.

Page 407. — **369.** 25 mi. **370.** $1388\frac{8}{9}$ mi. **371.** $28\frac{4}{5}$°. **372.** 18°. **373.** $7\frac{1}{5}$°. **374.** 20°; $66\frac{2}{3}$ mi. **375.** 35° 29′ 43″. **376.** 33° 32′ 28″. **377.** 90° 40′ 45″. **378.** 2° 5′ 3″. **379.** 9° 59′ 2″. **380.** 76° 32′ 23″. **381.** 26′ 47″. **382.** 1250 mi.

Page 408. — **383.** 15°. **384.** $1041\frac{2}{3}$ mi. **385.** 1 hr.; 7 A.M. **387.** 30° E.; 45° W.; $82\frac{1}{2}$° W.; $67\frac{1}{2}$° E. **388.** 1 P.M.; 11 A.M.; 4 P.M.; 9.20 A.M. **389.** 12 N't; 12 N't; 10 P.M.; 2 A.M. **390.** 15′. **391.** 15″.

Page 409. — **393.** 37° 30′; Roseville. **394.** 25° 7′ 30″. **395.** 15°. **396.** 60° 3′ 45″. **397.** 22° 23′ 45″. **398.** 37° 30′. **399.** 50° 5′. **400.** 85°. **401.** 86° 20′. **402.** 100° 15′ 18″ W. L. **403.** 53° W. L.

Page 410. — **404.** 102° 32′ W. L. **405.** 18° 33′ 50″ W. L. **406.** 109° 36″ W. L. **407.** 104° 19′ W. L. **408.** 56° 20′ W. L.; 51° 45′ W. L. **409.** 41° 12′ 30″ W. L. **410.** 22° 20′ E. L. **411.** 37° 28′ E. L. **412.** 32° 32′ W. L. **413.** 29° 42′ W. L. **414.** 32° 2′ 48″ W. L. **415.** 3° 33′ 50″ W. L. **417.** 2 hr. 1 min. 3 sec. **418.** 3 hr. 39 min. 14 sec. P.M. **419.** 1 hr. 42 min. $43\frac{11}{15}$ sec. **420.** 1 hr. 6 min. $12\frac{2}{3}$ sec.

Page 411. — **421.** 52 min. $8\frac{4}{5}$ sec. **422.** 1 hr. 3 min. $36\frac{14}{15}$ sec. **423.** 5 hr. 5 min. $22\frac{2}{5}$ sec. **424.** 6 hr. 45 min. $27\frac{1}{3}$ sec. **425.** 5 hr. 58 min. $3\frac{1}{2}$ sec. **426.** 6 hr. 52 min. 48 sec. **427.** 9 hr. 40 min. 32 sec. P.M. **429.** 1 hr.; 3 hr.; 2 hr. **431.** West; 97° 30′ W. L.

Page 412. — **1.** $\frac{1}{6}$; $\frac{1}{3}$; $\frac{1}{2}$; $\frac{1}{5}$; $\frac{1}{4}$. **3.** $1\frac{2}{3}\frac{3}{3}\frac{1}{7}\frac{3}{7}$%. **4.** 2. **5.** 13. **6.** 278. **7.** 3633 yd.; $660\frac{6}{11}$ rd.; $2\frac{11}{1760}\frac{5}{}$ mi. **8.** $924. **9.** $180.18.

Page 413. — **25.** 1 g.; 1 Kg. **26.** 9 Kl. **27.** 2.7 m. **28.** 8,000,000 cu. cm. **29.** $180. **30.** 9999 Km. from N. pole; 10,001 Km. from S. pole. **31.** 1 cm.; 1 m. **33.** $24,300; $2160. **35.** $120. **36.** $86\frac{2}{3}$ sq. in.

Page 414. — **40.** (1, 4, 9); (1, 4, 16); (1, 9, 16); (4, 9, 16); (1, 4, 25); (1, 9, 25); (4, 9, 25); (1, 16, 25). **41.** (1, 8); (8, 27); (1, 64); (8, 125). **42.** $82\frac{2}{5}$%; $121\frac{87}{103}$%. **43.** 140 lb. **44.** 300%. **45.** 144°; 216°; 45°. **46.** 136 in. **47.** 6000. **48.** 1440 rd.; 960 rd.; 1920 rd. **49.** 42,870 cu. yd. 10 cu. ft.

Page 415. — **51.** 16,971$\frac{3}{4}$ gal. **52.** 5 hr. **53.** $800. **54.** $9.60. **55.** 20 hr. **56.** $10. **57.** 80. **58.** $162. **59.** Man $210; Wife $60. **60.** A $6000; B $8000; C $10,000. **61.** 60 rd.

Page 416. — **65.** $6\frac{1}{2}$ in.; 1080 sq. in.; 1080 cu. in. **66.** $3\frac{1}{2}$ hr. **67.** $2651\frac{11}{14}$ lb. **68.** $1005\frac{5}{7}$ cu. in. **69.** $451\frac{11}{84}$ lb.

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